# Unveiling non-linearities via the Structure Analysis of Fourier Phase Maps

We present a method to detect non-linearities based on the characterization of the structural features of the Fourier phase maps. A Fourier phase map is a 2*D* set of points $M=\{\phi_{\vec{k}},\phi_{\vec{k}+\vec{\Delta}}\}$ where $\phi_{\boldsymbol{k}}$ is the phase of the ***k***-mode of the Fourier transform and **Δ** a phase shift. The phase maps are analyzed using the spectrum of weighted scaling indices to detect phase coupling. We propose a test of significance based on the comparison of properties of phase maps created from the original data and surrogate realizations. We show that the method reveals new features and assess the performance of surrogate data generating algorithms.

The Fourier phases are powerful indicators of the structure of a data set. Studies of phase coupling are found in astrophysics where the aim is to test for non-Gaussian signatures in the Cosmic Microwave Radiation Background[1] (CMB). We propose a method to detect non-linearities which uses the method of surrogates[2] to generate an ensemble of data sets which mimic the linear properties of the original data however wiping out higher-order correlations. Then, we create for both the original and the surrogate data phase maps which are subsequently characterized by means of the spectrum of weighted scaling indices[3]. The scaling index $\alpha_i$ is a local quantity that characterizes the scaling behavior of a point distribution in the neighborhood around $x_i$. For instance, for almost uniform 2*D* point distributions all α-values will be around 2. We apply our method to two different time series, namely the *z*-component of the Lorenz system in a chaotic regime and the logarithmic daily returns of the Dow-Jones for the period 1930-2003. The surrogate method destroys higher-order properties using of a phase randomization procedure. Thus, the phase maps of surrogate data should be more uniform. We propose a test of non-linearities based on the frequency distribution of α values *P*(α). We generate 20 surrogate data sets and create for the original

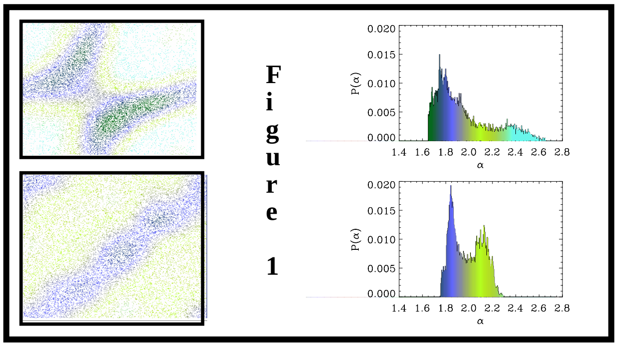


Figure 1

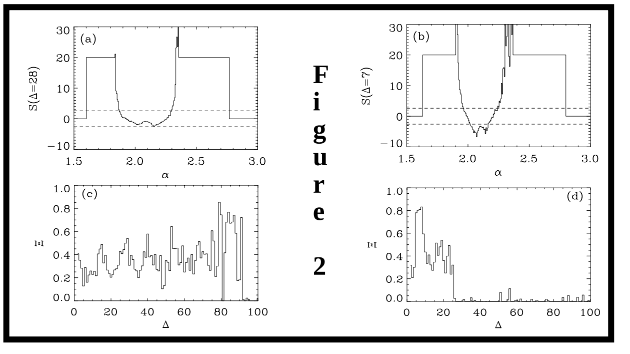


Figure 2

and the surrogate data phase maps for phase shifts $\Delta=\{\Delta_1,\ldots,\Delta_m\}$ where $\Delta_m$ is the maximum phase shift considered. The significance is defined through the following expression $S=\left(P_o(\alpha,\vec{\Delta}_l)-\langle P_s(\alpha,\vec{\Delta}_l)\rangle\right)/\sigma\left(P_s(\alpha,\vec{\Delta}_l)\right)$ where <> and σ are the mean value and standard deviation over the surrogate ensemble. The test result is positive if $S < -2.6$ for $1.9 < \alpha < 2.1$ or $S > 2.6$ for α elsewhere. This condition determines the region of the phase map where significant differences between the original and surrogate phase maps exist. If this region is tiny, we can not state that the original and the surrogate phase maps are different. Then, we define a new quantity Ξ through $\Xi=\sum_i P_o(\alpha_i,\Delta)$ where $\alpha_i$ are the values which fulfill the condition. Ξ is then the probability to significantly distinguish between the original and the surrogate phase maps. Upper row (lower row) of Fig.1 show color-coded α–images of the phase map for the Lorenz system (Dow Jones) and the *P*(α) distribution. It is possible to locate on the images the different α values. Note that *P*(α) indicates that the surrogate phase map is more uniform. Upper row of Fig. 2 show examples of *S* for the Lorenz system (left) and Dow Jones (right). The dashed lines at ± 2.6 indicate the confidence interval. In both cases, there is a wide range of α-values with high *S* values. Lower row of Fig. 2 show Ξ for the Lorenz system (left) and Dow Jones (right). For the Lorenz series Ξ ~ 0.4 while for the Dow Jones only Δ values lower than 25 lead to positive test results. In summary, the Lorenz system showed signatures of non-linear behavior at all Δ scales. However, for the Lorenz system surrogates are not always free from phase coupling. Then, our method can assess the quality of surrogate data. Our results indicate that a typical scale of non-linearities $\Delta_c$ exists for the Dow Jones. This may help to understand the financial-price dynamics. For weak non-linearities as in the CMB of radiation, the use of surrogate data sets and local structure measures to assess phase maps may be more appropriate than the use of global measures.

**References:**

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*R. Monetti, W. Bunk, F. Jamitzky, C. Räth, and G. Morfill*