

Bootstrap for 2pt Functions

Oliver Friedrich, SPV Stella Seitz

Lensing Seminar

July 18, 2014

Intro:
Bootstrapping of
2pt Data

Excursion:
Markov
Bootstrap in
Texture
Synthesis

A Markov
Bootstrap for 2pt
Correlation
Functions

Intro: Bootstrapping of 2pt Data

Excursion: Markov Bootstrap in Texture Synthesis

A Markov Bootstrap for 2pt Correlation Functions

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Bootstrap of Galaxy Surveys

1	2	3
4	5	6
7	8	9

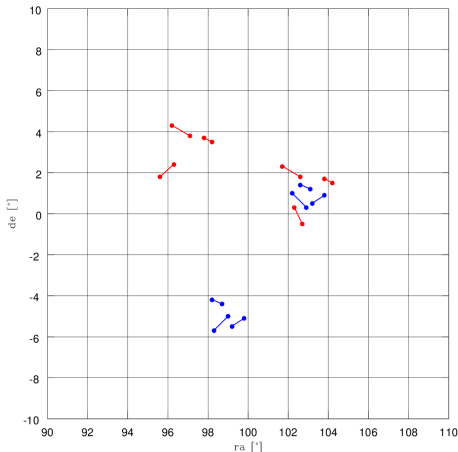
original sample

bootstrap sample

2	5	6
4	8	2
1	1	3

cut survey area into subpatches and resample them to generate new 'fake' observations.

Marked Bootstrap



to each subpatch
assign its correlations
with the whole patch
and resample only
these values

(Loh calls this
'marked bootstrap')

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Testing Validity of internal Covariance Estimations from Simulations

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- ▶ Simulate 300 Gaussian cosmic shear surveys,
⇒ measure sample covariance.
- ▶ Measure covariance internally from one simulation.

Cosmic Variance of ξ_+

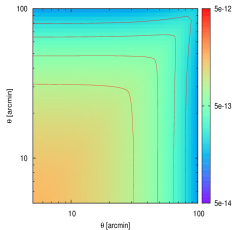
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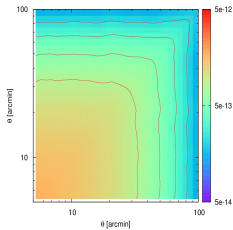
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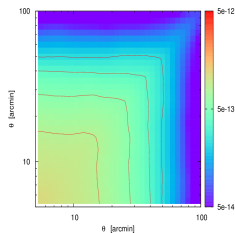
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analytic
covariance



from 300
simulations



internal estimation
(bootstrap)
in one simulation

Using different Sizes of Subpatches

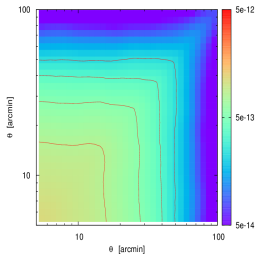
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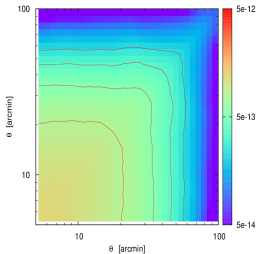
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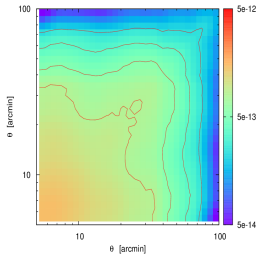
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225 subpatches



144 subpatches



36 subpatches

Markov Bootstrap

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- ▶ The bigger the subpatches - the noisier the bootstrap
- ▶ The smaller the subpatches - the bigger the patch-patch correlation
- ▶ Model the subpatches as a Markov-random-field:
Draw bootstrap samples such that internal correlations are respected.
(Modification of Markov-Bootstrap by Efros and Leung)

Outline

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Markov Resampling

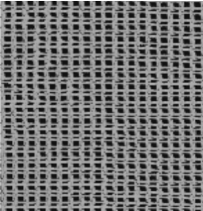
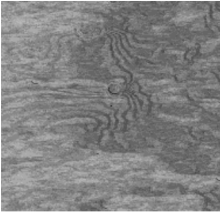
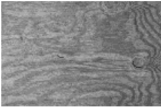
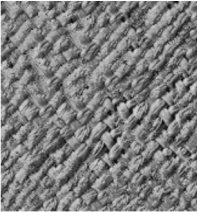
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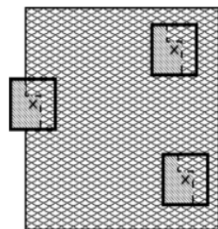
A Markov
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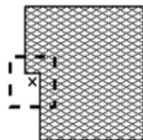
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Markov Resampling



Original



Synthesis

Draw new pixel p_i
from all p_j such that

$$d(i, j) < \epsilon,$$

where d compares
the neighborhood of
the pixels.

- ▶ In (appropriate) limit $\epsilon \rightarrow 0$ & $N_{pix} \rightarrow \infty$ the resampling follows the correct joint distribution of Pixels¹.
 - ▶ Pixel only carries one value!
- ⇒ Needs to be generalized to binned 2pt functions.

¹Proven in Levina and Bickel 2006

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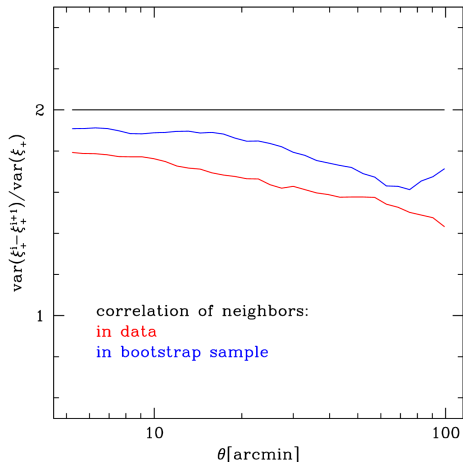
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Markov Bootstrap



Trying to mimic
neighbor correlations.

1D Explanation



- ▶ compute $\vec{d}_i := \vec{\xi}_i - \vec{\xi}_{i+1}$
- ▶ estimate $Cov_d = \langle \vec{d} \vec{d}^T \rangle$ from data

1D Explanation



- ▶ draw some value of χ^2
- ▶ draw $\vec{\xi}_?$ from $\vec{\xi}_i$ such that $\Delta\chi^2$ is minimal
($\chi^2 = d_?^T \cdot C^{-1} \cdot d_?$)

2D Generalization

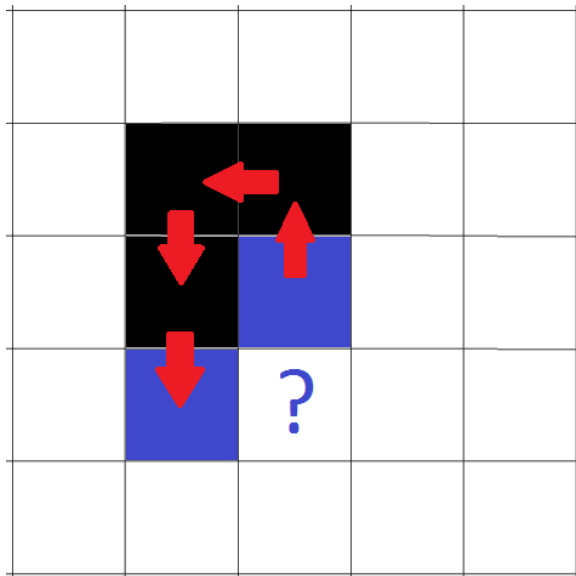
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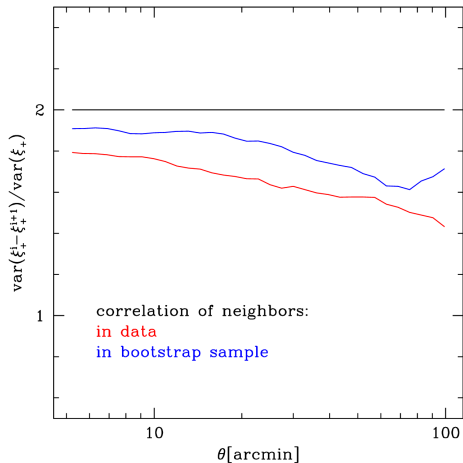
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Cosmic Variance of ξ_+

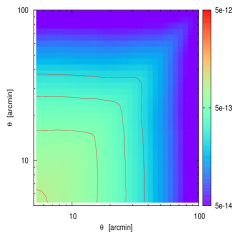
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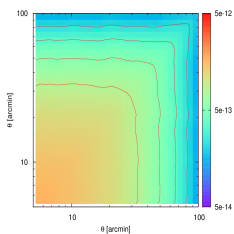
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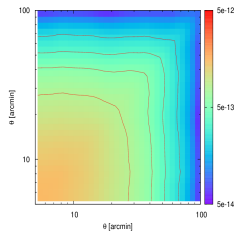
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standard
bootstrap
 30×30 subpatches



from 300
simulations



Markov
bootstrap
 30×30 subpatches

Cosmic Variance of ξ_+

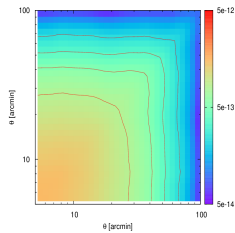
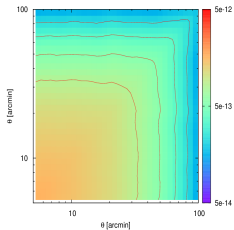
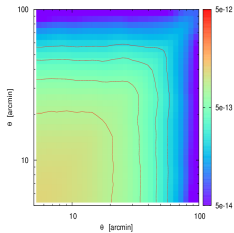
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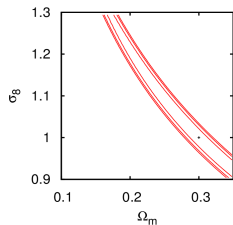
12x12 subpatches

from 300
simulations

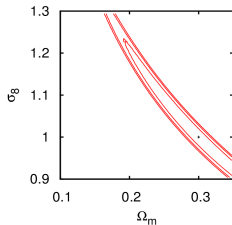
Markov
bootstrap

30x30 subpatches

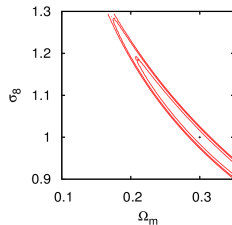
$\Delta\chi^2$ with different Covariances



from 300
simulations



Markov
bootstrap



standard
bootstrap

$\Delta\chi^2$ with different Covariances

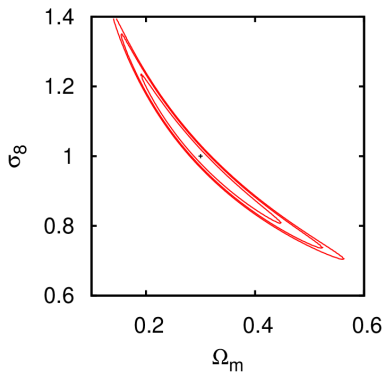
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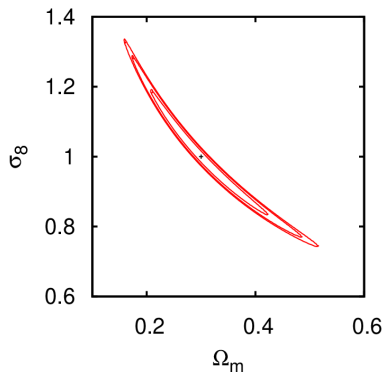
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Markov
bootstrap



standard
bootstrap

Compare Area inside 1σ Contour

- ▶ A_0 : area when using covariance of 300 independent simulations
- ▶ A_m : when using Markov bootstrap
- ▶ A_s : when using standard bootstrap
- ▶ $A_m/A_0 \approx 0.63$, $A_s/A_0 \approx 0.43$

- ▶ Texture Synthesis by Non-parametric Sampling
(Alexei A. Efros and Thomas K. Leung, IEEE International Conference on Computer Vision, 1999)
- ▶ Texture Synthesis and Non-parametric Resampling of Random Fields
(Elizaveta Levina and Peter J. Bickel, The Annals of Statistics, 2006)
- ▶ A valid and fast spatial bootstrap for correlation functions
(Ji Meng Loh, arXiv:0805.2325v1, 2008)