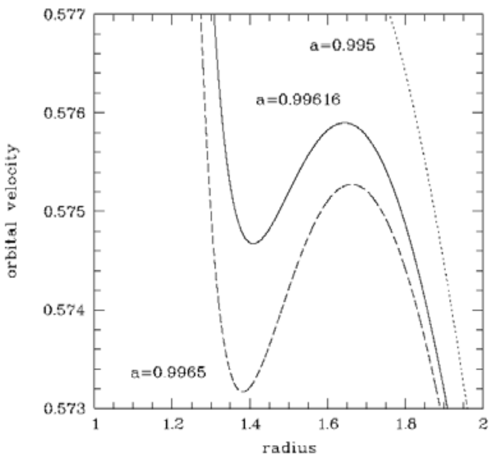
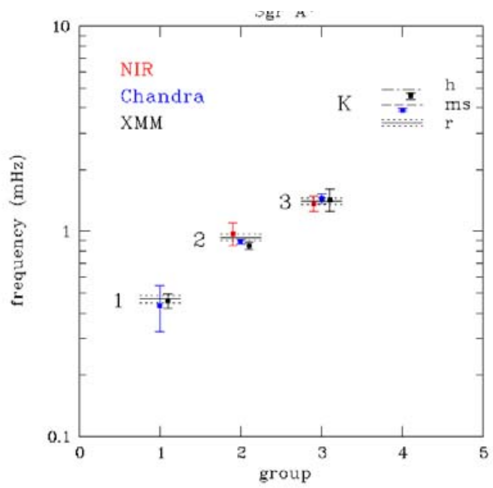
A new effect of General Relativity and the determination of mass and spin of black holes

The classical problem of a test particle orbiting a rotating black hole has long been solved. Stable circular orbits exist down to a minimal orbital radius *r*, the marginally stable circular orbit. As for Newtonian mechanics both the energy *E* and the angular momentum *L* of the particle are monotonic functions of *r* for the full range of the black hole spin parameter *a*. This means that there this is no preference for any specific value of *r* and *a*. The strictly monotonic behaviour with *r* has generally been assumed to hold for the orbital velocity $v^{(\Phi)}$ as well. But this is not the case. By a detailed analysis of the Boyer-Lindquist functions I have shown that the monotonic behaviour of $v^{(\Phi)}$ breaks down for $a > 0.9953$ at small, but stable *r* and $v^{(\Phi)}$ develops a minimum-maximum structure (c.f. Fig. 1). This can be taken as an indication that these orbits are no longer stable despite the fact that they satisfy the formal Bardeen stability criterion.



The min-max behaviour of $v^{(\Phi)}(r)$ suggests some oscillatory behaviour, which is supported by the finding that for *a=0.99616* the rate of change of $v^{(\Phi)}$ per unit length of *r* equals the radial epicyclic frequency at *r=1.546*. This resonance stabilizes the motion of the particle in an oscillating mode. Furthermore at exactly these *a* and *r* the vertical and the radial epicyclic frequencies are in a 3:1 resonance. Black holes with *a=0.99616* should show quasi-periodic oscillations with a 3:1 ratio, possibly a 3:2:1 structure, if harmonics or beat frequencies are effective.



I have associated the quasi-periodic oscillations found so far in three bright, independent flares from the Galactic Center black hole Sgr\,A* (Genzel et al. 2003 in the NIR, Aschenbach et al. 2004 in *Chandra* and *XMM-Newton* data) with this effect. These flares show the 3:2:1 frequency ratio (c.f. Fig. 2). Since *a* and *r* are fixed in general relativity units by the theory the mass of the black hole is given by picking just one of the three frequencies. For Sgr A* the black hole mass is (3.28±0.13)×106 M_O, and of course, *a=0.99616*. Group #4 shows the expected Kepler frequencies (K) at *r=1.546* (r), the marginally stable orbit (ms) and at the event horizon (h). The measurements indicate the presence of quasi-periodic light-variations from below the marginally stable orbit close to the event horizon.

For *a=0.99616* and *r=1.546* commensurable orbits may exist which would change the mass-frequency-relation. The analysis shows that for *a=0.99616* just one additional orbit with a commensurable orbital period exists at *r=3.919*. The ratio of the Kepler frequencies is 3:1 and the ratio of the frequencies of the vertical and radial epicyclic modes is 3:2. In this case quasi-periodic oscillations would occur at two frequencies with a ratio of 3:2. Twin high-frequency 3:2 quasi-periodic oscillations have been found in galactic black hole binaries of stellar mass, which for three of them have been roughly determined dynamically. For all three systems known the agreement is very good, but the accuracy of the mass determination is significantly improved down to the few percent level given by the frequency measurements. A general discussion of the implications of this new effect are given by Aschenbach, 2004, A&A, in press.

B. Aschenbach