Wave dispersion relation of two-dimensional plasma crystals in a magnetic fiel

The wave dispersion relation in a two-dimensional strongly-coupled plasma crystal is studied by theoretical analysis and molecular dynamics simulation taking into account a constant magnetic field parallel to the crystal normal. The expression for the wave dispersion relation clearly shows that high- and low-frequency branches exist as a result of the coupling of longitudinal and transverse modes due to the Lorenz force acting on the dust particles.

Micro particles that are introduced into a plasma environment charge up to a high negative value due to ion and electron bombardment. As a result of the strong Coulomb interaction between the particles often so-called “plasma crystals” are built up. The inter-particle potential is considered to be of the Yukawa type. There have been many theoretical and experimental studies on wave phenomena in “plasma crystals”. In these studies two wave modes, named “longitudinal dust lattice waves” and “transverse dust lattice waves”, have been identified and extensively analyzed. However, the influence of an external magnetic field on the wave dispersion has been considered barely, although magnetic fields play an important role for many common complex plasma environments. In this study, we analytically derive the wave dispersion relation in a two-dimensional plasma crystal under magnetic field influence, where the wave propagation and particle displacement are perpendicular to the magnetic field $\mathbf{B}$ [1]. The dispersion relation can be expressed by

$$\left[\omega^2 - 2\sum_{m=1,m\neq i}^{N} K_{x\,m}^{\;i} \sin^2\left(\frac{k_x x_0^m}{2}\right)\right]\left[\omega^2 - 2\sum_{m=1,m\neq i}^{N} K_{y\,m}^{\;i} \sin^2\left(\frac{k_x x_0^m}{2}\right)\right] - \omega_c^2\omega^2 = 0, \qquad (1)$$

where $\omega_c = |QB_z/m_d|$ is the cyclotron frequency of the dust particles.
In the limit of a vanishing magnetic field, ω_c = 0, we get the two well known, independent longitudinal and transverse wave modes from Eq.(1). When we apply a magnetic field though, the longitudinal and transverse modes in the plasma crystal are coupled due to the Lorentz force. Then, the dispersion relation has a cut-off at ω_c in the limit of long wave lengths where $k_x \to 0$.
Fig.1 show the wave spectrum due to thermal motion of the dust particles in a simulated plasma crystal under magnetic field, where dash lines are from Eq.(1). We can see a high-frequency ω_H and low-frequency branch ω_L. The profile of the dispersion relations that we obtained from the thermal particle motion are in good agreement with our theoretical predictions given in Eq.(1).

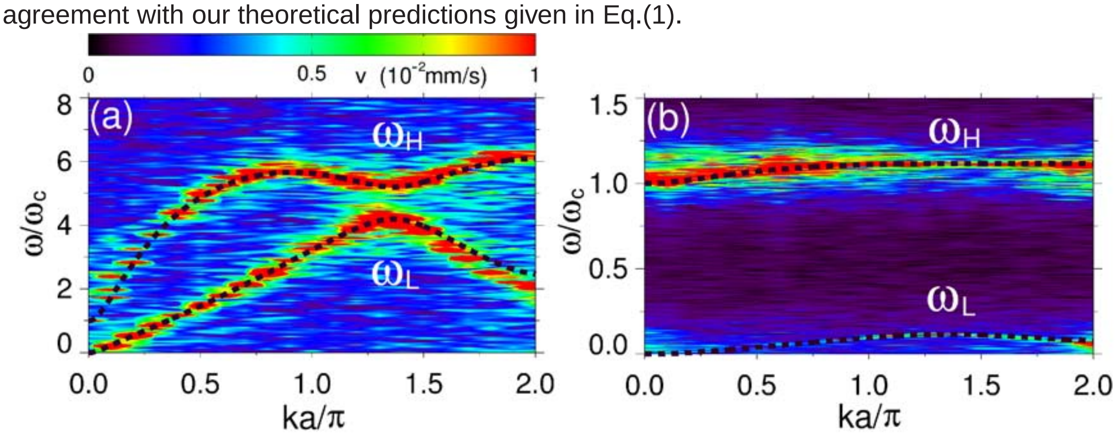

Figure. 1 ; Dependence of the dispersion relation on the magnetic field strength which is represented by the parameter ω_c/ω_0, where ω_c is the dust cyclotron frequency and ω_0 the dust plasma frequency defined as $\omega_0 = \sqrt{Q^2/4\pi\varepsilon_0 m_d a^3}$, where a is the mean inter-particle distance. Figure (a) and (b) are for $\omega_c/\omega_0 = 0.52$ ($\omega_c/\omega_0 < 1$; a weakly-magnetized plasma crystal), and for $\omega_c/\omega_0 = 6.85$ ($\omega_c^2/\omega_0^2 >> 1$; a strongly-magnetized plasma crystal), where a screening parameter $\kappa = a/\lambda_D$ is 1.0.

[1] G. Uchida, U. Konopka, and G. Morfill, Phys. Rev. Lett. **93**, 155002 (2004).

G. Uchida, U. Konopka, and G. Morfill