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The gamma-ray background: a consequence of metagalactic cosmic ray origin?

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Abstract. An estimate is made of the γ-ray spectrum resulting from an extragalactic, evolving source model of cosmic ray origin in the absence of intergalactic gas. The experimental situation does not at present appear to allow definite rejection of this theory, since the calculated spectrum lies between the extreme observational limits. It is possible that such a theory can explain the cosmic ray spectrum, the isotropy of cosmic rays and the diffuse γ-ray spectrum.

1. Introduction

There is at present no generally agreed theory for the origin of high energy cosmic rays, despite considerable advances in knowledge about energetic galactic and extragalactic objects. It is not even clear whether the important sources of cosmic rays are inside or outside the galaxy, and conflicting views are held by a number of authors such as: Ginzburg and Syrovatskii (1964) (galactic origin) and Brecher and Burbidge (1972) (extragalactic origin).

In view of this uncertainty we have re-examined a suggestion by Hillas (1968) that the whole primary spectrum can be explained by metagalactic origin in an expanding universe with source evolution. The basic assumptions of the model are that the primary particles are predominantly protons, that their production spectrum is described by a power law with constant index, and that the total output from all sources varies with redshift according to a power law. The present cosmic ray spectrum is considered a result of interactions with the relict radiation in the past history of the universe back to some starting time, and this radiation is assumed to have a Planck spectrum with $T = 2{\cdot}7(1+z)$ K.

A prominent feature of the cosmic ray spectrum, the sharp change in spectral index at 3×10^{15} eV, is explained by assuming a sudden switching on of cosmic ray sources at some value of redshift, $z_{\rm m}$. The value of $z_{\rm m}$ has to be chosen so that the energy losses due to pair creation on the relict radiation can produce the 'kink' in the right position at 3×10^{15} eV. The model is attractive in that it leads to the rapid change of slope in a reasonably convincing fashion: the more usual explanation in terms of lack of galactic containment runs into severe problems when the containment model is examined quantitatively (Bell *et al* 1972) and required to fit the primary spectrum. An extragalactic origin would also explain the high degree of isotropy of primaries which is maintained up to at least 10^{19} eV (Karakula *et al* 1972).

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The relatively accurate data about the primary spectrum allow a good estimate of the model parameters (for any particular cosmological model assumed). The aim of the present paper is to derive these parameters by a simple analytical method and to estimate the diffuse gamma-ray spectrum to be expected in this model. A brief account of our calculation and conclusions has been presented in an earlier paper (Strong *et al* 1973). Since there appears to be no firm evidence for the existence of intergalactic matter, we have assumed the intergalactic gas density to be essentially zero for the purpose of this calculation. Also, there is no way of knowing the proton/electron ratio in the sources and we have neglected the electron component (the significance of these assumptions will be seen later). The only additional component required in our treatment is the starlight background, which can be estimated with sufficient accuracy for our purpose by considering the contribution from known sources.

2. The model and derivation of parameters

The production spectrum is assumed to follow

$$G(E_p, z)\,dz = BE_p^{-\gamma} f(z)\,dz \tag{1}$$

with

$$f(z) \begin{cases} = H_0^{-1} \dfrac{(1+z)^\beta}{(1+z)^2(1+2q_0 z)^{1/2}} & z < z_m \\ = 0 & z > z_m \end{cases}$$

where E_p is proton energy in electron volts and q_0 is the cosmological deceleration parameter. $G(E_p, z)$ is the production spectrum per unit z defined for co-moving coordinates, in which the effect of expansion on number density (ie density proportional to $(1+z)^{+3}$) has been removed. The formula corresponds to source evolution of the type given by Longair (1966, 1970) in which the number or efficiency of sources varies with time as $(1+z)^\beta$ up to some maximum z, and is assumed to cut off at this z. The additional terms in $f(z)$ above are necessary to convert from production per unit time to production per unit z,

$$\frac{dt}{dz} = \frac{H_0^{-1}}{(1+z)^2(1+2q_0 z)^{1/2}}.$$

Throughout we have assumed $q_0 = \frac{1}{2}$ for the geometry of the model; if $q_0 = 0$ the only effect is to increase β by $\frac{1}{2}$, leaving $f(z)$ unchanged. It should be stressed that in our treatment β is a fitted parameter, to be compared later with values from source counts.

The parameters of equation (1) can be obtained from the known primary cosmic ray spectrum which can be approximated in the relevant range by:

$$j(E_p) \begin{cases} = 4\times 10^{14} E_p^{-2.6}\ \mathrm{cm^{-2}\,s^{-1}\,sr^{-1}\,eV^{-1}} & \text{for } E_p < 3\times 10^{15}\ \mathrm{eV} \quad (2a) \\ = 7.7\times 10^{23} E_p^{-3.2}\ \mathrm{cm^{-2}\,s^{-1}\,sr^{-1}\,eV^{-1}} & \text{for } E_p > 3\times 10^{15}\ \mathrm{eV} \quad (2b) \end{cases}$$

with E_p measured in electron volts. The value of γ in equation (1) can easily be obtained from (2*a*) since the spectral shape below the 'kink' is not affected by the relict radiation, so $\gamma = +2.6$.

The proton spectrum at $z = 0$ is given by the solution of the energy transfer equation and is

$$j(E_p) = \int_0^{z_m} G(E(E_p, z), z) \frac{\partial E}{\partial E_p}\bigg|_z \, dz \tag{3}$$

where $E(E_p, z)$ is the energy which a proton of energy E_p (now) had at redshift z, and is the solution of

$$\frac{dE(E, z)}{dz}\bigg|_{tot} = \frac{dE(E(1+z), 0)}{dz}\bigg|_{pp-pmp} \times (1+z)^3 + \frac{E}{(1+z)}(\text{redshift}) \tag{4}$$

with boundary condition $E(E_p, 0) = E_p$ (eg Blumenthal 1970, Hillas 1968); pp represents pair production and pmp photomeson production. In the region of the 'kink', just above 3×10^{15} eV, the spectrum is determined by the onset of pair production on the relict radiation at high redshift, and it is sufficient to represent the energy losses in this region by

$$\frac{dE}{dz}\bigg|_{pp} \begin{cases} = 0 & \text{for } E < \dfrac{7 \times 10^{17}\ \text{eV}}{1+z} \\ \to \infty & \text{for } E > \dfrac{7 \times 10^{17}\ \text{eV}}{1+z}. \end{cases} \tag{5}$$

This is justified since, while redshift and pair creation losses are the same order of magnitude at $z = 0$, the pair creation losses increase faster than $(1+z)^3$, so protons lose energy rapidly down to $7 \times 10^{17}/(1+z)$ eV in a small interval of z, after which only redshift losses are important.

The value of z_m can be found as follows. The lowest proton energy affected by relict radiation at z_m is $7 \times 10^{17}/(1+z_m)$ eV, as a result of the redshift between z_m and $z = 0$ this energy is reduced for observers now by a factor $(1+z_m)$, so finally

$$3 \times 10^{15} = \frac{7 \times 10^{17}}{(1+z_m)^2},$$

that is, (6)

$$z_m = 14{\cdot}3.$$

The change in slope at 3×10^{15} eV is found experimentally to be $\Delta\gamma = 3{\cdot}2 - 2{\cdot}6 = 0{\cdot}6$. We now relate this to the model parameters. Substitution of (5) in (3) leads to

$$j(E_p) = \int_0^{z_u} G(E_p(1+z), z)(1+z)\, dz \tag{7}$$

where $z_u = z_m$ for $E_p < 3 \times 10^{15}$ eV, and

$$(1+z_u)^2 = \frac{7 \times 10^{17}}{E_p} \qquad \text{for } E_p > 3 \times 10^{15}\ \text{eV}.$$

Thus using equation (1) for $G(E, z)$ we get

$$j(E_p) \begin{cases} = \dfrac{BH_0^{-1}E_p^{-\gamma}}{\beta - \gamma - \frac{1}{2}} \{(1+z_m)^{\beta-\gamma-\frac{1}{2}} - 1\} & \text{for } E < 3 \times 10^{15}\ \text{eV} \\ = \dfrac{BH_0^{-1}E_p^{-\gamma}}{\beta - \gamma - \frac{1}{2}} \left\{\left(\dfrac{7 \times 10^{17}}{E}\right)^{\frac{1}{2}(\beta-\gamma-\frac{1}{2})} - 1\right\} & \text{for } E > 3 \times 10^{15}\ \text{eV}. \end{cases} \tag{8}$$

The required $\Delta\gamma$ can thus be obtained by putting

$$\Delta\gamma = 0.6 = \tfrac{1}{2}(\beta-\gamma-\tfrac{1}{2})$$

so for $\gamma = 2.6$

$$\beta = 4.3.$$

Using equations (8) and (2) we find

$$B = \frac{H_0(\beta-\gamma-\frac{1}{2})\times 4\times 10^{14}}{(1+z_m)^{\beta-\gamma-\frac{1}{2}}-1}. \tag{9}$$

It should be stressed that the accuracy in determining the parameters of the primary spectrum is relatively high. The error in $\Delta\gamma$ is less than 0·1 and the error in determination of the position of the 'kink' does not exceed 40%. Therefore β and z_m satisfy

$$4.1 < \beta < 4.5 \qquad (q_0 = \tfrac{1}{2})$$

$$11 < z_m < 18.$$

Figure 1 shows the result of evaluating $j(E_p)$ from equation (3) numerically, using $\beta = 4.5$ and $z_m = 15$ and we also show a summary of the experimental data. As was shown by Hillas (1968) good agreement is obtained up to about 3×10^{19} eV, at which energy the cut off due to photomeson production on the relict radiation sets in.

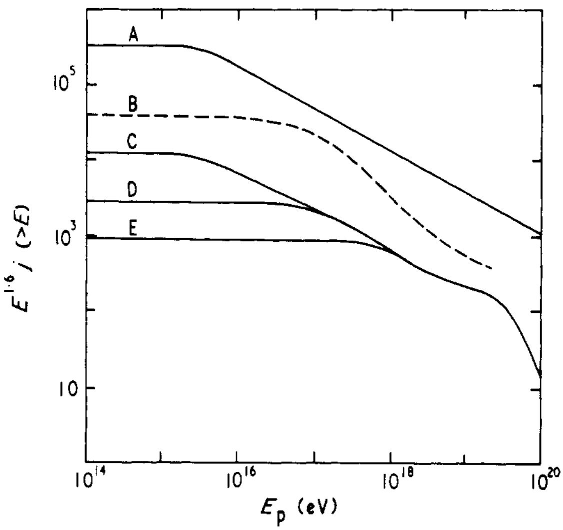

Figure 1. Primary spectra on Hillas' theory for various parameters, compared with (unnormalized) experimental summary. Curve A, experimental; B, $z_D = 10$; C, $z_m = 15$; D, $z_m = 3$ and E, $z_m = 1$. Power law evolution: $\beta = 4.5$, exponential evolution: exp $(1+z_D)z(1+z)^{-1}$.

It has been shown (Rowan-Robinson 1971) that source evolution of the form $\exp\{z(1+z_D)/(1+z)\}$ with $z_D = 10$ and no sharp cut off can represent the source count data as well as Longair's power law form. In figure 1 we show the proton spectrum to be expected in this case; it is clear that the change of slope is much more smooth and presents a much worse fit to the experimental data than the power law form with sharp cut off.

3. Comparison of evolution parameters with source count results

The variation in source efficiency with z was introduced by analogy with results from radio source counts (eg Longair 1966, 1970). Values of β obtained in this way are in rough agreement with the value obtained above, eg Longair (1966) found $\beta = 5{\cdot}7$ (density evolution) or 3·3 (luminosity evolution) compared with our value $4{\cdot}3 \pm 0{\cdot}2$. The value of z_m on the other hand is in clear disagreement since Longair found $2{\cdot}3 < z_m < 4$.

Evaluating the coefficient B in the production spectrum using equation (9), we find that in the case of evolutionary effects with $\beta = 4{\cdot}3$ the required source efficiency now is less by a factor of about 40 than that without evolution (ie $\beta = 0$). It is well known that the total efficiency of radiogalaxies as sources of cosmic rays (derived from radio observations), eg Longair (1970), is at least three orders of magnitude lower than that needed under the assumption that all cosmic rays are extragalactic in origin. This estimate however is based on the assumption that the proton/electron ratio is the same as observed in the solar system. This is not really justified, since the observed electrons are known to be galactic in origin, while protons on the present hypotheses are extragalactic, so the ratio of the two types of primary need have no resemblance to their ratio in sources.

4. Implications of the present model

All universal theories of cosmic rays, and the present one is no exception, have some difficulties. The main problems are the lack of the expected cut off at about 3×10^{19} eV due to the contemporary relict radiation, and the probability of an excess flux of extragalactic x rays and γ rays. The lack of a black-body cut off above 3×10^{19} eV would appear to support a local origin at high energies, but the situation is not clear since detailed analysis of the arrival directions of cosmic rays by a number of workers (eg Karakula *et al* 1972) show that primaries of energies a few times 10^{18} eV present almost perfect isotropy.

At present the number of air shower events above about 3×10^{19} eV is very small (~ 30) and their energy estimates are subject to rather large errors so that the presence of a cut off in this model is perhaps not a conclusive objection to the model at the present time.

On the basis of assumptions about the density of intergalactic matter and the electron to proton ratio, Longair (1970) and Stecker and Silk (1969) have demonstrated that the experimental intensities of the x-ray background and diffuse γ rays are lower than those expected if all primaries are extragalactic.

The discrepancies can be removed, however, by assuming that the density of extragalactic gas is low enough and that the proton/electron ratio is high.

The situation with regard to extragalactic matter appears to be as follows. Considering all the (detected) matter in galaxies and averaging over the Hubble radius, the mean universal density is of the order of 10^{-7} atoms/cm^3. If the mean interstellar gas density in a galaxy is about 2 % of the mean matter density this means that the effective universal gas density would be of the order of 2×10^{-9} atoms/cm^3. In fact if a large fraction of this gas is in clouds having such magnetic fields that the cosmic ray protons responsible for producing γ rays of the energy of interest here (protons of a few GeV) cannot penetrate them then the effective gas density could be much less than 2×10^{-9} atoms/cm^3. It can be shown (appendix 3) in the evolving universe model of § 2 that the density required (present epoch) in order that universal proton interactions should give rise to the measured diffuse γ background is of the order of 3×10^{-9} atoms/cm^3. There is thus no obvious inconsistency in taking a mean universal gas density low enough for γ production in p–p collisions to be negligible. Such a reduced density would also make negligible the contribution from bremsstrahlung to the x-ray diffuse background (Stecker and Silk 1969 have shown that bremsstrahlung from extragalactic electrons is a strong contender for explanation of the x-ray background).

Concerning the proton/electron ratio, Longair (1970) has shown that a value of 10^4–10^5 is required in the sources if the intensity of diffuse x rays from inverse Compton interactions of electrons on black-body photons is not to be greater than that observed. It is not impossible that the ratio is above this limit.

Another way out would be to assume that only cosmic rays above say 10^{14} eV are of extragalactic origin (eg Beresinsky and Zatsepin 1970). In this case both of the problems referred to above are removed since the energy flux of primaries above 10^{14} eV is about three orders of magnitude less than the total flux. In this case the problem of joining the galactic and extragalactic parts of the spectrum would arise. Some experimental observations indeed show features which could be interpreted as such a point of joining, although the data can also be interpreted in other ways.

As pointed out in § 1, it is still possible to obtain a test of the theory of extragalactic origin of cosmic rays from γ-ray observations even when ρ_{IG} is assumed to be zero and the electron component in the sources is assumed negligible. The following sections are concerned with estimates of the expected γ-ray flux under these conditions.

5. γ-ray flux expected in present model

5.1. The interaction process

As has been discussed in § 2 the difference between the production spectrum out to z_m and the one actually observed is due to energy losses from electromagnetic interactions of protons with the relict radiation. The whole of this energy difference must reappear in the form of an electromagnetic component. For the primary spectrum given by equation (2) the energy flux expected in the form of photons is $1{\cdot}7\times10^5$ eV cm^{-2} s^{-1} sr^{-1} (see § 5.2.4). This value is obtained from the observed energy spectrum of cosmic rays alone, so it is not sensitive to the details of the model.

The main problem is the determination of the form of the energy spectrum of these γ rays. The spectrum depends on the presence of starlight in metagalactic space, the role of starlight being to convert photons into electrons via electron pair creation in γ–γ collisions. The electrons will lose their energy via the inverse Compton effect (ICE) on the relict radiation (synchroton losses for electrons being unimportant compared with ICE losses provided the metagalactic magnetic field is less than 10^{-7} G for the energies of interest).

The question of the present day starlight spectrum is discussed in appendix 1. However, since we are interested in processes occurring at large z, for which nothing is known about the starlight spectrum, it is clearly necessary to make some assumptions about how the starlight spectrum varies with z. We have approached the problem by treating two extreme cases:

(A) Starlight density is assumed sufficiently large at all z (up to z_m) such that the

γ_{bb}–γ_s pair production/ICE cycling reduces all the photons to energies below the pair-production threshold in a small interval of z. The γ spectrum is obtained for each z, redshifted to $z = 0$ and the results from all z are summed.

(B) Starlight density is assumed sufficient only to produce interactions on a scale of order of cH_0^{-1}; most γ_{bb}–γ_s interactions will then occur at small z ($z \sim 0$ to 3). This model applies particularly to the case where the optical galaxies were formed at small z, which is reasonable on general astrophysical grounds.

Figure 2 sketches the essential processes involved in treatments (A) and (B).

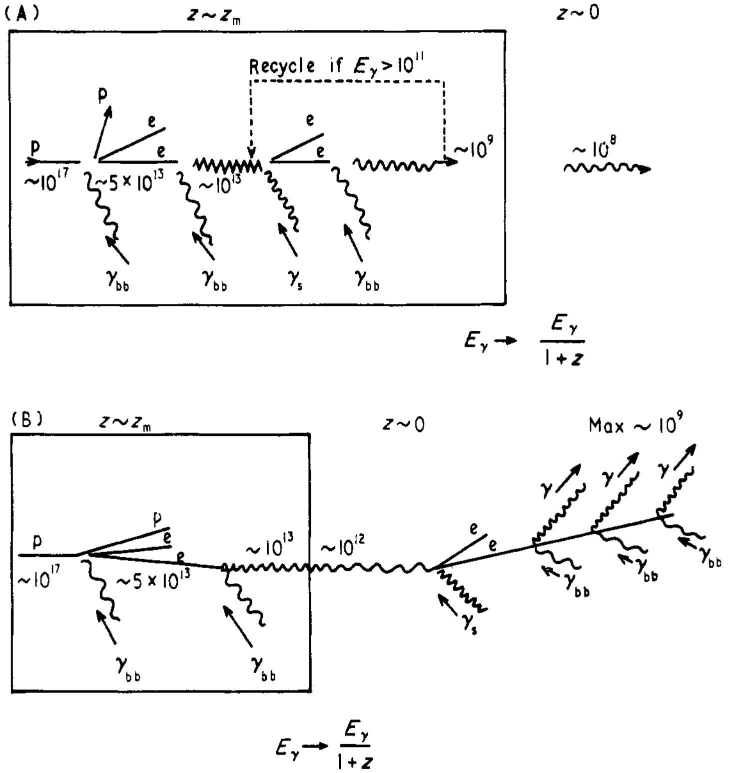

Figure 2. Summary of treatments (A) and (B) for the γ spectrum.

5.2. *Treatment* (*A*)

5.2.1. Description of simplifying assumptions. The final γ spectrum contains contributions from all z up to z_m and in principle it is necessary to solve for the electron–photon cascades at all z in an expanding universe. In these calculations however we have assumed both the initial $\gamma p \rightarrow e^+e^-p$ reaction and the shower resulting from this to be 'point processes' in z, and finally redshifted the spectra from each z to $z = 0$.

This procedure is fully justified for the (γ, p) reaction, since above the threshold the *attenuation* length falls rapidly to $5 \times 10^{27}/(1+z)^3$ cm and the proton energy satisfies (for $q_0 = \frac{1}{2}$),

$$\frac{1}{E}\frac{dE}{dz} \sim \frac{cH_0^{-1}(1+z)^3}{5 \times 10^{27}(1+z)^{2.5}} \sim 2(1+z)^{1/2}.$$

The proton thus loses most of its energy in a small z interval. The attenuation length for electrons on the microwave background is always much less than cH_0^{-1} for the energy range of interest, and so ICE can be treated as a point process in z. This can be demonstrated as follows. Photons with energy E_γ at $z = 0$ are produced by electrons of energy $E_e = (\frac{3}{4}E_\gamma m_e^2/\epsilon_0)^{1/2}$ where $\epsilon_0 = 2{\cdot}7kT$ and this result is independent of the value of z at which the electrons were produced. The interaction length for electrons $\Lambda_i(z) \sim 6\times 10^{21}/(1+z)^3$ cm and

$$v(E_e, z) = \frac{E_\gamma}{E_e}(z) = 3{\cdot}2\times 10^{-15}(1+z)E_e$$

so that electrons lose most of their energy to γ's in Δz given by

$$\Delta z = \left(\frac{1}{E}\frac{\mathrm{d}E}{\mathrm{d}z}\right)^{-1} = \frac{\Lambda_i(z)}{v(E_e, z)}\frac{(1+z)^2(1+2q_0z)^{1/2}}{cH_0^{-1}} = \frac{2\times 10^8(1+z)^{-1\cdot 5}}{E_e(\mathrm{eV})}.$$

Thus for 100 MeV photons, $E_e = 1{\cdot}7\times 10^{11}$ eV and $\Delta z = 10^{-3}(1+z)^{-1\cdot 5}$.

Since the interaction length for pair production on starlight is much greater than the attenuation length for electrons we conclude that most of the energy resides in photons during the energy degradation process.

We next note that electrons from (γ_{bb}p) reactions at given z are produced at roughly constant energy since

$$\frac{\langle E_e\rangle}{E_p} \sim \frac{E_{\mathrm{th}}}{E_p}\left(\frac{m_e}{m_p}\right)$$

where E_{th} is the threshold for pair production ($\simeq 7\times 10^{17}/(1+z)$ eV). Thus

$$\langle E_e\rangle \sim 4\times 10^{14}/(1+z).$$

We also note that the *shape* of the final γ spectrum produced by any given electron will not depend greatly on the initial electron energy, and therefore it is sufficient to take some representative initial electron energy for each z, calculate the resulting γ spectrum and then normalize to the total energy going into electrons at that z.

Another simplifying assumption we make is that all γ's above some threshold $E_{\gamma\mathrm{th}}$ will always undergo pair production on starlight, and that below $E_{\gamma\mathrm{th}}$ no further interactions occur. $E_{\gamma\mathrm{th}}$, which is a function of the starlight spectrum is assumed independent of z. Arguments given in appendix 1 suggest $E_{\gamma\mathrm{th}}$ should be of the order of 10^{11} eV, and this is the value used in our calculations. It is also clear that the value of $E_{\gamma\mathrm{th}}$ is not very sensitive to changes in the starlight spectrum (which will certainly change with z), and this helps to justify our assumption of constant $E_{\gamma\mathrm{th}}$.

5.2.2. Method of calculation for particular z. The method for calculating the result of a shower at any z is as follows. Starting from a single electron at E_{e0}, the γ spectrum formed as it loses all its energy by IC on the relict radiation is shown in appendix 2 to be

$$N_{\gamma 1}(E_\gamma) = \frac{1}{v(E_e)E_\gamma}\frac{\mathrm{d}(\lg E_e)}{\mathrm{d}(\lg E_\gamma)} \qquad \text{for } E_\gamma < v(E_{e0})E_{e0} \tag{10}$$

where $v(E_e) = \langle E_\gamma\rangle/E_e$ for IC, all evaluated at $E_e = E_\gamma/v(E_e)$.

The electron spectrum resulting from interactions with starlight is then obtained, assuming that each γ produces two electrons each of energy $\frac{1}{2}E_\gamma$ (ie that γ–γ interactions occur near the threshold CM energy). Thus

$$N_{e1}(E_e)\begin{cases} = 4N_{\gamma 1}(2E_e) & \text{if } \frac{1}{2}v(E_{e0})E_{e0} \geqslant E_e \geqslant \frac{1}{2}E_{\gamma th} \\ = 0 & \text{elsewhere.} \end{cases} \tag{11}$$

The electrons now undergo IC on the relict radiation as before, producing a γ spectrum:

$$N_{\gamma 2}(E_\gamma) = N_{\gamma 1}(E_\gamma)\int_{E_{e\,min}}^{\infty} N_{e1}(E_e)\,dE_e \tag{12}$$

where

$$E_{e\,min}\begin{cases} = \dfrac{E_\gamma}{v(E_e)} & \text{for } E_\gamma > \frac{1}{2}E_{\gamma th}v(\frac{1}{2}E_{\gamma th}) \\ = \frac{1}{2}E_{\gamma th} & \text{for } E_\gamma < \frac{1}{2}E_{\gamma th}v(\frac{1}{2}E_{\gamma th}). \end{cases}$$

The electron spectrum $N_{e2}(E_e)$ from this spectrum is then calculated as before using (11), and the process is continued until the γ spectrum is zero above $E_{\gamma th}$. The γ spectra from each generation below $E_{\gamma th}$ are then added to give the resultant γ spectrum. This spectrum can then be redshifted back to $z = 0$ according to $N^1(E_\gamma) = (1+z)N^1(E_\gamma(1+z))$ where $N(E_\gamma)$, $N^1(E_\gamma)$ are spectra before and after redshifting respectively. We write the γ spectrum at $z = 0$ resulting from one electron of $E_{e0} = 10^{15}$ eV at z as $\Delta N(E_\gamma, z)$.

5.2.3. Summation over z. The summation of contributions to the spectrum from all z requires weighting according to the energy going into pairs at each z, which depends on both source evolution and the reduction in (γ, p) threshold proton energy. Thus at each z, the energy $\Delta E(z)$ going into electron pairs per unit z is proportional to

$$(1+z)^{\beta-2\cdot 5}\int_{E_{pth}/(1+z)}^{\infty} N(E_p)\left(E_p - \frac{E_{pth}}{(1+z)}\right)dE_p.$$

For $N(E_p) \propto E_p^{-\gamma}$, $\Delta E(z) \propto (1+z)^{\beta+\gamma-2-2\cdot 5} = (1+z)^{2\cdot 4}$ putting $\gamma = 2\cdot 6$ and $\beta = 4\cdot 3$. The observed γ spectrum from all z per 10^{15} eV of energy going into electrons (this energy being evaluated at the redshift of electron production), is therefore

$$N(E_\gamma) = \frac{\int_0^{z_m}(1+z)^{2\cdot 4}\,\Delta N(E_\gamma, z)\,dz}{\int_0^{z_m}(1+z)^{2\cdot 4}\,dz}. \tag{13}$$

As explained in § 5.2.5, the contributions $\Delta N(E_\gamma, z)$ can be well approximated by

$$\Delta N(E_\gamma, z)\begin{cases} = \Delta N(E_\gamma, 0)(1+z)^{-0\cdot 93} & \text{for } E_\gamma < \dfrac{E_{\gamma th}}{1+z} \\ = 0 & \text{for } E_\gamma > \dfrac{E_{\gamma th}}{1+z}. \end{cases} \tag{14}$$

Substitution of equation (14) in (13) leads to

$$N(E_\gamma)\begin{cases} = 0\cdot 11\,\Delta N(E_\gamma, 0) & \text{for } E_\gamma < \dfrac{E_{\gamma th}}{1+z_m} = 6\cdot 5\times 10^9\text{ eV} \\ = 1\cdot 3\times 10^{-4}\,\Delta N(E_\gamma, 0)\left\{\left(\dfrac{10^{11}}{E}\right)^{2\cdot 47} - 1\right\} & \text{for } E_{\gamma th} > E_\gamma > \dfrac{E_{\gamma th}}{(1+z_m)}. \end{cases} \tag{15}$$

5.2.4. Normalization of spectrum

The spectrum (15) is finally normalized so that the energy in γ's is equal to the energy difference between the observed cosmic ray proton spectrum above 3×10^{15} eV and that expected from a continuation of the $E^{-2\cdot6}$ spectrum above this energy, ie the spectrum expected if black-body radiation is absent. Thus for a change of slope from $-2\cdot6$ to $-3\cdot2$ at 3×10^{15} eV,

$$W = \left(\frac{1}{0\cdot6}-\frac{1}{1\cdot2}\right) j(E_p = 3\times10^{15})E_p^2 = 1\cdot7\times10^5 \text{ eV cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1}$$

where W is the energy flux in γ's and $j(E)$ is given by equation (2). The normalized spectrum together with observational points is shown in figure 3.

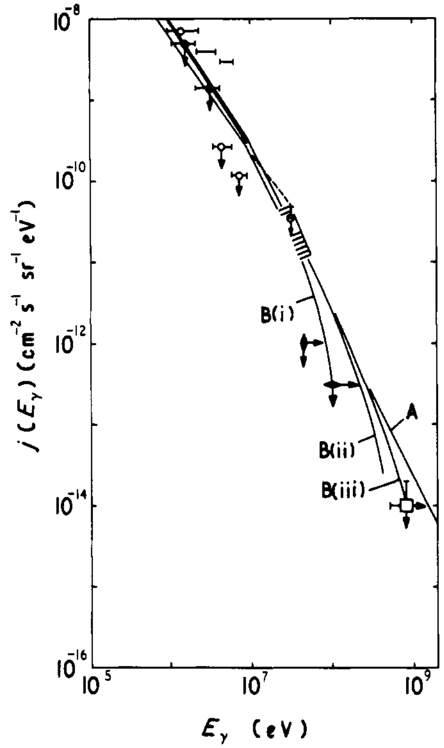

Figure 3. Gamma-ray spectrum expected from Hillas' model for treatments (A) and (B). ● OSO III (Kraushaar *et al* 1972), ◆ Cosmos 208 (Bratolubova-Tsulukidze *et al* 1970), □ Proton 2 (Bratolubova-Tsulukidze *et al* 1970), ● Cosmos 163 (Golenetskii *et al* 1971), ├─┤ ERS 18 (Vette *et al* 1970), ╥╥╥╥ Mayer-Hasselwander *et al* (1972), ○ Daniel *et al* (1972), ⊖ Share *et al* (1972), --- Apollo 15 (Trombka 1972).

5.2.5. Calculation of $N(E_\gamma, z)$. Calculations were made as described in § 5.2.2 for $E_0 = 10^{15}$ eV, $z = 0$ ($T_{bb} = 2\cdot7$ K) and $E_0 = 10^{14}$ eV, $z = 9$ ($T = 27$ K), the factor 10 in E_0 allowing for the difference in (γ, p) threshold energy at these two redshifts. In each case $E_{\gamma th}$ was taken as 10^{11} eV and three generations were sufficient to change all energy into γ's of energy less than 10^{11} eV.

It was found that in both cases $\Delta N(E, z)$ was well approximated by: (i) a power law spectrum exponent $-1\cdot5$ for $E_\gamma < \frac{1}{2}E_{\gamma th}v(\frac{1}{2}E_{\gamma th}) = 8\times10^6$ eV; (ii) a power law spectral exponent $-1{\cdot}93$ for $8\times10^6 < E_\gamma < 10^{11}/(1+z)$ eV. Examination of these spectra shows that $\Delta N(E_\gamma, z=9)$ can be obtained from $\Delta N(E_\gamma, 0)$ simply by 'redshifting' the part of the latter spectrum with exponent $-1{\cdot}93$ by a factor $(1+z)=10$ in energy, the $E_\gamma^{-3/2}$ section then being fitted where the change in exponent occurs. This procedure simply implies that the whole spectrum $\Delta N(E_\gamma, 0)$ be multiplied by $(1+z)^{-0{\cdot}93}$ and leads to equation (14) above.

5.3. Treatment (B)

In this treatment we assume that the γ's produced as a result of the first ICE interaction are redshifted to $z=0$ before possibly undergoing pair production on starlight. As before we treat the $(\mathrm{p}, \gamma_{\mathrm{bb}})$ and ICE interactions as point processes. Now equation (13) shows that the main contribution to the spectrum comes from z near z_m (a result of rapid evolution and decreasing threshold for $(\mathrm{p}, \gamma_{\mathrm{bb}})$ reactions with increasing z), and it is a good approximation to assume all the energy to be injected at z_m.

We expect protons at z_m to produce electrons of roughly

$$\frac{10^{18} m_\mathrm{e}}{(1+z_\mathrm{m})m_\mathrm{p}} \sim \frac{5\times10^{14}}{(1+z_\mathrm{m})}\,\mathrm{eV}$$

(see argument in § 5.2.1). These electrons lose energy by ICE to give a γ spectrum with maximum energy of about $1{\cdot}5\times10^{14}/(1+z_\mathrm{m})$ eV which on redshifting to $z=0$ becomes $E^1_{\gamma\max} = (1{\cdot}5\times10^{14})/(1+z_\mathrm{m})^2 \sim 7\times10^{11}$ eV. Under these conditions we expect most of the photons to undergo one pair production on starlight (since $E_{\gamma\mathrm{th}} \sim 10^{11}$ eV). After pair production and subsequent ICE on relict radiation the maximum energy becomes $E_{\gamma\max} = \frac{4}{3}(E^1_{\gamma\max}/2m_\mathrm{e})^2 kT_0 \sim 4\times10^8$ eV. In view of the uncertainties involved in this estimate, we use $E_{\gamma\max}$ as a variable parameter, calculate for different values of $E_{\gamma\max}$ and show that the conclusions do not depend critically on this parameter.

Consider first the spectrum at z_m. As shown in appendix 2, electrons of given energy produce a γ spectrum of slope $-\frac{3}{2}$ by ICE provided $\langle E_\gamma\rangle = KE_\mathrm{e}^2$. After redshifting to $z=0$, γ's with $E_\gamma > E_{\gamma\mathrm{th}}$ will undergo pair production on starlight. The resulting spectrum when these electron pairs have undergone ICE can be shown to be

$$J(E_\gamma) = f\frac{W}{2}E_{\gamma\max}^{-1/4}\frac{1-(E_\gamma/E_{\gamma\max})^{1/4}}{1-(E_{\gamma\min}/E_{\gamma\max})^{1/4}}E_\gamma^{-7/4} \qquad \text{for } E_{\gamma\min} < E_\gamma < E_{\gamma\max} \tag{16}$$

and

$$J(E_\gamma) = f\frac{W}{2}E_{\gamma\max}^{-1/4}E_{\gamma\min}^{-1/4}E_\gamma^{-3/2} \qquad \text{for } E_\gamma < E_{\gamma\min}$$

where $E_{\gamma\min}$ is the energy of photons produced by ICE on electrons of energy $\frac{1}{2}E_{\gamma\mathrm{th}}$, ie $E_{\gamma\min} = \frac{1}{2}E_{\gamma\mathrm{th}}v(\frac{1}{2}E_{\gamma\mathrm{th}}) = 8\times10^6$ eV for $E_{\gamma\mathrm{th}} = 10^{11}$ eV. W is the total energy flux expected in γ's and is identical to that defined in § 5.2.4. f is a factor between 0 and 1 which allows for the fact that in this treatment only a fraction of the γ's is processed by pair-production on starlight—those with $E_\gamma < E_\mathrm{th}$ after redshifting from z_m will not undergo pair production. It can be shown that

$$f = 1 - 53/E_{\gamma\max}^{1/4} \qquad \text{(energy in eV)}.$$

The spectra represented by (16) are shown in figure 3 for $E_{\gamma\max} = 10^8$ eV, 5×10^8 eV and 10^9 eV.

6. The experimental situation

The observations of the diffuse γ spectrum in the 1–100 MeV range are at present in a rather conflicting state. Thus Golenetskii (1971) claims that the whole region 30 keV–100 MeV can be satisfactorily fitted by a spectral index $-2\cdot4$ (differential). This follows from results of Cosmos 135 and 163 (which are lower than those of Vette *et al* (1970) in the 1–6 MeV range) and from Cosmos 208 (Bratolubova-Tsulukidze *et al* 1970) which gives a value at 50 MeV. The OSO III measurement at 100 MeV (Kraushaar *et al* 1972) is also consistent with this $E^{-2\cdot4}$ interpretation, as is a result from Proton-2 taken from Stecker *et al* (1971). A recent result from a balloon experiment by Mayer-Hasselwander *et al* (1972) gives a value in the 30–50 MeV range, a factor of about ten above the Cosmos 208 results. Pinkau (private communication) considers that a spectral index -2 in the range above 1 MeV is possible.

Another balloon experiment (Share *et al* 1972) has recently lent some support to these higher values and is in fact in good agreement with the Mayer-Hasselwander result. Further support seems to be given by results from Apollo 15 (Trombka 1972).

7. Conclusions

Figure 3 shows that for treatment (A) the Hillas model predicts γ fluxes at 50–100 MeV considerably in excess (factor of order 10) of the upper limits set by Cosmos and OSO III experiments. For treatment (B), the excess lies at lower energies, but is present for any reasonable values of the parameter $E_{\gamma\,\mathrm{max}}$. If these experimental points are correct we would appear to have clear evidence against all cosmic rays being of universal origin. If on the other hand the intensities found by the balloon experiments at 30–50 MeV, are correct, there is no conflict between our prediction and the observations. Indeed, it is tempting to suggest the present model as an explanation for the observed γ-ray spectrum in the 1–100 MeV region. In so far as problems of background make the balloon experiments a little less reliable than the satellite measurements the evidence tends to indicate that here is a conflict between the predictions of the model and observation and thus that all cosmic rays are not of universal origin. However, this conclusion could be reversed if later work were to show that the satellite measurements have underestimated the γ-ray intensity.

It is worth noting that our treatment applies equally to a 'hybrid' model of the type considered by Beresinsky and Zatsepin (1970), in which a Hillas type origin takes over from a galactic origin above some energy, say 10^{15} eV. In this case, arguments based on primary proton interactions with intergalactic matter, and of primary electrons with the relict radiation, are much less strong (since the energy available is several orders of magnitude less), while the processes considered in the present work are unaffected.

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Appendix 1. The starlight spectrum

We have used the theoretical spectra of Partridge and Peebles (1967), in which the starlight background is derived from various models of normal galaxy evolution in an expanding universe. As they suggest we adopt their model 2 for $\lambda < 1\,\mu\text{m}$ and model 4 for $\lambda > 1\,\mu\text{m}$ as shown in figure 4 together with a 6000 K grey-body spectrum of 10^{-2} eV cm^{-3} for comparison.

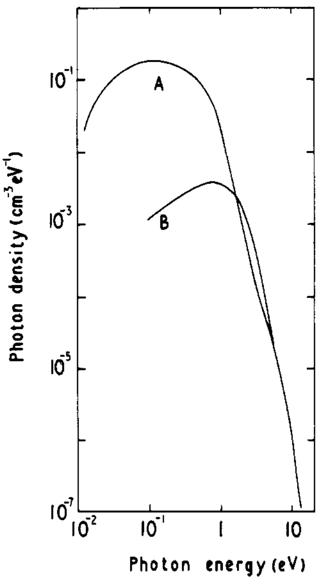

Figure 4. Extragalactic starlight spectrum at present time (curve A), from Partridge and Peebles (1967) and 6000 K grey-body spectrum (10^{-2} eV cm^{-3}) (curve B) for comparison.

The peak of the starlight spectrum occurs near 0·2 eV, compared to 0·8 eV for a Planck spectrum at 6000 K, the difference being due to the redshifting of light from distant galaxies into the infrared.

The pair-production interaction lengths λ_i for the starlight spectra are shown in figure 5, together with λ_i for 2·7 K black-body radiation. For the starlight model adopted the total interaction length does not fall below about 2×10^{26} cm for all $E_\gamma > 5 \times 10^{11}$ eV, so the energy of initial electrons will be all in particles of less than 5×10^{11} eV in distances small compared to cH_0^{-1}. We require an estimate of the photon energy $E_{\gamma\text{th}}$ for which no more interactions will occur. Since the maximum distance that a particle can travel (for $q_0 = \frac{1}{2}$) is of the order of $\frac{2}{3}cH_0^{-1}$, we take $E_{\gamma\text{th}}$ to be the energy at which

$$\lambda_i = \tfrac{2}{3}cH_0^{-1} \sim 8 \times 10^{27}\ \text{cm} \qquad (\text{for } H_0 = 75\ \text{km s}^{-1}\ \text{Mpc}^{-1}).$$

This value gives $E_{\gamma\text{th}} = 10^{11}$ eV and this is adopted in subsequent calculations. $E_{\gamma\text{th}}$ is not very sensitive to the value of λ_i used, so that even if we require λ_i smaller by a factor 10, $E_{\gamma\text{th}}$ is only displaced to $2{\cdot}5 \times 10^{11}$ eV. Also, $E_{\gamma\text{th}}$ is not very sensitive to the assumed starlight spectrum as can be seen by comparing the two starlight curves of figure 5.

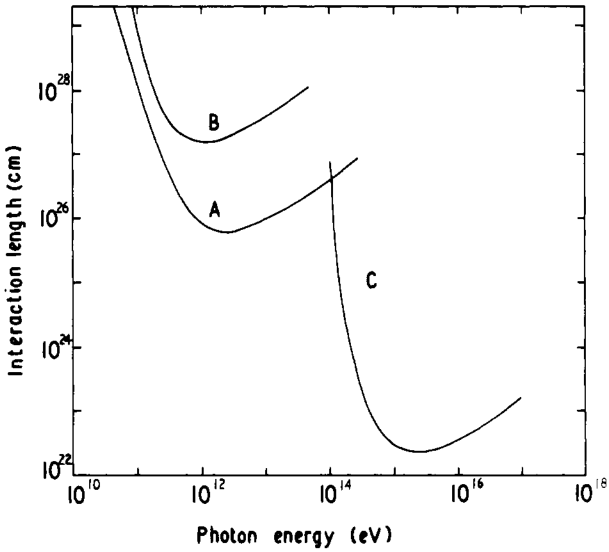

Figure 5. Pair-production interaction lengths for the two starlight fields of figure 4 (curves A and B) and for black-body radiation at 2·7 K (curve C).

Appendix 2. γ-spectrum from one electron losing energy by IC

Define $v(E_e) = \langle E_\gamma \rangle / E_e$ for inverse Compton collisions on the background radiation. Assume that for given E_e, all γ's are produced with unique energy E_γ. From energy conservation,

$$E_\gamma N_{1\gamma}(E_\gamma)\,\mathrm{d}E_\gamma = \mathrm{d}E_e,$$

that is,

$$N_{1\gamma}(E_\gamma) = \frac{1}{E_\gamma}\frac{\mathrm{d}E_e}{\mathrm{d}E_\gamma} = \frac{1}{v(E_e)E_\gamma}\frac{\mathrm{d}(\lg E_e)}{\mathrm{d}(\lg E_\gamma)} \tag{A.1}$$

where $v(E_e)$ is evaluated at $E_e = E_\gamma / v(E_e)$. We write (A.1) in this form because $\mathrm{d}(\lg E_e)/\mathrm{d}(\lg E_\gamma)$ is a slowly varying function of energy, approaching $\frac{1}{2}$ for energies $E_e < 10^{14}$ eV and $T = 2{\cdot}7$ K. In this case, $v(E_e) = KE_e$, where $K = 3{\cdot}2 \times 10^{-15}\,\mathrm{eV}^{-1}$, giving from (A.1)

$$N_{1\gamma}(E_\gamma) = \frac{E_\gamma^{-3/2}}{2\sqrt{K}}. \tag{A.2}$$

In our calculations we obtained $N_{1\gamma}(E_\gamma)$ from (A.1), the $v(E_e)$ function being derived from the appropriate differential cross section for IC on black-body radiation.

Appendix 3. γ-ray production from inelastic p–p interactions

We calculate the γ-ray flux above 100 MeV resulting from the model derived in § 2. This flux is

$$I(>100\,\mathrm{MeV}) = c\iint j(E_p, z)n\sigma(>100(1+z)|E_p)(1+z)^{-3}\,\mathrm{d}E_p\,\mathrm{d}t$$

where $j(E_p, z)$ is the proton flux in the past per unit proper volume and n is the intergalactic matter proper density. Putting $j(E_p, z) = j_0(E_p)F(z)$ where $j_0(E_p)$ is the present value, and $n = n_0(1+z)^3$, we get

$$I(>100\,\mathrm{MeV}) = \frac{n_0 c}{4\pi}\int F(z)q(E_\gamma(1+z))\,\mathrm{d}t \qquad \text{(A.3)}$$

where

$$q(E_\gamma) = 4\pi\int j_0(E_p)\sigma(>E_\gamma|E_p)\,\mathrm{d}E_p.$$

Proceeding as in § 2 we find

$$j(E_p, z) = j(E_p, 0)(1+z)^{2+\gamma}\frac{(1+z_m)^{\beta-\gamma-\frac{1}{2}}-(1+z)^{\beta-\gamma-\frac{1}{2}}}{(1+z_m)^{\beta-\gamma-\frac{1}{2}}-1}.$$

Using the form of $q(E_\gamma)$ given by Cavallo and Gould (1971) together with $\beta = 4{\cdot}3$, $z_m = 14{\cdot}3$ and $H_0 = 75\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, equation (A.3) gives

$$I(>100\,\mathrm{MeV}) = 10^4 n_0\,\mathrm{cm^{-2}\,s^{-1}\,sr^{-1}}$$

so that for $I(>100\,\mathrm{MeV}) \simeq 3\times10^{-5}\,\mathrm{cm^{-2}\,s^{-1}\,sr^{-1}}$ (Kraushaar *et al* 1972) we have

$$n_0 < 3\times10^{-9}\,\mathrm{cm^{-3}}.$$

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