

Cosmic censorship for phantom energy

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There is threat of a black hole transformation into the naked singularity by accretion of phantom energy

The third law of black hole thermodynamics is violated in the test fluid approximation for the process of phantom energy accretion onto a rotating or electrically charged black hole

We demonstrate this by using new analytical solutions for

- ① spherically symmetric stationary accretion of perfect fluid with arbitrary equation of state in the Reissner-Nordström metric
- ② stationary accretion of ultra-hard fluid onto the Kerr-Newman black hole

The black hole mass is continuously decreasing in phantom energy accretion process, but both the angular momentum and electric charge are remaining constant

In result, a black hole reaches the extreme state during a finite time with a threat of transformation into the naked singularity and violation of the cosmic censorship conjecture

Our speculative assumption, however, is that the cosmic censorship conjecture remains valid even for phantom energy case, if one takes into account the back reaction of an accreting fluid onto a near extreme black hole

Some hint for the validity of this hypothesis comes from the specific case of the ultra-hard fluid accretion onto the electrically charged or rotating black hole

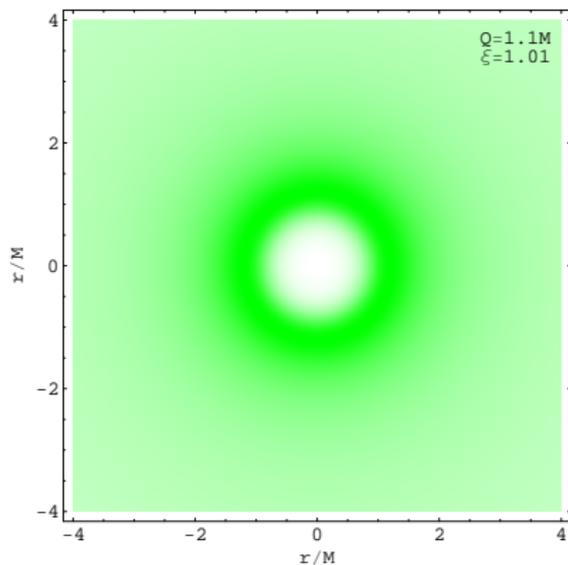
In this case the energy density of accreting fluid diverges at the event horizon of extreme black hole, thus violating the test fluid approximation

Some examples of exact analytic solutions

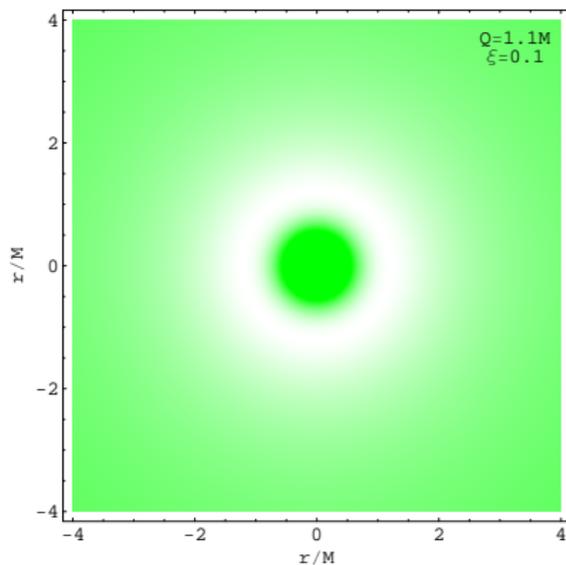
Static Chaplygin atmosphere around Reissner-Nordström naked singularity

$$\rho = -\alpha/\rho, \quad \xi = \rho_\infty^2/\alpha, \quad \alpha > 0$$

Chaplygin dark energy

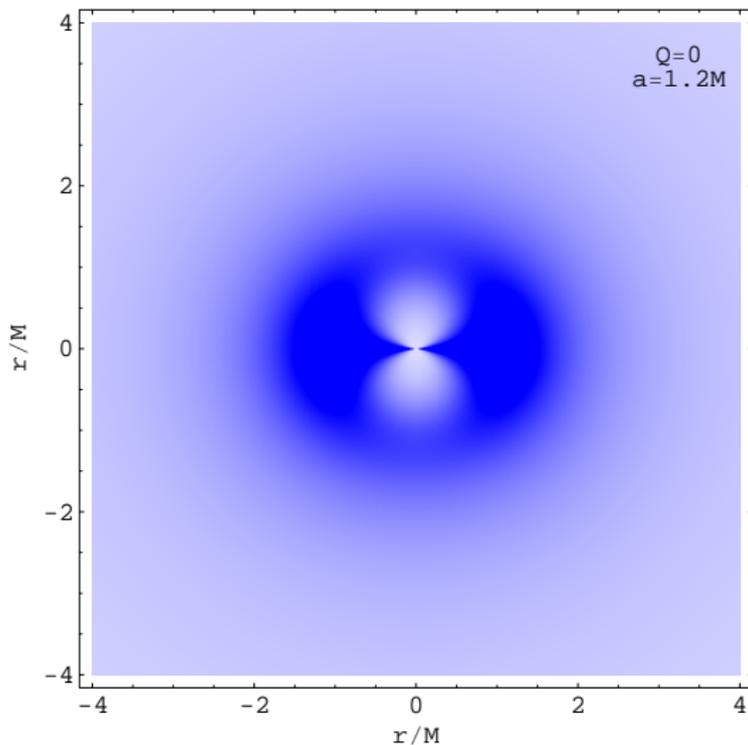


Chaplygin phantom

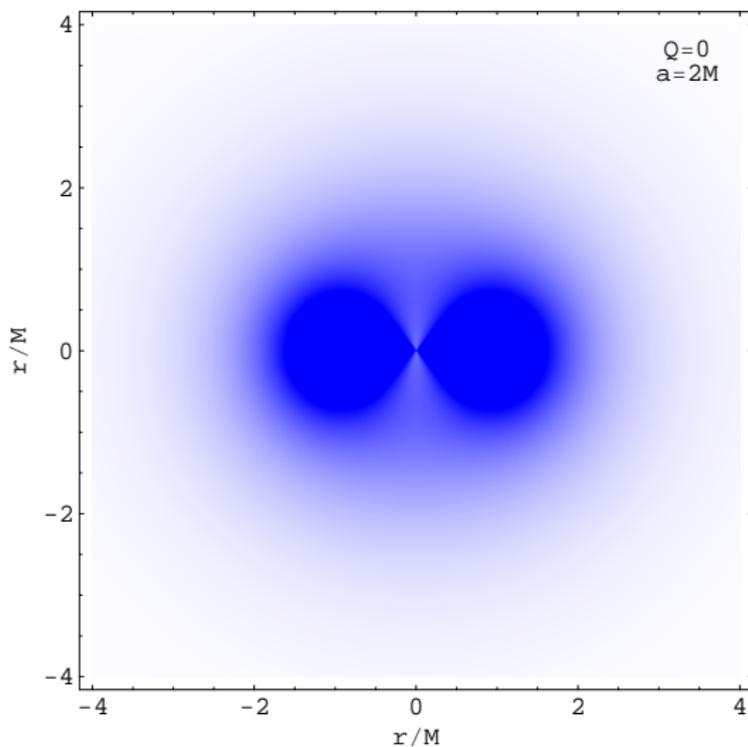


Density plots of radial distribution $\rho(r)$

Stationary atmosphere of ultra-hard perfect fluid (massless scalar field) around the Kerr naked singularity



Scalar atmosphere around the Kerr naked singularity



Energy density distribution of static scalar field $\phi = \phi(\mathbf{r})$ with a finite total mass $M_f \ll M$. There is no perfect fluid analogue to this solution.

Approaching to the extreme black hole state

$$Q = \text{const}, \quad J = Ma = \text{const}, \quad a^2 + Q^2 \rightarrow M^2$$

Reissner-Nordström case

$$\int_0^{t_{\text{NS}}} \dot{M} dt = Q - M(0), \quad t_{\text{NS}} = \frac{q^3 - 3q^2 + 2 - 2(1 - q^2)^{3/2}}{3q^4} \tau$$

$$q = Q/M(0), \quad \tau = -\{4\pi[\rho_\infty + p(\rho_\infty)]M(0)\}^{-1}$$

Kerr case

$$\int_0^{t_{\text{NS}}} \dot{M} dt = \sqrt{J} - M(0), \quad t_{\text{NS}} = \frac{1}{6\tilde{a}^{1/2}} \left[1 - \frac{1 - \sqrt{1 - \tilde{a}^2}}{\tilde{a}^{3/2}} + 2F\left(\frac{1}{2} \arccos \tilde{a}, 2\right) \right] \tau$$

$\tilde{a} = J/M^2(0)$, $F(\phi, k)$ — elliptic integral of the first kind

Time t_{NS} is finite, and so the third law of black hole thermodynamics is violated in the test fluid approximation!

But, the test fluid approximation may be violated at extreme case

Conclusions and questions

- New analytical solutions for stationary distribution of perfect fluid in the Reissner-Nordström and Kerr-Newman metrics
- Electrically charged or rotating black holes evolve during a finite time to the extreme state by accretion of test phantom fluid $a^2 + Q^2 = M^2$
- Static atmosphere of fluid around electrically charged or rotating naked singularities without influx $a^2 + Q^2 > M^2$
- Divergence of fluid energy density at the event horizon of extreme black hole due to violation of test fluid approximation!? $\rho(r_+) \rightarrow \infty, a^2 + Q^2 \rightarrow M^2$
- Back reaction of fluid on the near extreme black hole prevents its transformation into the naked singularity in accordance with the cosmic censorship conjecture!? $a^2 + Q^2 \leq M^2$
- Babichev, Chernov, Dokuchaev & Eroshenko, arXiv:0806.0916
ibid. arXiv:0807.0449