Bootstrap for 2pt Functions

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Lensing Seminar

July 18, 2014

Intro: Bootstraping of 2pt Data

Excursion: Markov Bootstrap in Texture Synthesis

A Markov Bootstrap for 2pt Correlation Functions

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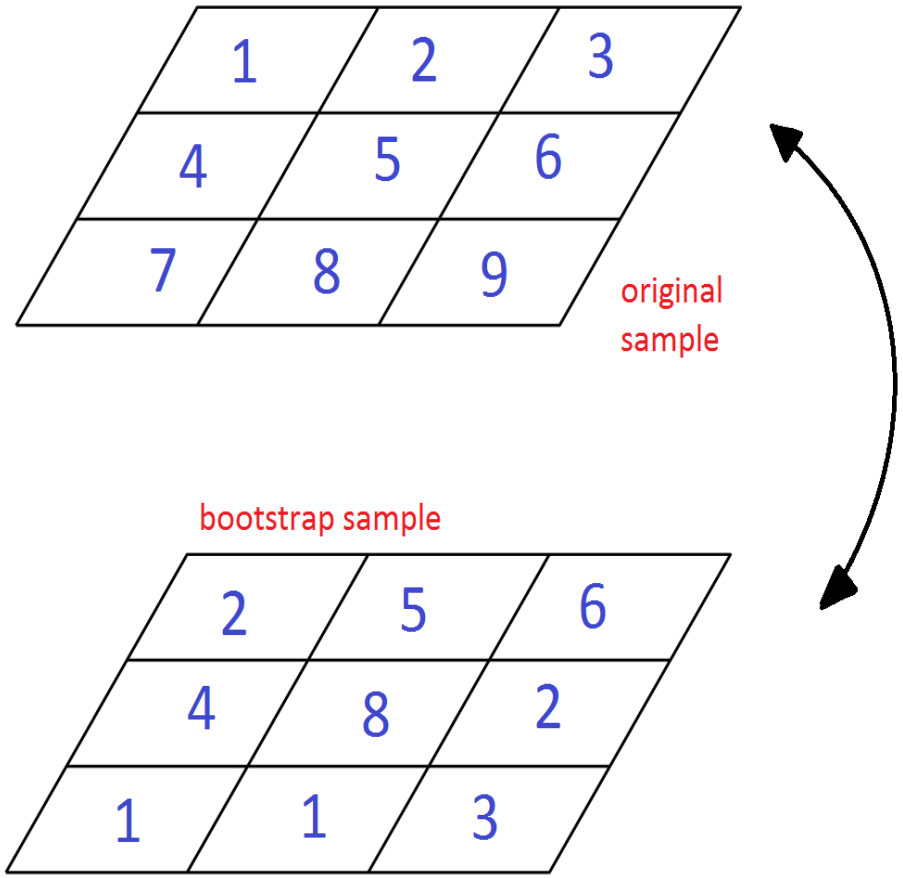
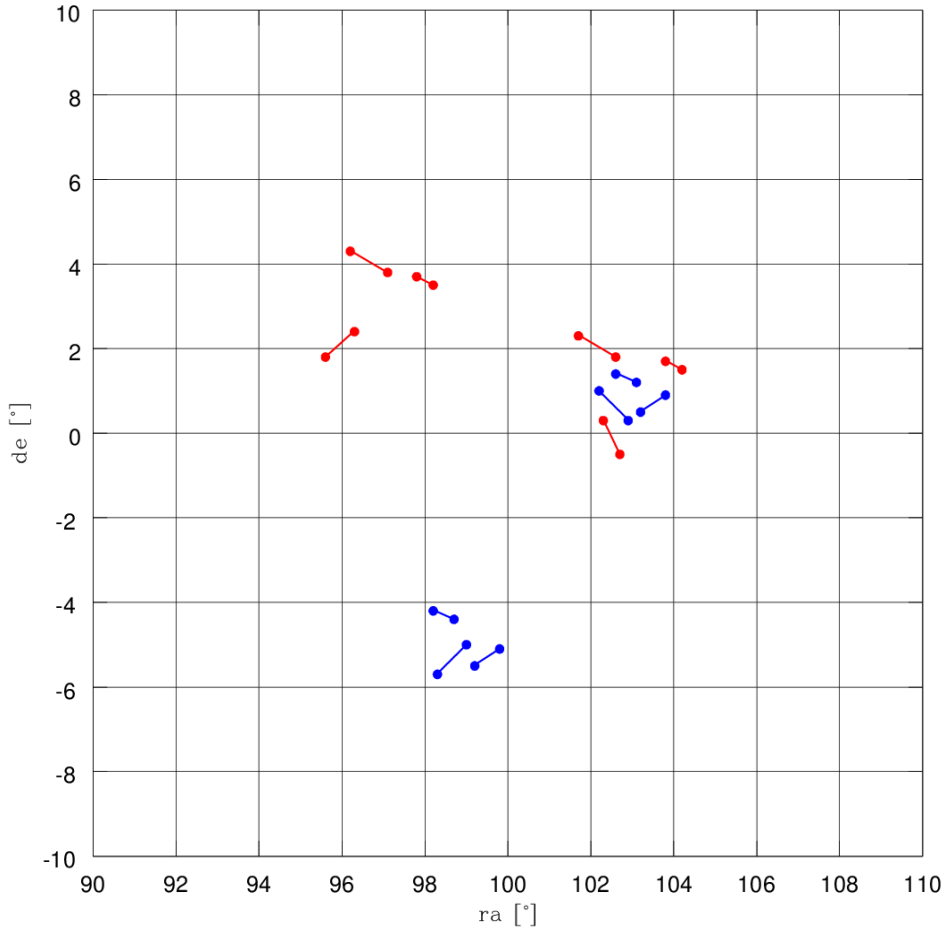
Bootstrap of Galaxy Surveys



cut survey area into subpatches and resample them to generate new 'fake' observations.

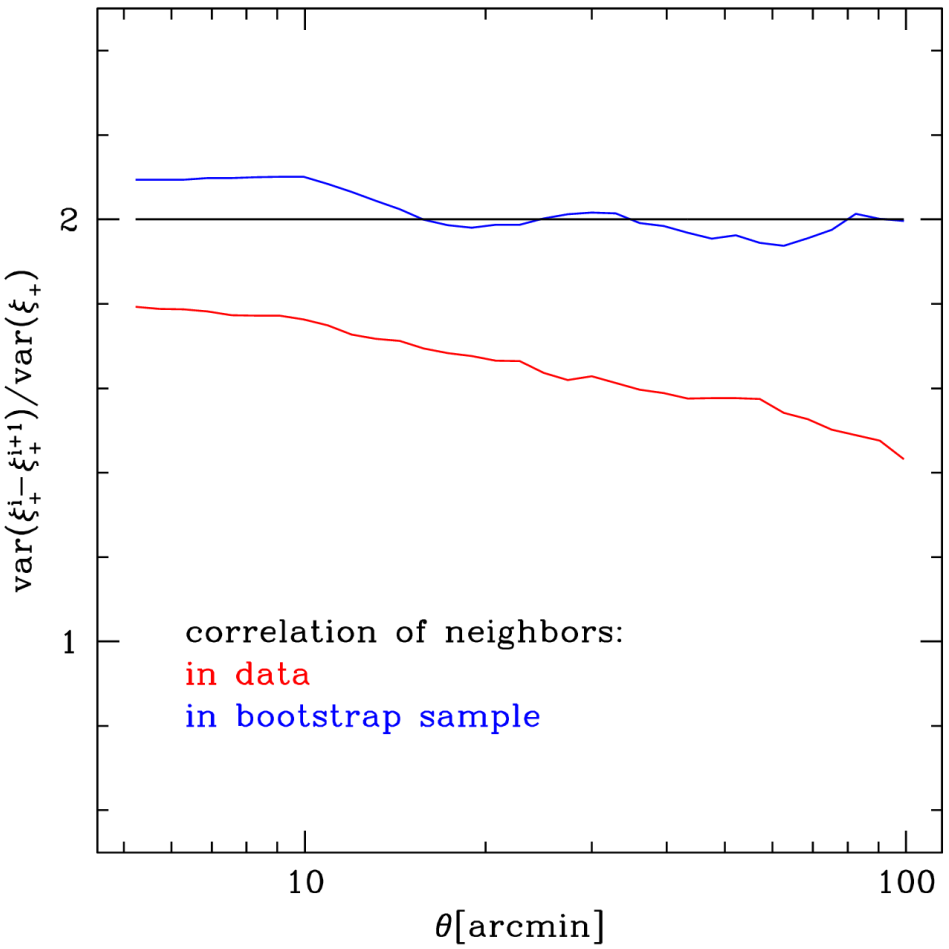
Marked Bootstrap



to each subpatch assign its correlations with the whole patch and resample only these values

(Loh calls this 'marked bootstrap')

Correlation of neighboring Subpatches



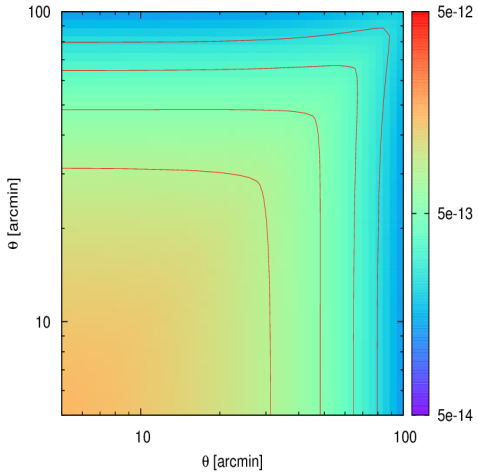
Problem: close subpatches are not independent

$\Rightarrow$ bootstrap resampling in principle invalid..

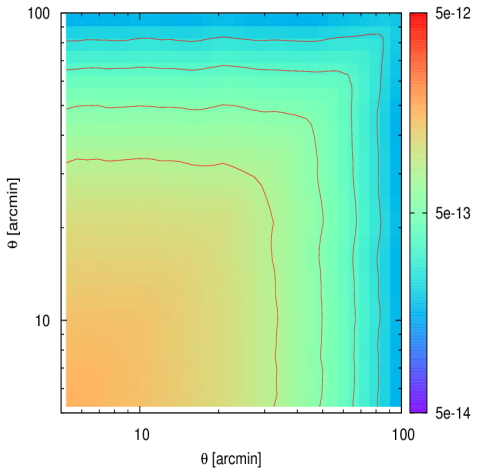
Testing Validity of internal Covariance Estimations from Simulations

- Simulate 300 Gaussian cosmic shear surveys,
  $\Rightarrow$ measure sample covariance.
- Measure covariance internally from one simulation.

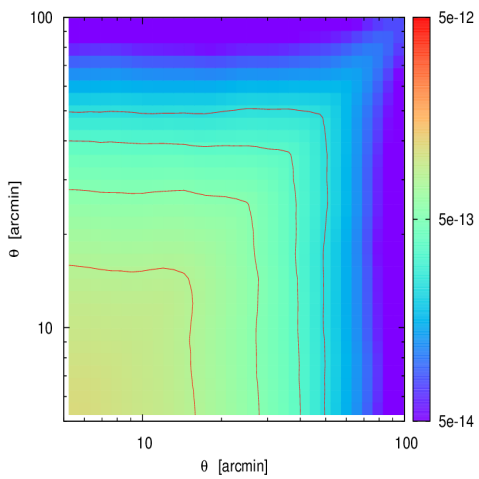
Cosmic Variance of ξ_+



analytic covariance

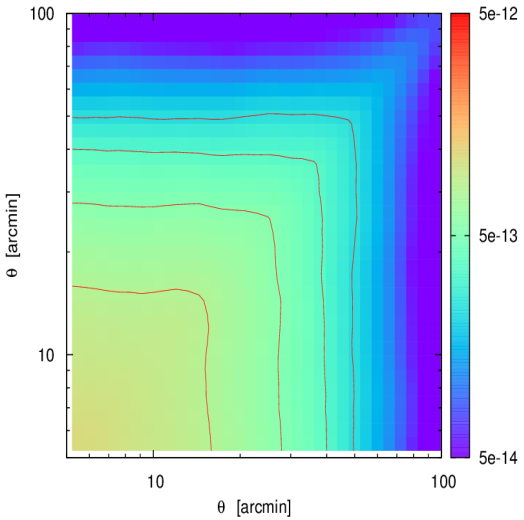


from 300 simulations

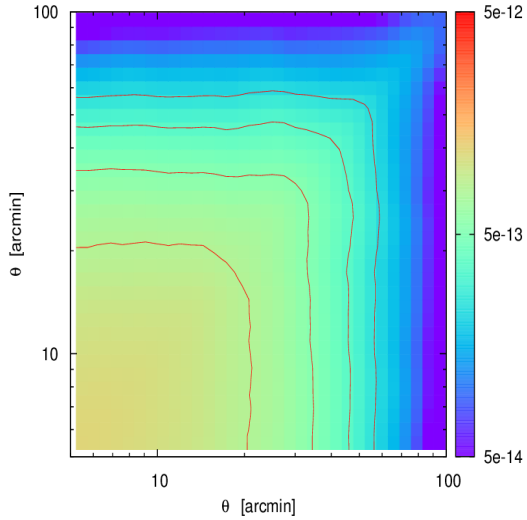


internal estimation (bootstrap) in one simulation

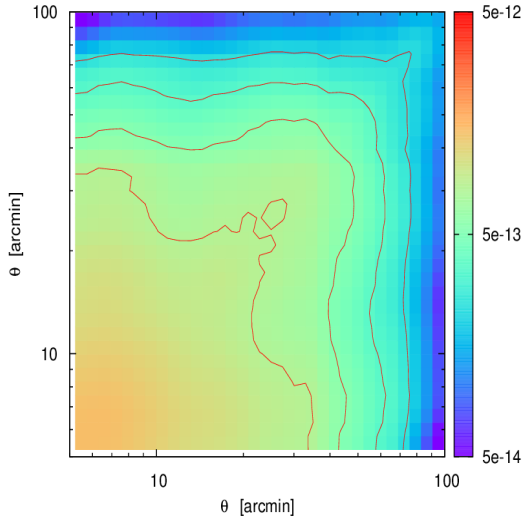
Using different Sizes of Subpatches



225 subpatches



144 subpatches



36 subpatches

Markov Bootstrap

- The bigger the subpatches - the noisier the bootstrap
- The smaller the subpatches - the bigger the patch-patch correlation
- Model the subpatches as a Markov-random-field:
  Draw bootstrap samples such that internal correlations are respected.
  (Modification of Markov-Bootstrap by Efros and Leung)

Outline

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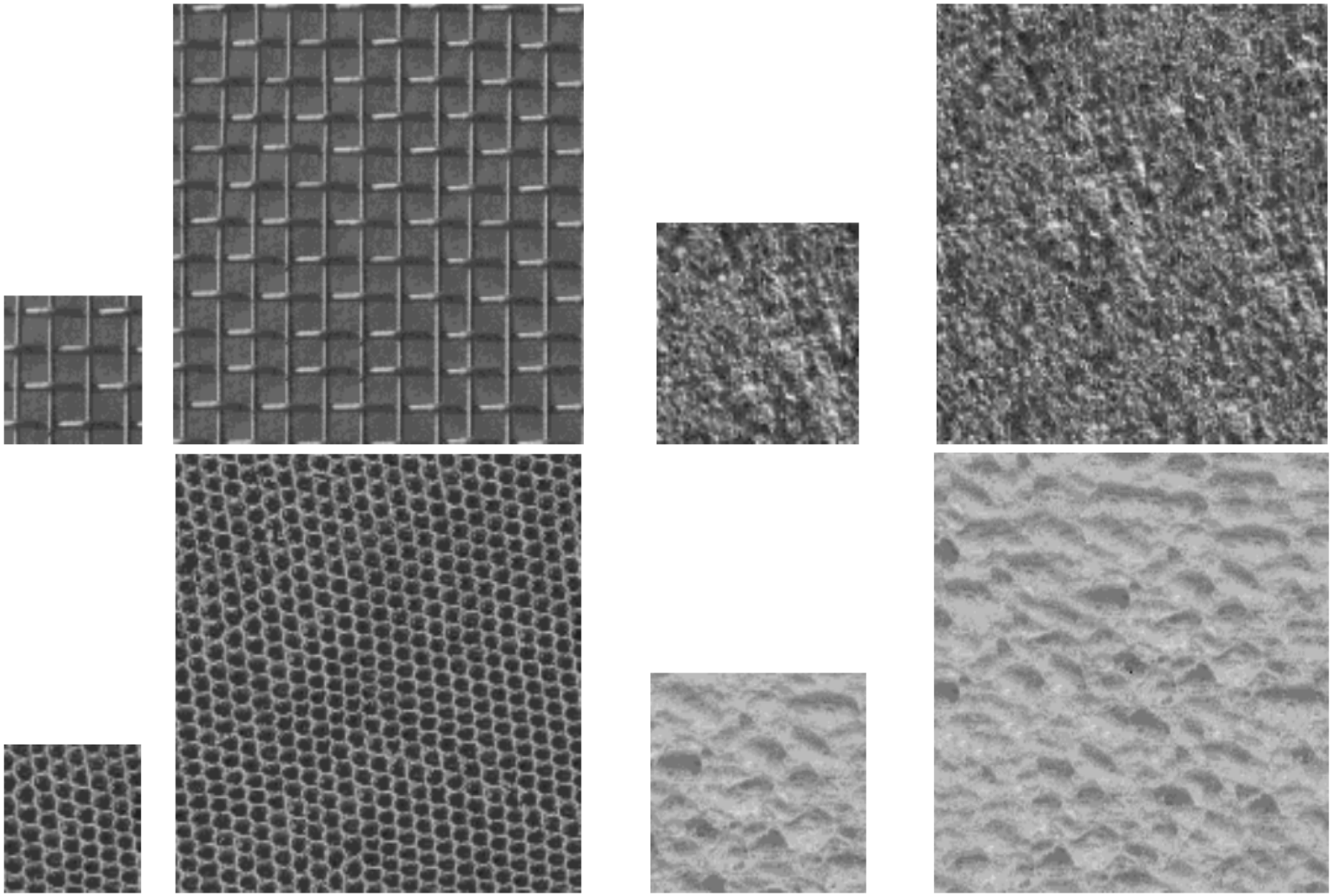
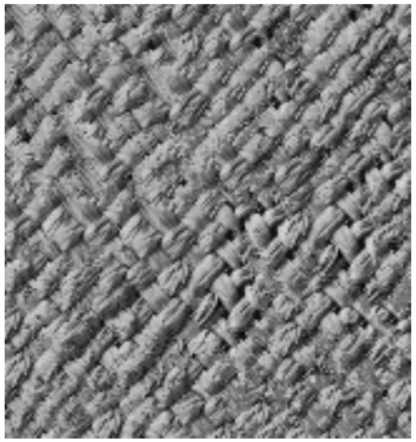
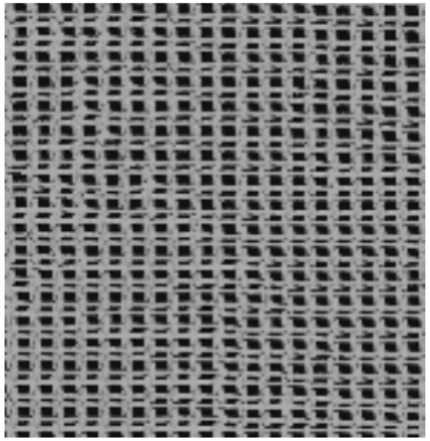
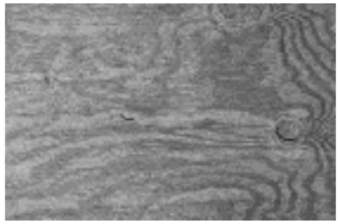
Markov Resampling

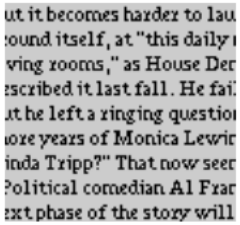


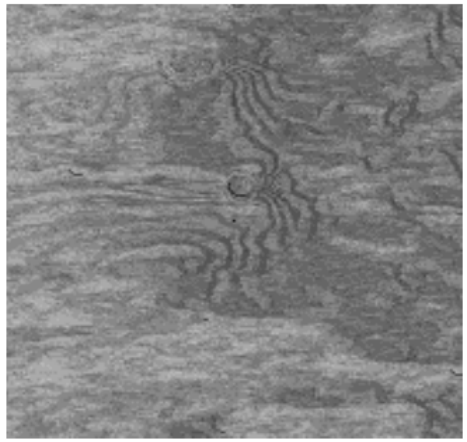
Markov Resampling

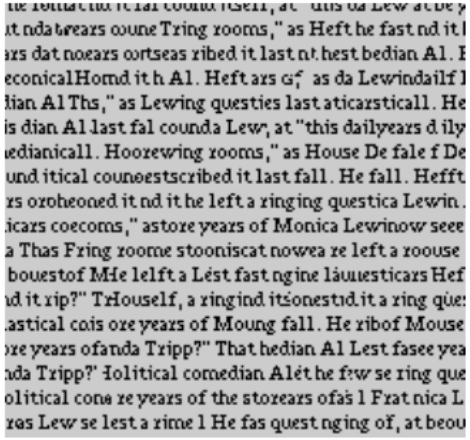




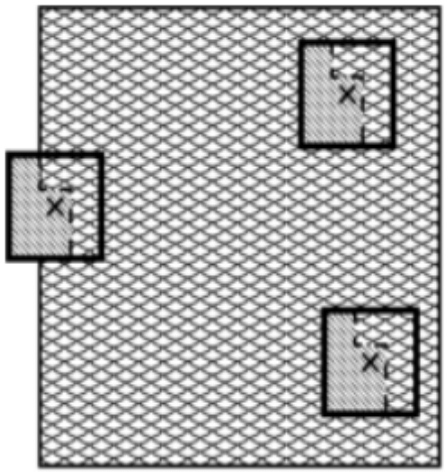




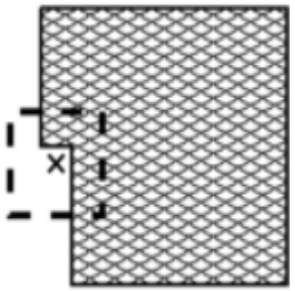




Markov Resampling



Original



Synthesis

Draw new pixel p_i from all p_j such that

$$d(i,j) < \epsilon,$$

where d compares the neighborhood of the pixels.

Markov Bootstrap

- In (appropriate) limit $\epsilon \to 0$ & $N_{pix} \to \infty$ the resampling follows the correct joint distribution of Pixels[1].
- Pixel only carries one value!

$\Rightarrow$ Needs to be generalized to binned 2pt functions.

[1] Proven in Levina and Bickel 2006

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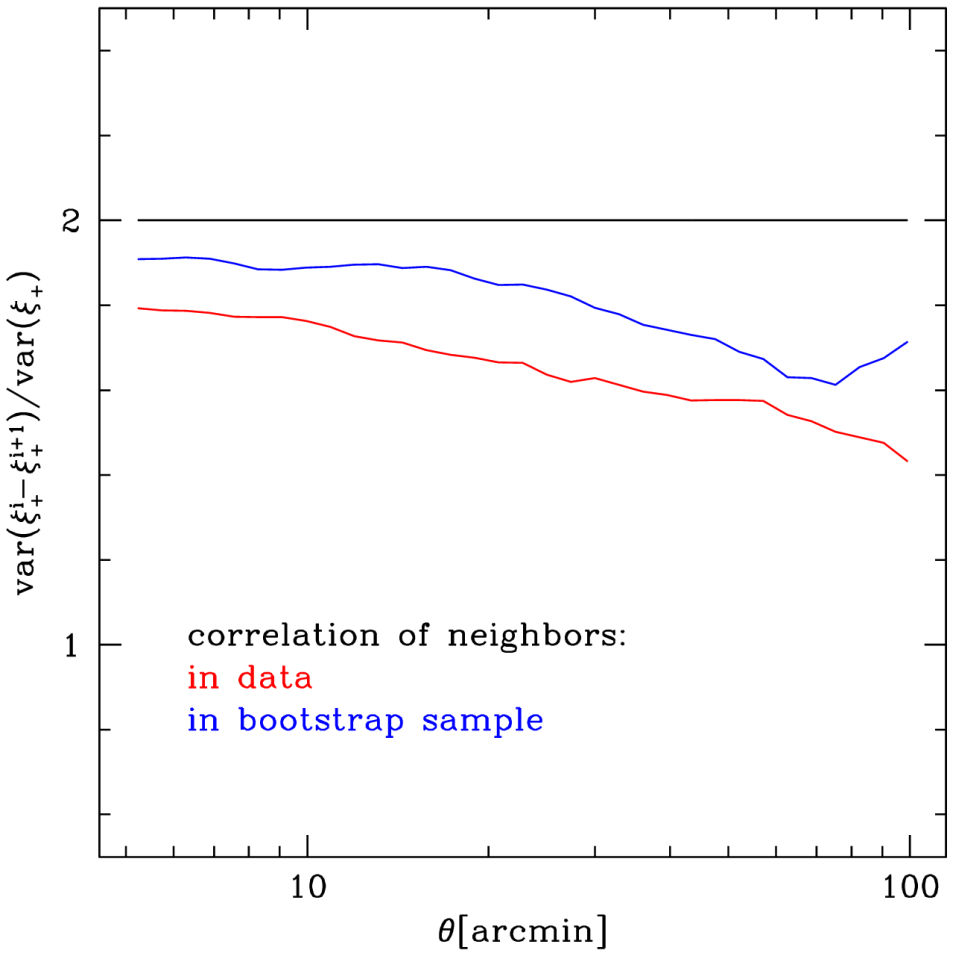
Markov Bootstrap



Trying to mimic neighbor correlations.

1D Explanation

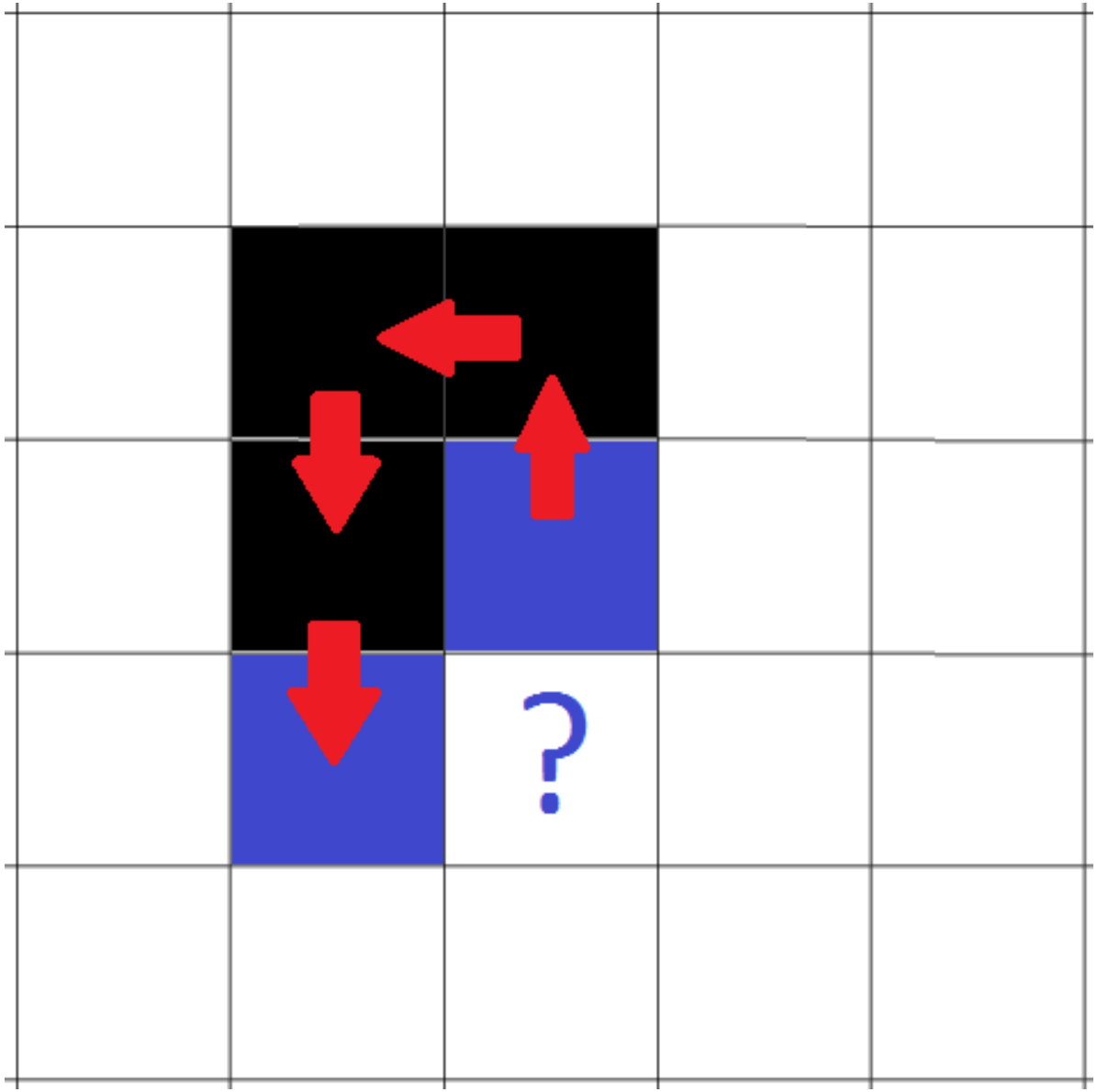
- compute $\vec{d}_i := \vec{\xi}_i - \vec{\xi}_{i+1}$
- estimate $Cov_d = \langle \vec{d}\ \vec{d}^T \rangle$ from data

1D Explanation

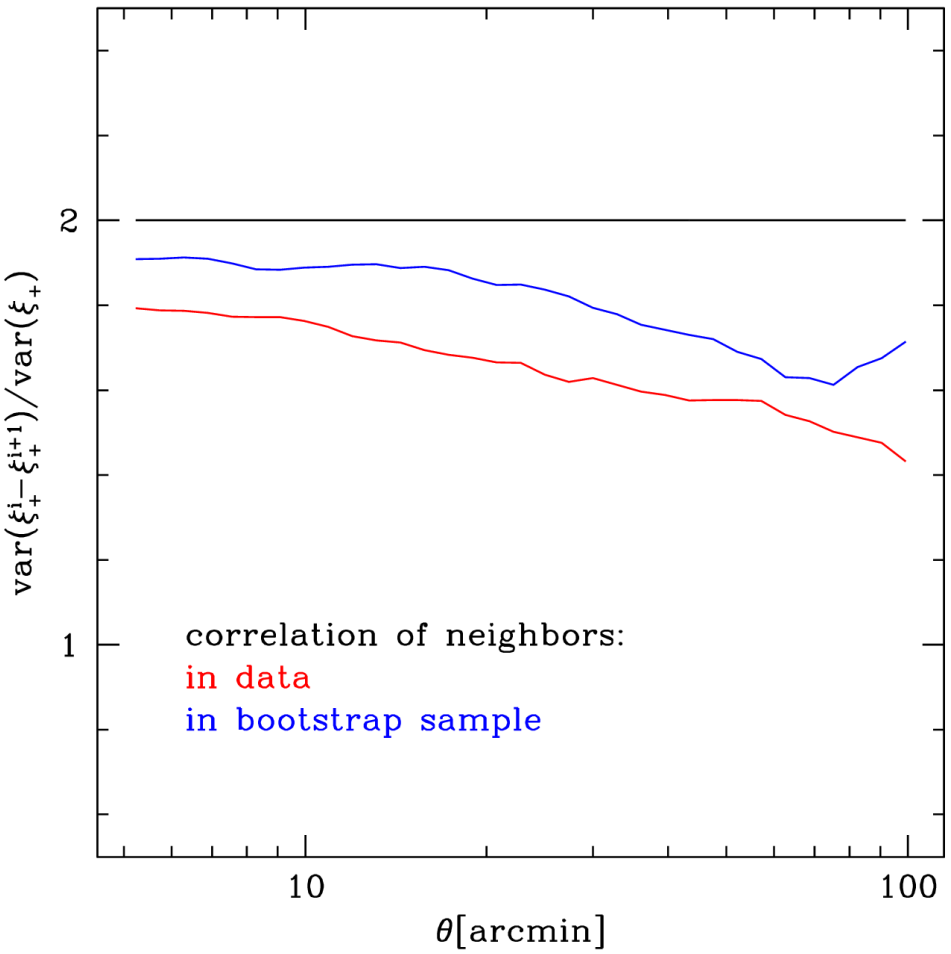
- draw some value of χ^2
- draw $\vec{\xi}_?$ from $\vec{\xi}_i$ such that $\Delta\chi^2$ is minimal

$(\chi^2 = d_?^T \cdot C^{-1} \cdot d_?)$

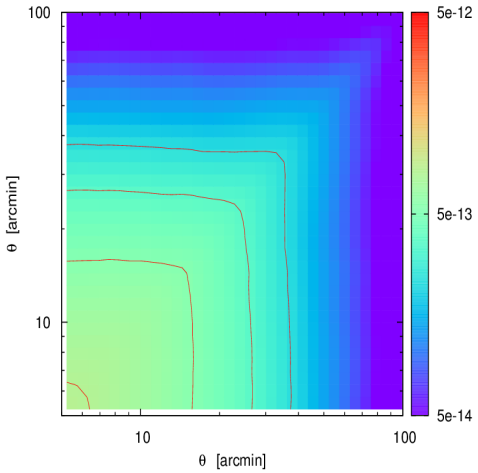
2D Generalization



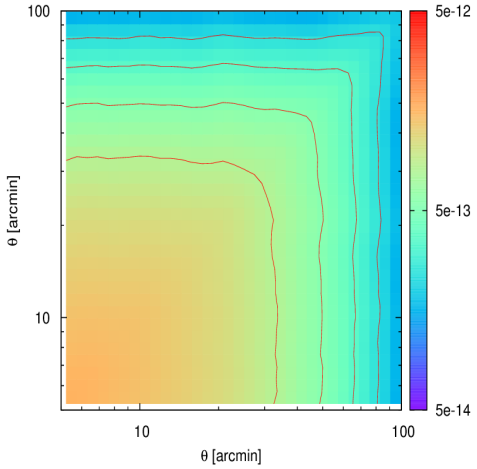
Markov Bootstrap



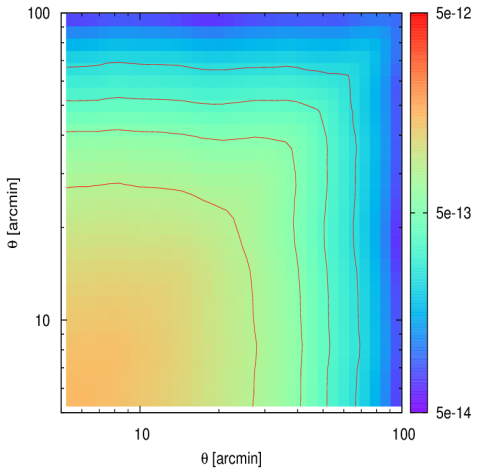
Cosmic Variance of ξ_+



standard bootstrap 30×30 subpatches



from 300 simulations



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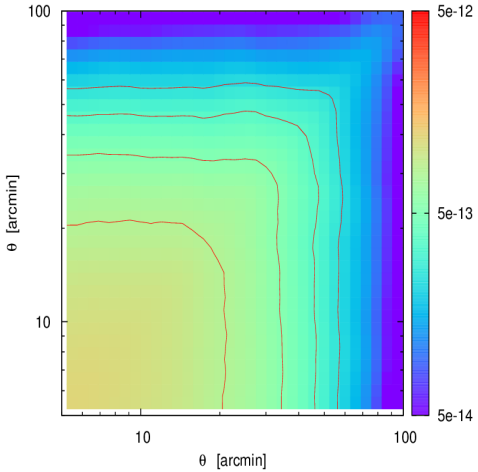
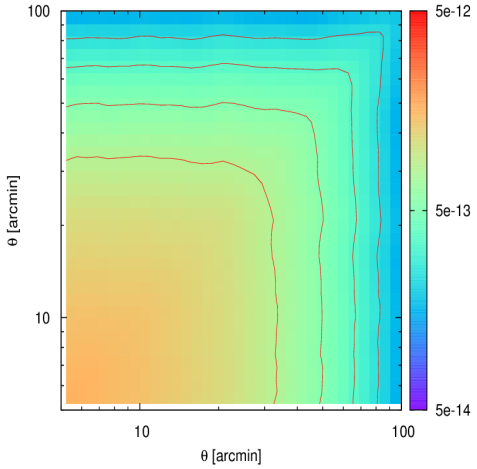
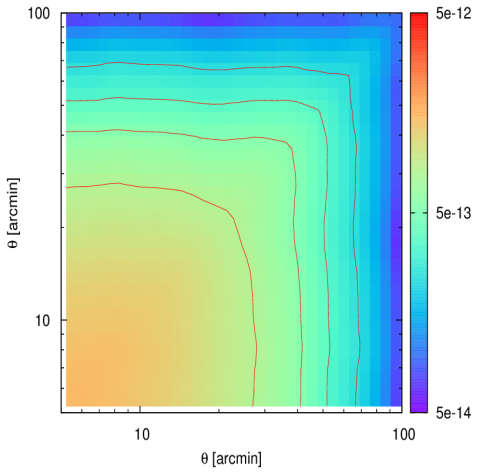


standard bootstrap 12×12 subpatches

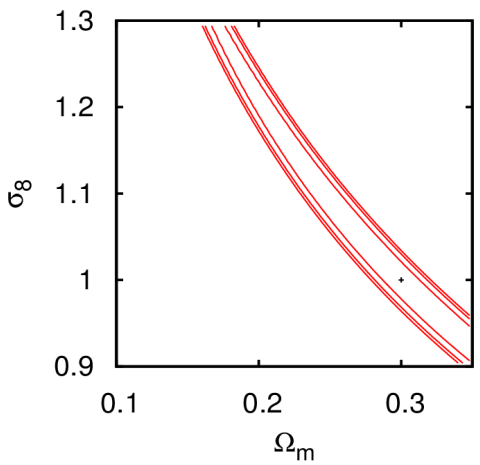


from 300 simulations

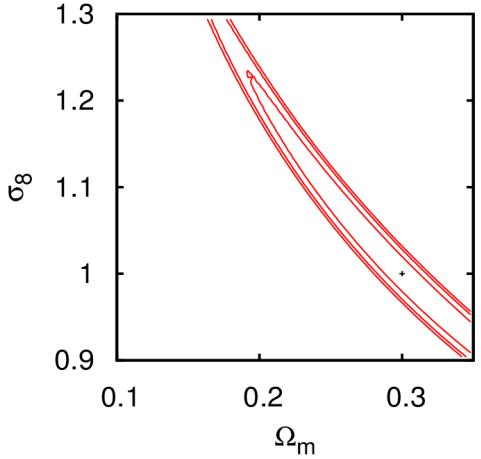


Markov bootstrap 30×30 subpatches

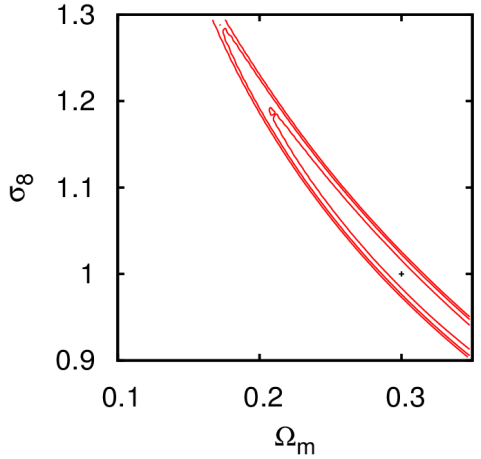
$\Delta\chi^2$ with different Covariances



from 300 simulations

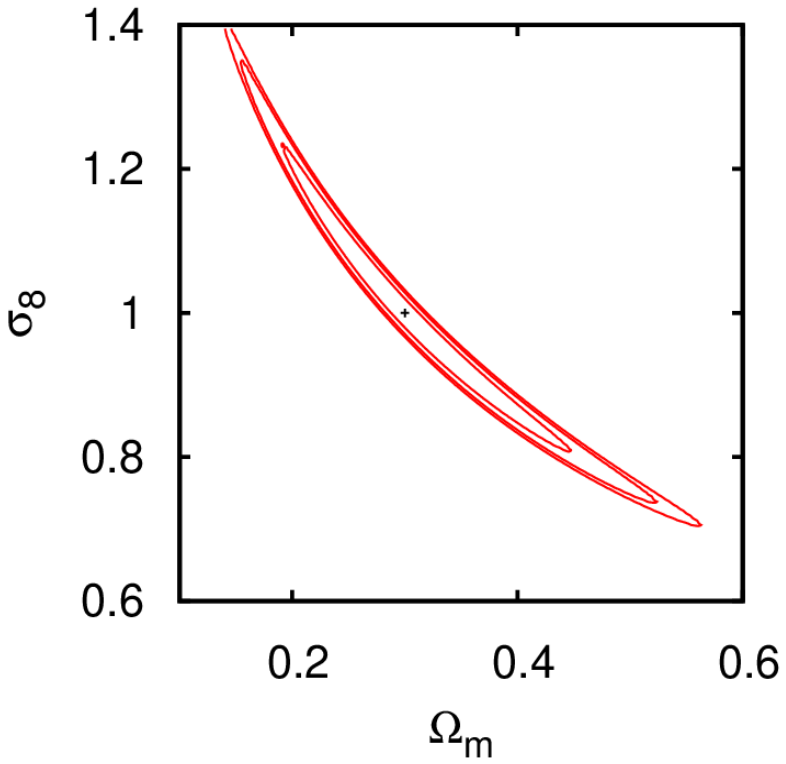


Markov bootstrap



standard bootstrap

$\Delta\chi^2$ with different Covariances



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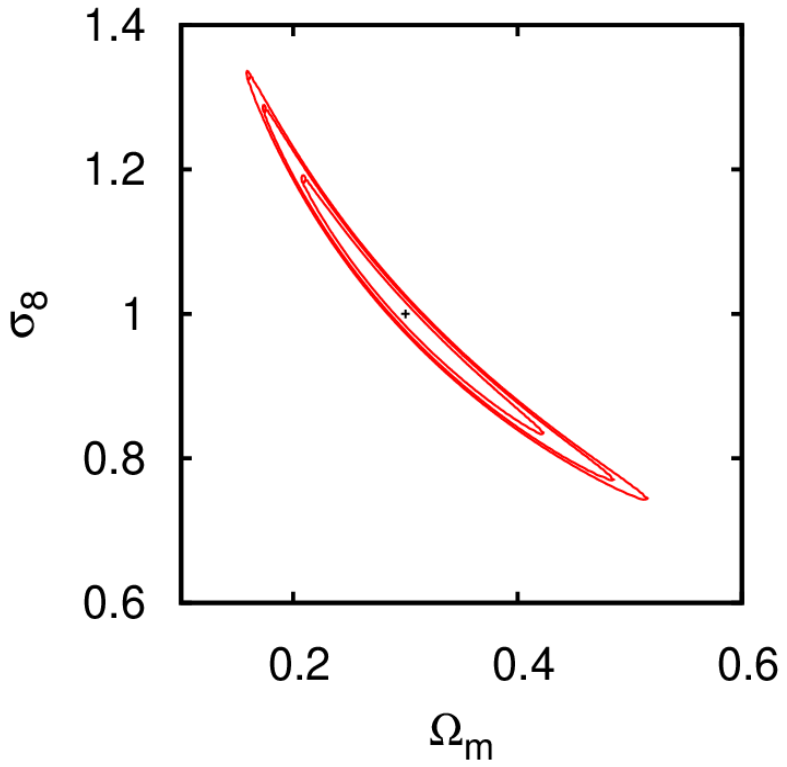

standard bootstrap

Compare Area inside 1σ Contour

- A_0: area when using covariance of 300 independent simulations
- A_m: when using Markov bootstrap
- A_s: when using standard bootstrap
- $A_m/A_0 \approx 0.63$, $A_s/A_0 \approx 0.43$

Sources

- Texture Synthesis by Non-parametric Sampling

  (Alexei A. Efros and Thomas K. Leung, IEEE International Conference on Computer Vision, 1999)

- Texture Synthesis and Non-parametric Resampling of Random Fields

  (Elizaveta Levina and Peter J. Bickel, The Annals of Statistics, 2006)

- A valid and fast spatial bootstrap for correlation functions

  (Ji Meng Loh, arXiv:0805.2325v1, 2008)