CAN SLOW ROTATING NEUTRON STAR BE RADIO PULSAR?

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Abstract

It is shown that the curvature radius of magnetic field lines in the polar region of rotating magnetized neutron star can be significantly less than usual curvature radius of dipole magnetic field. Magnetic field in polar cap is distorted by toroidal electric currents flowing in the neutron star crust. These currents close up the magnetospheric currents driven by generation process of electron-positron plasma in pulsar magntosphere. Due to the curvature radius decrease electron-positron plasma generation becomes possible even for slow rotating neutron stars, $P \cdot B_{12}^{-2/3} < 10sec$. P is the period of star rotation, B_{12} is the magnitude of the magnetic field on the star surface B in the units of $10^{12}Gauss, B_{12} = B/10^{12}G$.

1 Introduction

Discovery of radio pulsar PSR J2144-3933, having the period $P = 8.51sec$ [1], was a surprise for physics community dealing with radio pulsars. Radio pulsars with such long periods have never been observed before. According to observations, pulsars lie bellow the line $P \cdot B_{12}^{-8/15} = 1sec$ on the diagram $P - B_{12}$. This line is known as death line for radio pulsars. The death line means that slow rotating neutron stars cannot be radio sources. We can interpret observation of the source with order of the period $P \approx 10sec$ as a neutron star with a super strong magnetic field $B_{12} \approx 10^2$. Such magnetic field exceeds so called critical magnetic field $B > B_\hbar = m_e^2 c^3/e\hbar = 4.4 \cdot 10^{13}Gauss$. For $B = B_\hbar$ the electron cyclotron energy $\hbar\omega_c = \hbar eB/m_e c$ becomes equal to the electron rest energy $m_e c^2$. Neutron stars with strong magnetic field $B > B_\hbar$ are called magnetars [2], but they are not sources of radio emission. The reason for that is the suppression of electron-positron generation by strong magnetic field in pulsar magnetosphere. Besides, the estimation of the value of magnetic field B for pulsar PSR J2144-3933, obtained from the value of spin down of its rotation $\dot{P}$, $B_{12} \simeq (P \cdot \dot{P}_{-15})^{1/2}$, $\dot{P}_{-15} = \dot{P}/10^{-15}sec/sec$, points out the normal value of its magnetic field, $B_{12} \simeq 2$. However, this estimation follows from the assumption that pulsar rotation energy losses are the magnetodipole losses, when the energy of a magnetized rotating neutron star radiates in the form of vacuum electromagnetic wave having a frequency equal to star rotation frequency, $\Omega = 2\pi/P$. Here it is also assumed that such unknown parameters as neutron star radius R, its momentum of inertia I and its inclination angle of magnetic dipole axis to the rotation axis χ are equal to $R = 10^6 cm, I = 10^{45} g \cdot cm^2, \chi = \pi/2$. Real values of R, I and χ can differ from these values, but the difference is not big enough to create a magnetic field with magnitude B two order times exceeding the normal value $B_{12} \simeq 1$.

The death line defined by the relation $P \cdot B_{12}^{-8/15} = 1sec$ seams to be incorrect, but reflects observation selection. Indeed, pulsar PSR J2144-3933 is the weakest among observable sources. Its small spin down luminosity $\simeq 3 \cdot 10^{28} erg/sec$ and little distance to it $\simeq 180pc$ [3,4] makes impossible observation of such sources at Galactic distances.

In radio pulsar theories the death line is defined by the conditions of electron-positron generation in the polar region of the neutron star magnetosphere. The mechanism of radio emission in the pulsar magnetosphere is not established now, but there is no doubt that radio emission is related to existence of dense plasma in pulsar magnetosphere. Dense plasma means that electron concentration n_e exceeds significantly the so called Goldreich-Julian density $n_{GJ} = \Omega B/2\pi ce$, which for a usual pulsar ($P \simeq 1sec, B_{12} \simeq 1$) near its surface is $n_{GJ} \simeq 10^{11} cm^{-3}$. Therefore, for the radio emission to be generated it is necessary to produce plasma, which under the condition $n_e \gg n_{GJ}$ have to arise in pulsar magnetosphere. The elementary act of birth of an electron-positron pair e^-, e^+ is a one-photon generation of a pair by a gamma quantum in strong magnetic field, $\gamma + B \rightarrow e^- + e^+ + B$. For such a process to take place, the transverse photon momentum (perpendicular to the magnetic field) must exceed the threshold value $2m_e c$. This means that the energetic gamma quanta with energies larger than $1MeV$ have to be generated in the magnetosphere. The source of gamma photons are electrons and positrons thierself, but they must be energetic enough to radiate gamma quanta. Thus, there must be regions in the pulsar magnetosphere with strong electric field, where electrons and positrons are accelerated to large energies. Charged particles move along magnetic field lines in strong magnetic field. They can either radiate photons if magnetic field lines have a curvature, in this case they are called curvature photons, or scatter thermal photons, radiating from a star surface, by Compton process. In both cases gamma quanta move initially almost along the trajectory of radiated particle, i.e. tangential to magnetic field lines. For the cascade generation of e^-, e^+ pairs to take place, i.e. for each primary accelerated charge particle to produce many secondary particles, radiated gamma photons must bear a large number of e^-, e^+ pairs. For that a gamma quantum has to obtain the threshold value of its transverse momentum. That is possible only if magnetic field lines have a finite value of radius of their curvature ρ_c. Then the distance travelled by the photon before the birth of e^-, e^+ pair, i.e. its free path l, will be proportional to the curvature radius of the magnetic field line $l \propto \rho_c(2m_ec^2/\varepsilon_{ph})$ (ε_{ph} is the photon energy). Magnetic field strength in the pulsar magnetosphere decreases rapidly with the distance from the star center r, $B \propto (r/R)^{-3}$. Therefore, large free paths of photons with respect to e^-, e^+ pair production do not result in effective electron-positron plasma production in the polar region of the pulsar magnetosphere. It is clear that the curvature radius ρ_c of magnetic field lines in the polar region of the pulsar magnetosphere, from which the radio emission escapes, is an important characteristic of cascade process of plasma generation. For small curvature radius plasma production becomes more effective.

The polar region of magnetosphere is the projection of the light surface, i.e. the outer boundary of the magnetosphere, where the corotation velocity of magnetospheric plasma approaches the speed of light c, to the star surface along the magnetic field lines. That is the region around the axis of the magnetic dipole with the dimension of r_0, $r_0 \simeq R(\Omega R/c)^{1/2} \ll R$. There the curvature of magnetic field, considering it as dipole, is small. On the dipole axis the curvature radius is equal to infinity, $\rho_c \to \infty$. On the edge of polar cap $r = r_0$ the curvature radius is finite, but large

$$(\rho_c)_{dip} = \frac{4}{3}R\left(\frac{\Omega R}{c}\right)^{-1/2} \gg R.$$

For a normal pulsar ($P \simeq 1sec, R \simeq 10^6 cm$) ρ_c is of the order of $10^8 cm$. This value of the curvature radius fixes the death line on the diagram $P - B_{12}$, $\quad P \cdot B_{12}^{-\alpha} < 1sec$. Here α is the power index depending on the concrete model of plasma generation. In order to expand the region of radio pulsar existence on the plane $P - B_{12}$ one needs first of all to diminish the value of curvature radius of magnetic field lines ρ_c in the polar region of the magnetosphere.

For that Arons [5] suggested that the dipole center can be displaced with respect to the star center at some distance. It is clear that approaching the dipole center to the star surface we decrease the curvature radius. In order to explain the phenomena of pulsar PSR J2144-3933 Arons shifted the dipole on the distance comparable with the star radius. However, the reason for such asymmetry for the distribution of the star matter is not clear.

Here we suggest that the magnetic field lines in the polar region are strongly distorted in more natural manner. That is for the sack of electric currents flowing in the star crust. They close the magnetospheric electric currents arising in the magnetosphere at process of electron- positron plasma generation. Due to the fact that Hall conductivity significantly exceeds Pedersen conductivity in the neutron star crust, the electric current falling onto the star surface from the magnetosphere is amplified in toroidal direction distorting the magnetic field under the surface. Therefore, the magnetic field curvature changes namely in the region of plasma generation.

2 Electric currents

While generating an electron-positron plasma in the polar region of the neutron star magnetosphere, there arises an electric current j, flowing along open field lines of the magnetic field. The open field lines are the lines of magnetic field, which are not closed inside of the magnetosphere, but cross the light surface $R_L = c/\Omega$ and escape to infinity. The projection of open magnetic field lines on the neutron star surface is the polar cap of the size of $r_0 \simeq R\,(\Omega R/c)^{1/2}$. The electric current j streams to one direction in the center of polar cap, and to opposite direction near its edge. The total current $\int j\,ds$ is equal to zero. This current realizes the spin down of a star, flowing in the crust across the magnetic field. The magnetospheric plasma at $n_e \gg n_{GJ}$ totally screens the magnetodipole radiation. At $j = 0$ the flux of electromagnetic energy does not cross the light surface. Only the electric current j generates in the pulsar magnetosphere toroidal magnetic field B_φ, which creates the radial flux of the electromagnetic energy from the magnetosphere. The value of j is of the order of the characteristic Goldriech-Julian current $j_{GJ} = cen_{GJ}$. Let us introduce a demensionless value of the electric current $i = j/j_{GJ}$. According to the paper [6] the value of i is less than unit, $i \leq 1$. On the outer edge of the magnetosphere, near the light surface, the electric current j is closed. Also it must be closed in the neutron star crust, hitting to it from the magnetosphere. Under strong magnetization of the matter of the crust $\omega_c\tau_e \gg 1$ (τ_e is the electron relaxation time), the electric conductivity of the crust is strongly anisotropic. The biggest component of the crust conductivity is its longitudinal part, along the magnetic field, $\sigma_\|$. The value of the conductivity, which is perpendicular to the magnetic field, along the electric field, $\sigma_\perp$, is $(\omega_c\tau_e)^2$ times less than the longitudinal one, $\sigma_\| = \sigma_\perp(\omega_c\tau_e)^2$. Besides, Hall conductivity $\sigma_\wedge$ exists also, which results in the electric current oriented orthoganal to both magnetic $\mathbf{B}$ and electric $\mathbf{E}$ fields, $\mathbf{j}_H = \sigma_\wedge[\mathbf{EB}]/B$. The Hall conductivity $\sigma_\wedge$ is less than the longitudinal one $\sigma_\|$, but is much larger than the perpendicular one $\sigma_\perp$, $\sigma_\wedge = \sigma_\|/(\omega_c\tau_e) = \sigma_\perp(\omega_c\tau_e)$. The electric current $\mathbf{E}$ appears in the crust when the current j inroads into it from the magnetosphere. The field is potential $\mathbf{E} = -\nabla\Psi$, because we consider all quantities in the frame rotating with the star, there everything is stationary. The electric current flowing in the crust $\mathbf{j} = -\hat{\sigma}\nabla\Psi$ has a large toroidal component $j_\varphi \simeq j(\sigma_\wedge/\sigma_\perp) \gg j$. This is due to the fact that the electric field which is orthogonal to the magnetic field induces first of all the Hall electric current. The toroidal current j_φ will be an additional source of the poloidal magnetic field. Structure of the electric currents arising in the crust is drawn on the figure 1.

Now we will discuss qualitatively how the magnetic field in polar cap is disturbed by electric currents flowing in the neutron star crust. The toroidal current $j_\varphi = j(\sigma_\wedge/\sigma_\perp) = i(\sigma_\wedge/\sigma_\perp)(B\Omega/2\pi)$ streams in the crust near the star surface and occupies the region with size of $r_0 = R\,(\Omega R/c)^{1/2}$. The poloidal magnetic field δB generated by this current is $\delta B = iB(\sigma_\wedge/\sigma_\perp)(\Omega R/c)^{3/2}$. The disturbed magnetic field in the magnetosphere under the star surface can be

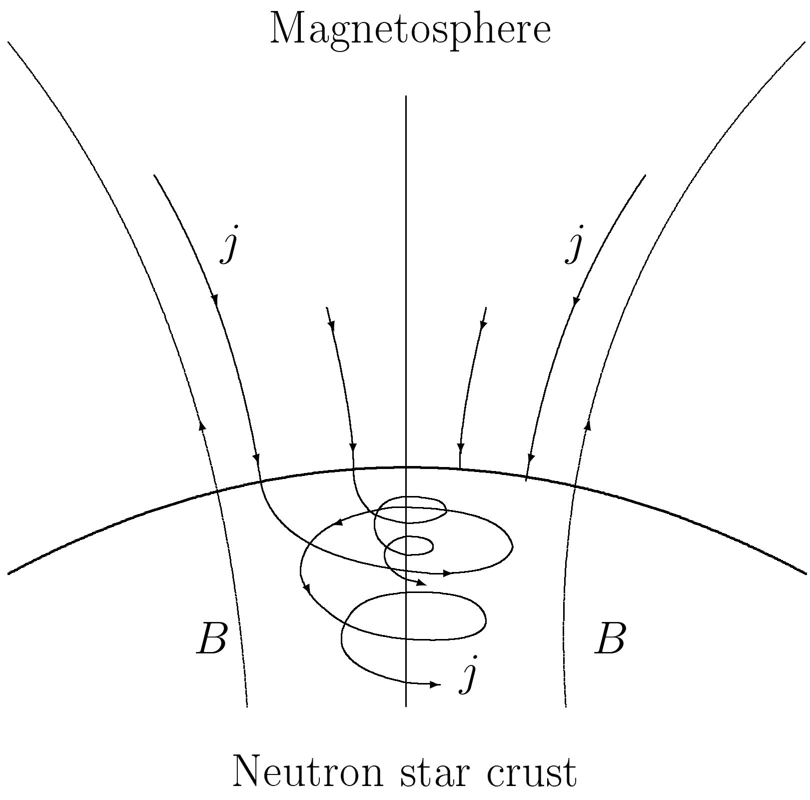

Figure 1: Electric currents in the crust

considered approximately as a field of magnetic dipole with dipole momentum

$$\delta\mu \simeq \delta B r_0^3 = i\frac{\sigma_\wedge}{\sigma_\perp}\left(\frac{\Omega R}{c}\right)^3 BR^3 = i\frac{\sigma_\wedge}{\sigma_\perp}\left(\frac{\Omega R}{c}\right)^3 \mu$$

Here μ is the magnetic momentum of the basic dipole placed in the neutron star core, $\mu = BR^3 \simeq 10^{30} Gauss \cdot cm^3$. Despite the large parameter $\sigma_\wedge/\sigma_\perp \gg 1$, the disturbed magnetic momentum is small, $\delta\mu \ll \mu$, because of small value of $\Omega R/c \simeq 10^{-4}$. The ratio $\sigma_\wedge/\sigma_\perp = \omega_c\tau_e$ is [7]

$$\omega_c\tau_e = 4\cdot 10^3 B_{12} T_6^{-1};$$

$$\omega_c = 1,8\cdot 10^{19} B_{12} sec^{-1},\ \ \tau_e = 2,2\cdot 10^{-16} T_6^{-1} sec.$$

The value T_6 is the star surface temperature in the units of $10^6 K$, $T_6 = T/10^6 K$. Thus, we can neglect the disturbed magnetic field at large distances from the polar cap, $r \gg r_0$. However, there are two factors which do not permit us to neglect the disturbed magnetic field near the star surface $r \simeq r_0$. First, the curvature of the magnetic field δB is large, $\rho_c \simeq r_0$, and second, the transverse component of the basic magnetic field in the polar region is much less than its longitudinal component. This is the reason of large curvature radius of the dipole magnetic field near its axis, $\rho_c \gg R$. The magnetic momenta μ and $\delta\mu$ have one axis, therefore vertical B_z and radial B_ρ components of the magnetic field are

$$\begin{aligned} B_z &= \mu\left(3\frac{z^2}{r^5} - \frac{1}{r^3}\right) + \delta\mu\left(3\frac{\delta z^2}{\delta r^5} - \frac{1}{\delta r^3}\right); \\ B_\rho &= 3\mu\frac{\rho z}{r^5} + 3\delta\mu\frac{\rho\delta z}{\delta r^5}. \end{aligned} \tag{1}$$

The quantity δz is the coordinate along the axis of the dipole μ, measured from the star surface, $\delta z = z - R$, ρ is the radial coordinate, it is the distance from axis z, the quantity δr is $\delta r = \left(\rho^2 + \delta z^2\right)^{1/2}$.

The electric current streaming in the crust changes a bit the vertical magnetic field B_z. From the first equation of Expression (1) it follows that $\delta B_z/B_z \simeq (\delta\mu/\mu)(R/r_0)^3 = i(\sigma_\wedge/\sigma_\perp)(r_0/R)^3 \ll 1$. The radial magnetic field may be disturbed significantly, $\delta B_\rho/B_\rho \simeq (\delta\mu/\mu)(R/r_0)^4 = i(\sigma_\wedge/\sigma_\perp)(r_0/R)^2 = i(\sigma_\wedge/\sigma_\perp)(\Omega R/c)$. So, for the estimation of the curvature of magnetic field in the polar region we can consider that the vertical component of the magnetic field is not disturbed, $B_z = \mu\left(2z^2 - \rho^2\right)/r^5 \simeq 2\mu/R^3$. The expression for the curvature radius

$$\rho_c^{-1} = \left(1 + \frac{B_\rho^2}{B_z^2}\right)^{-3/2}\left[\frac{\partial}{\partial z} + \frac{B_\rho}{B_z}\frac{\partial}{\partial\rho}\right]\left(\frac{B_\rho}{B_z}\right) \tag{2}$$

contains the magnetic field line inclination B_ρ/B_z. Substituting Expression (1) into (2) we get

$$\rho_c = \frac{4}{3}\frac{R^2}{r_0}\left[1 + 23i\frac{\sigma_\wedge}{\sigma_\perp}\frac{r_0}{R}\right]^{-1}.$$

If the value of electric current i flowing in the pulsar magnetosphere is not small, then the curvature radius of magnetic field in the polar cap may differ strongly from the curvature radius of the dipole magnetic field. Under the condition $23i(\sigma_\wedge/\sigma_\perp)(r_0/R) = 23i(\sigma_\wedge/\sigma_\perp)(\Omega R/c)^{1/2} \geq 1$ the curvature radius is defined by the electric currents only,

$$\rho_c \simeq \frac{4}{69}\frac{R^3}{r_0^2}\left(i\frac{\sigma_\wedge}{\sigma_\perp}\right)^{-1}, \tag{3}$$

and may be compatable with the neutron star radius.

3 Distortion of a dipole

Previously we gave the qualitative picture of a magnetic field line distortion by the electric currents flowing in the neutron star crust. Here we describe quantitatively the discussed phenomenon. As it was shown before the magnetic field distortion under the star surface may be large, so it is impossible to consider the field of the current in crust as a disturbance of basic magnetic dipole.

We have the following physical problem. The cylindrically symmetric electric current $\mathbf{j}_0(\mathbf{r})$ falls down from the magnetosphere ($h > 0$) onto the conducting matter ($h < 0$). The boundary between the magnetosphere and the crust is $h = 0$. As the size of the polar cap is much smaller than the star radius, $r_0 << R$, we will consider the boundary as flat. Let us introduce cylindrical coordinates h, ρ, φ. The electric currents in the magnetosphere are along the magnetic field lines, which are not known yet. Total current is equal to zero, $\int_0^{r_0} \mathbf{j}_0(\rho)\rho d\rho = 0$. Initially we have the magnetic field of the dipole configuration with its center placed on the axis $\rho = 0$ at the distance $h = -R$ from the surface. The matter of the crust in the magnetic field posses a strongly anisotropic electric conductivity ($\sigma_\|, \sigma_\wedge, \sigma_\perp$), and $\sigma_\| >> \sigma_\wedge >> \sigma_\perp$. The current in the crust is closed, i.e. flows perpendicular to the magnetic field. It means that the electric field must appear in the crust $\mathbf{E} = -\nabla\Psi$. $\Psi(h,\rho)$ is the electric potential. Due to the Hall conductivity $\sigma_\wedge$ the toroidal electric current j_ϕ must arise in the crust. This current changes the poloidal magnetic field in the crust and in the magnetosphere under the star surface.

The closed, $\nabla\mathbf{j} = 0$, cylindrically symmetric current is presented in the form

$$\mathbf{j} = [\nabla J_1 \nabla\varphi] + J_2\nabla\varphi. \tag{4}$$

Here J_1 and J_2 are the scalar functions of (h,ρ); φ is the azimuthal angle. The similar relation can be written for the magnetic field

$$\mathbf{B} = [\nabla f \nabla\varphi] + g\nabla\varphi.$$

The quantity $f(h,\rho)$ is the flux of the poloidal magnetic field, $g(h,\rho)/\rho$ is the toroidal magnetic field. From the Maxwell equation $curl\mathbf{B} = 4\pi\mathbf{j}/c$ we find the relations between the quantities J_1, J_2, f and g

$$J_2 = -\frac{c}{4\pi}\tilde{\triangle}f, \quad J_1 = \frac{c}{4\pi}g; \quad \tilde{\triangle} = \rho\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\right) + \frac{\partial^2}{\partial h^2}.$$

Introducing the unit vector $\mathbf{b}$ along the magnetic field,

$$\mathbf{b} = \rho \frac{[\nabla f \nabla \varphi] + g \nabla \varphi}{\left[(\nabla f)^2 + g^2\right]^{1/2}},$$

we present the electric current in the form

$$\mathbf{j} = -\mathbf{b}(\sigma_{\parallel} - \sigma_{\perp})(\mathbf{b}\nabla\Psi) - \sigma_{\perp}\nabla\Psi - \sigma_{\wedge}\left[\nabla\Psi\mathbf{b}\right]. \tag{5}$$

Equating the Expressions (4) and (5), we obtain equations connecting the flux of the poloidal magnetic field f and the toroidal magnetic field g with the electric potential Ψ

$$\begin{aligned} \frac{c}{4\pi}\tilde{\triangle} f &= \frac{\rho g(\sigma_{\parallel} - \sigma_{\perp})}{[(\nabla f)^2 + g^2]}\left[\nabla\Psi\nabla f\right]_{\phi} - \frac{\rho\sigma_{\wedge}}{\left[(\nabla f)^2 + g^2\right]^{1/2}}(\nabla\Psi\nabla f); \\ \frac{c}{4\pi}\left[\nabla g\nabla\varphi\right] &= -\frac{\rho(\sigma_{\parallel} - \sigma_{\perp})\left[\nabla\Psi\nabla f\right]_{\phi}}{[(\nabla f)^2 + g^2]}\left[\nabla f\nabla\varphi\right] - \sigma_{\perp}\nabla\Psi - \frac{\rho g\sigma_{\wedge}}{\left[(\nabla f)^2 + g^2\right]^{1/2}}\left[\nabla\Psi\nabla\varphi\right] \end{aligned} \tag{6}$$

Let us express the quantity $\left[\nabla\Psi\nabla f\right]_{\phi}$ from the first equation of the system (6),

$$\left[\nabla\Psi\nabla f\right]_{\phi} = \frac{\left[(\nabla f)^2 + g^2\right]}{\rho g\sigma_{\parallel}}\left\{\frac{c}{4\pi}\tilde{\triangle} f + \frac{\rho\sigma_{\wedge}}{\left[(\nabla f)^2 + g^2\right]^{1/2}}(\nabla\Psi\nabla f)\right\}.$$

The conductivity along the magnetic field is large, $r_0\sigma_{\parallel}/c >> 1$. It means that the electric field is almost parallel to the magnetic field, $\Psi = \Psi(f) + \delta\Psi, \quad \delta\Psi \propto \sigma_{\parallel}^{-1}, \quad \delta\Psi << \Psi(f)$.

Substituting the expression for $\left[\nabla\delta\Psi\nabla f\right]_{\phi}$ to the second equation of (6), we obtain the equation not containing the quantity $\sigma_{\parallel}$

$$\frac{c}{4\pi} g\left[\nabla g\nabla\varphi\right] = -\left\{\frac{c}{4\pi}\tilde{\triangle} f + \rho\sigma_{\wedge}\left[(\nabla f)^2 + g^2\right]^{1/2}\frac{d\Psi}{df}\right\}\left[\nabla f\nabla\varphi\right] - \sigma_{\perp} g\frac{d\Psi}{df}\nabla f. \tag{7}$$

Here we use a useful coordinate system f, x^1, φ, $\nabla x^1 = [\nabla f\nabla\varphi]$. The coordinate x^1 is directed along lines of the poloidal magnetic field lying on the surfaces $f = const$. The function g depends on the coordinates f, x^1, and is found from the component of Equation (7) directed along ∇f,

$$g = \frac{4\pi}{c}\frac{d\Psi}{df}\int_0^{x^1}\rho^2(f, x^{1'})\sigma_{\perp}(f, x^{1'})dx^{1'} + F(f).$$

We consider the coordinate x^1 to be equal to zero at the level $h = 0$. But $h = 0$ is the boundary between the magnetosphere and the star crust. Then the function $F(f)$ is defined by the electric currents flowing in the magnetosphere

$$F(f) = \frac{4\pi}{c}\int_0^f j_1(f')df',$$

where the quantity $j_1(f)$ is the current density as a function of the magnetic flux f, $j_1 \nabla x^1 = j_0 \mathbf{e}_h$. Total current equals to zero $\int_0^{f_0} j_1(f')df' = 0$. The value of f_0 is the boundary magnetic surface limiting the region where the current exists, i.e. the boundary of the polar cap. Namely the quantity f_0 is the outer parameter of the problem, it is the flux of the magnetic field passing through the light surface. As a result the expression for g has the form

$$g = \frac{4\pi}{c}\left[\frac{d\Psi}{df}\int_0^{x^1}\rho^2\sigma_\perp dx^{1'} + \int_0^f j_1 df'\right].$$

The toroidal magnetic field must be equal to zero on the boundary $f = f_0$, $g = 0$, because the total electric current equals to zero inside the polar cap. It means that the quantity $d\Psi/df$ must turn to zero on the boundary $f = f_0$

$$\frac{d\Psi}{df} = const(f - f_0)^\alpha.$$

The equation defining the poloidal magnetic field is extracted from the component of Equation (7) directed along ∇x^1

$$\tilde{\triangle} f + g\frac{\partial g}{\partial f} + \frac{4\pi\rho\sigma_\wedge}{c}\left[(\nabla f)^2 + g^2\right]^{1/2}\frac{d\Psi}{df} = 0.$$

Non-disturbed dipole magnetic field is defined by the first term of this equation $\tilde{\triangle} f = 0$, the second term describes the field distortion by the electric currents flowing along the spiral magnetic field lines. This effect exists in all magnetosphere in the region of open lines. However, the second term is small near the star surface where the magnetic field strength is large [6]. And finally the third term is due to the Hall currents flowing in the crust. The characteristic value of the Hall conductivity $\sigma_\wedge$ is [7]

$$\sigma_\wedge = 10^{18} B_{12}^{-1}\rho_5^{2/3}\,\mathrm{sec}^{-1},$$

where ρ_5 is the density of the crust matter in the units of $10^5 g \cdot cm^{-3}$. The characteristic values in our problem are

$$\rho \to r_0, h \to r_0, f \to f_0, x^1 \to f_0/r_0, j_1 \to j_0 r_0^2/f_0, \Psi \to j_0 r_0/\sigma_\perp, g \to j_0 r_0^2/c.$$

Because of that the therm $g\partial g/\partial f$ is small, it is $i^2(\Omega R/c)^3$ times smaller than the rest therms. As a result we obtain the following equation for the flux of the poloidal magnetic field

$$\tilde{\triangle} f + \frac{4\pi\rho^2\sigma_\wedge B}{c}\frac{d\Psi}{df} = 0. \tag{8}$$

The product $\sigma_\wedge B$ depends only on the matter density in the crust which changes with the depth h. The real dependence of conductivity on the crust density, $\rho_5 \propto (-h)^3$, comes to the strong dependence of the Hall conductivity $\sigma_\wedge$ on the depth h, $\sigma_\wedge \propto [1 - \gamma(h/r_0)^3]^{2/3}$, where the parameter γ is large, $\gamma \approx 10^2 - 10^3$.

Equation (8) is valid in the region $h < 0$ in the star crust where the electric currents flow, $f < f_0$. In other regions $h > 0$ and $f > f_0, h < 0$ the equation for the poloidal magnetic field is $\tilde{\triangle} f = 0$. This equation contains not only the dipole field but also all multipole components. The function $f(\rho, h)$ must be continuous on the boundaries $h = 0, f = f_0$. The boundary $f(\rho, h) = f - 0$ itself is unknown, it has to be found from the solution of the problem. Thus, the problem to find the poloidal magnetic field flux $f(\rho, h)$ in all space can be formulated as

$$\tilde{\triangle} f = -\Theta(f_0 - f)\Theta(-h)\frac{4\pi\rho^2\sigma_\wedge B}{c}\frac{d\Psi}{df}, \quad (9)$$

where $\Theta(y)$ is the step function, $\Theta(y) = 1, y > 0$; $\Theta(y) = 0, y < 0$. The boundary conditions for Equation (9) are

1. the limiting value of f at $\rho \to 0$ and
2. the approaching of the solution to the dipole field $f \propto \rho^2/[\rho^2+(h+R)^2]^{3/2}$ at large distances $\rho/r_0, h/r_0 >> 1$.

4 Calculations

We solved Equation (9) numerically. For calculations we select the following parameters: $d\Psi/df \propto (f-f_0)^2$, $R/r_0 = 100$, $\gamma = 10^3$, $(4\pi/3)i(\sigma_\wedge/\sigma_\perp)\,(P/1sec)^{-1} = 1$. The result of calculations are drawn on the figures. The figures 2 show the magnetic field lines of the poloidal component for positive value of the electric current i in the center of the polar cap (fig. 2a) and for the negative current (fig. 2b). The position of the edge of the cap ($\rho = r_0$) corresponds to the value of $\rho = 5$ on the figures. The dashed lines are the pure dipole field lines. We see that the distortion of the magnetic field in the cap is significant. The figures 3 show the curvature, $(r_0/\rho_c)^{-1}$, of the poloidal magnetic field lines on the star surface versus the distance from the cap center ρ ($i > 0$ - 3a, $i < 0$ - 3b). Even for small values of the electric current $|i| \simeq 10^{-2}$ for the selected parameters the curvature radius of the magnetic field lines differs noticeably from the curvature radius of the dipole field (the dashed lines on the figures 3). Let us note that the quantity $23i(\sigma_\wedge/\sigma_\perp)(r_0/R) \simeq 1$ for PSR J2144-3933 ($P = 8.51, B_{12} = 2$) and for selected parameters. And the curvature radius estimation obtained in section 2 is confirmed by the calculations.

5 Conclusions

Here we discussed the reason of change of the curvature of magnetic field lines in the polar cap, where generation of electron-positron plasma takes place. The curvature radius defines the free path of a fast particle with respect to the pair birth. The less curvature radius is, the less the free path becomes. The less the free path is, the less the height of the polar cap is, and consequently, the less the electric voltage in the cap is. According to the paper of Ruderman & Sutherlend [8] the value of the voltage difference ψ between the star and magnetosphere in

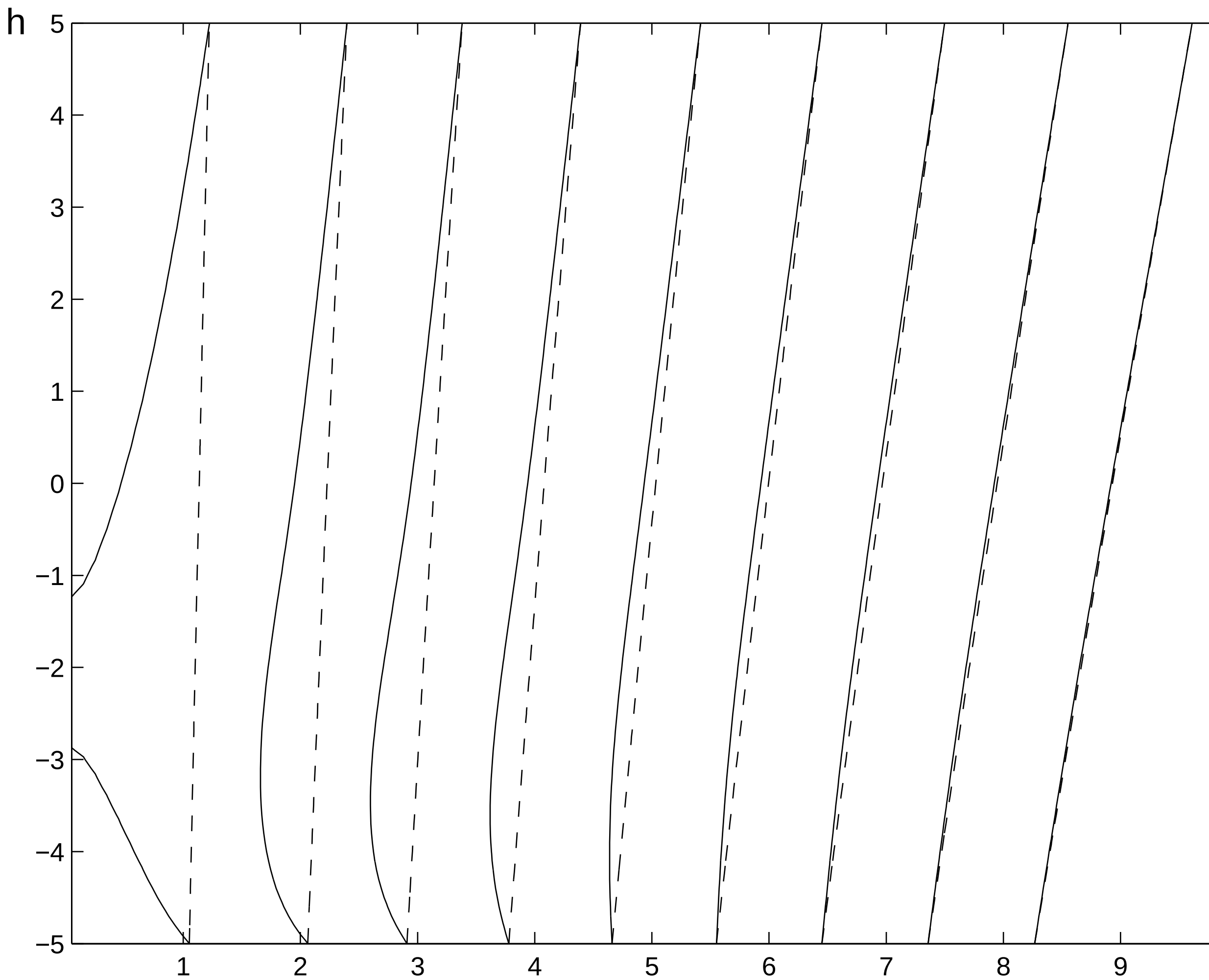

Figure 2: Poloidal magnetic field lines in the polar cap, for positive magnetospheric current. Dashed lines are the dipole field

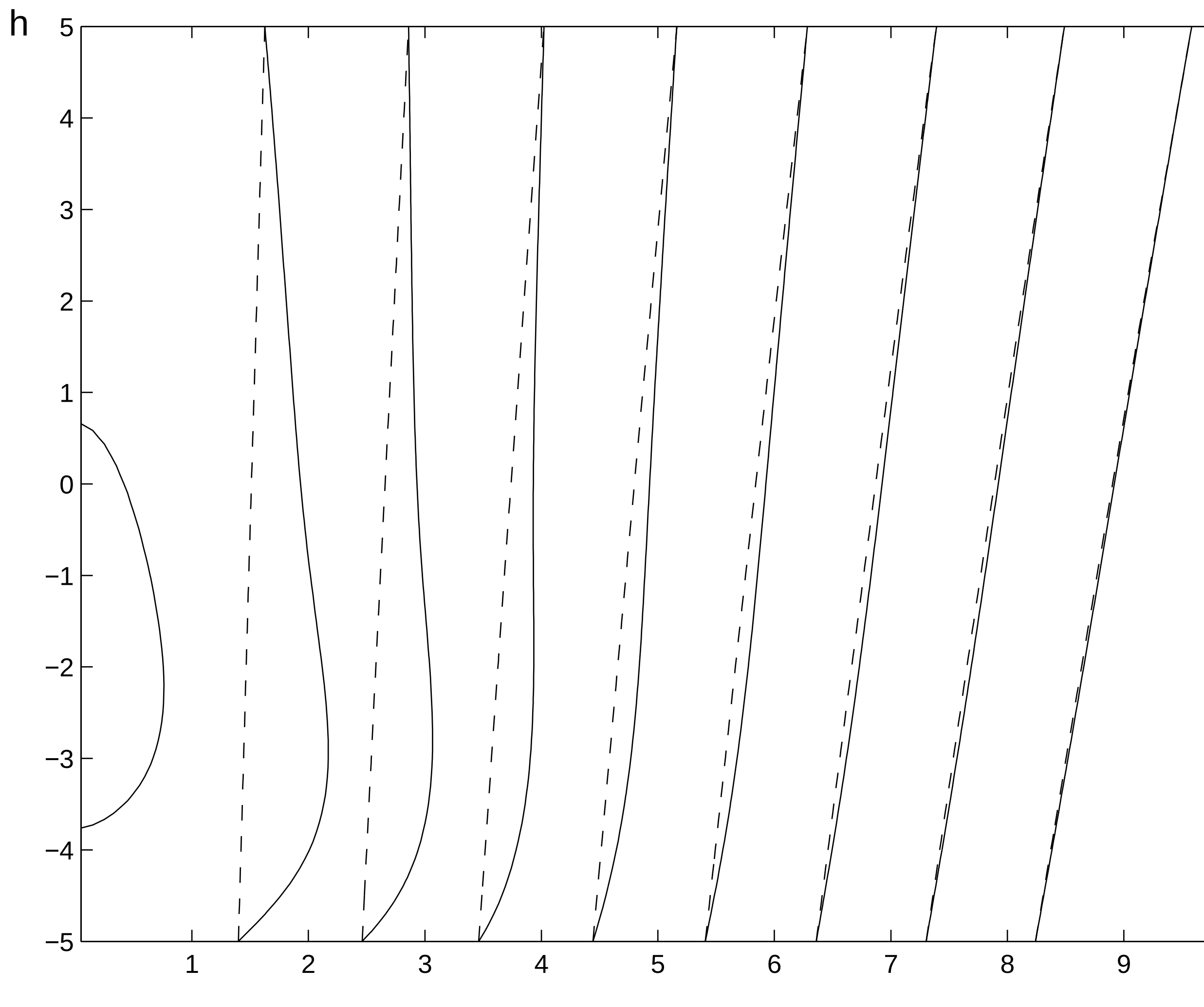

Figure 3: Poloidal magnetic field lines in the polar cap, for negative magnetospheric current. Dashed lines are the dipole field

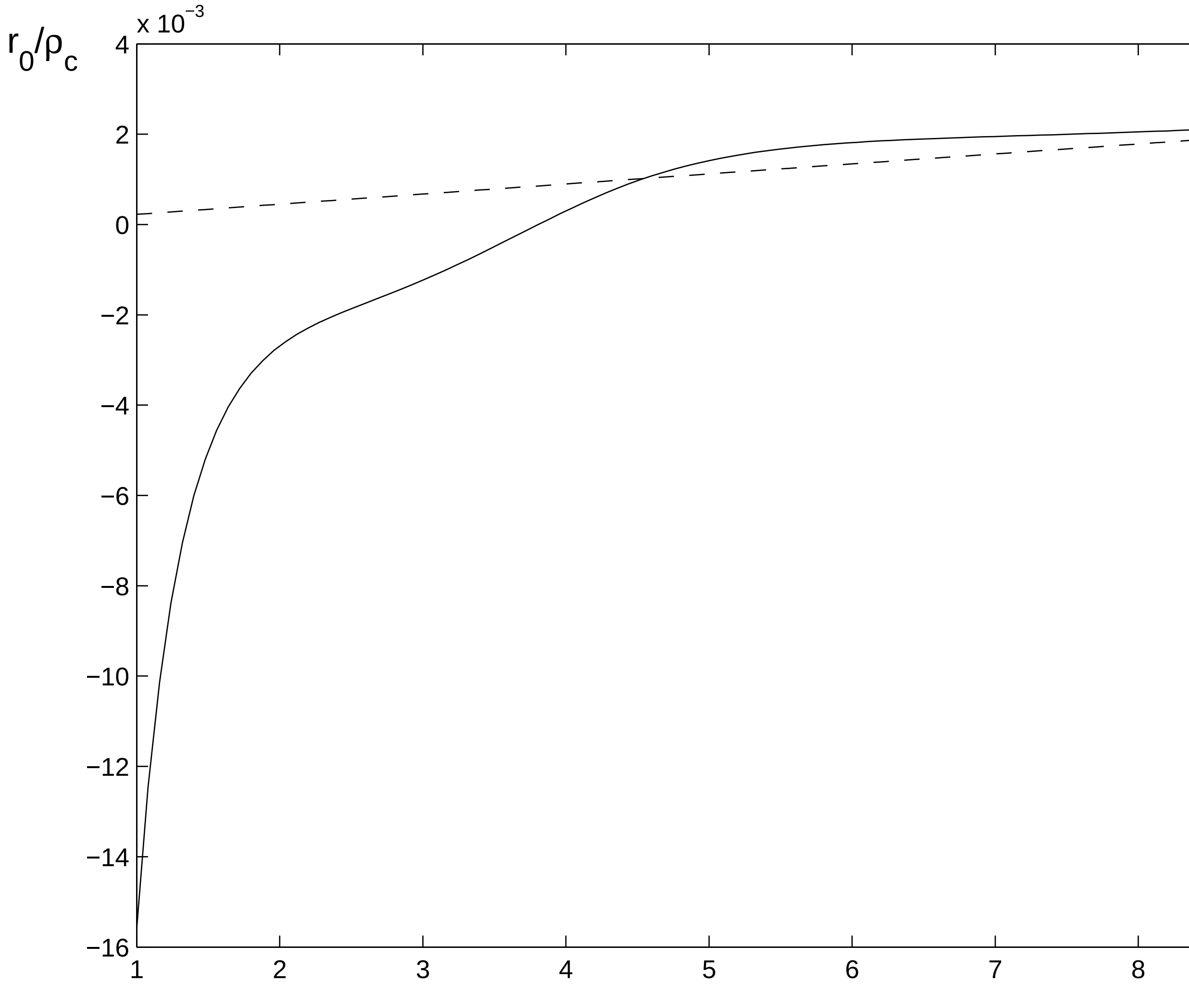

Figure 4: Curvature of the poloidal magnetic field lines on the star's surface, for positive magnetospheric current. The dashed line is the curvature of the dipole field

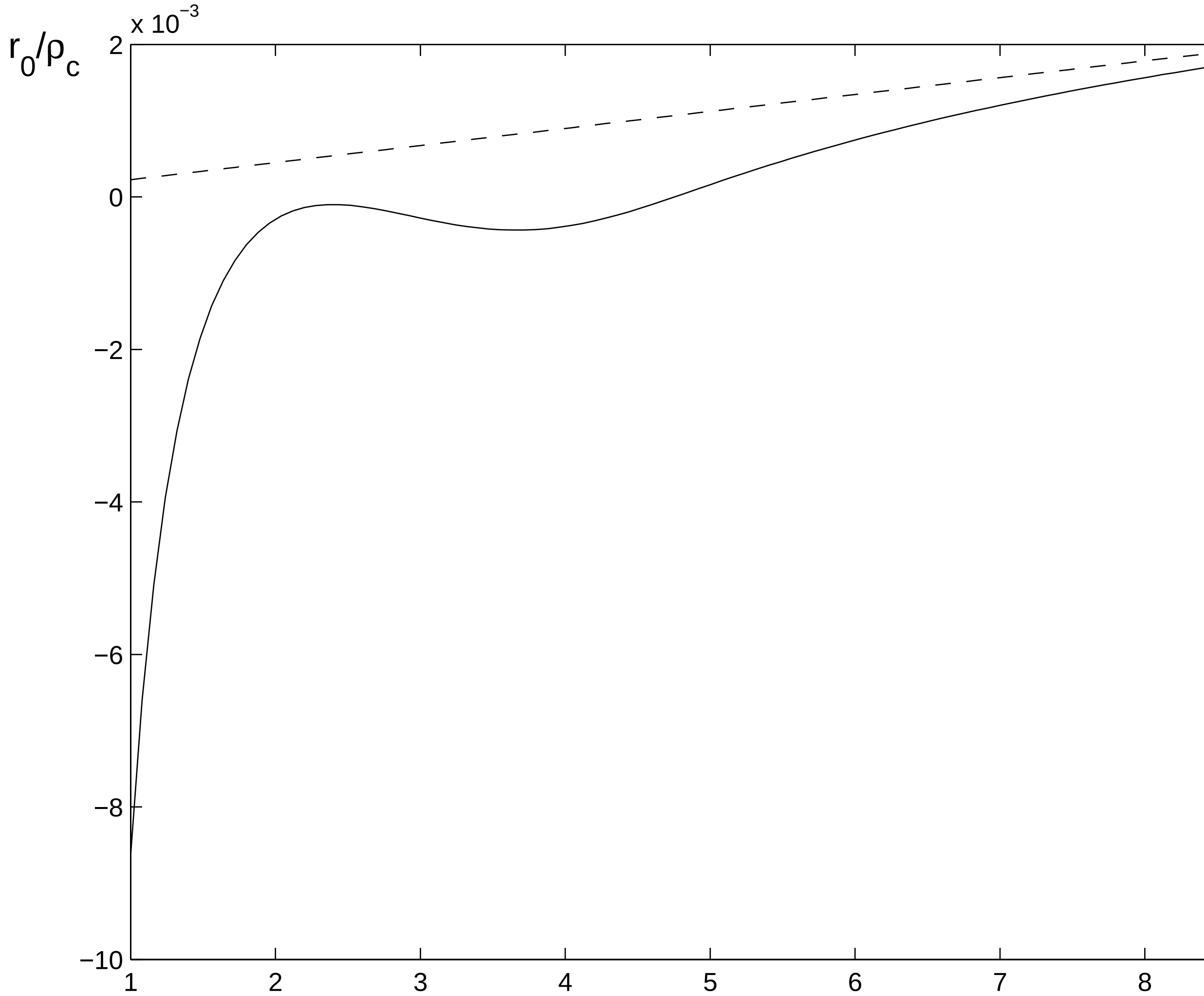

Figure 5: Curvature of the poloidal magnetic field lines on the star's surface, for negative magnetospheric current. The dashed line is the curvature of the dipole field

the polar region is proportional to the curvature radius ρ_c, $\psi \propto \rho_c^{4/7} B_{12}^{-1/7}$. This is the case when an electron-positron plasma is generated in the polar cap under the star surface and small amount of primary electrons is ejected from the crust. On the other hand, the maximum value of this potential is $\psi_m \simeq \pi e n_{GJ} r_0^2$ [6]. For standard value of the curvature radius of the dipole field in polar region we obtain the usual criteria for the polar cap ignition $P \cdot B^{-8/15} < 1sec$ ($\psi < \psi_m$). Let us note that the power $-8/15$ just coincides with the slope of the death line on the diagram $\log P - \log B_{12}$ for observed pulsars. If we use our results for the minimum value of the curvature radius of the field distorted by the crust current (see formula (3), $i = 1$, $\rho_c \propto P B_{12}^{-1} T_6$) we find another criteria $P \cdot B_{12}^{-2/3} < 10 T_6^{-2/9} sec$. This condition explains well existence of the slow rotating pulsar PSR J2144-3933.

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