Is the torus topology compatible with the WMAP data?

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Introduction

It is a mathematical fact that fixing the spatial curvature does not determine uniquely the topology und thus the shape of the universe. Only if it is assumed that the spatial section is simply connected, it follows that the topology is (in the flat case) given by the infinite Euclidean space. Exactly this assumption is made in the concordance model [4].
It is the purpose of this poster to demonstrate that the WMAP data [5] are compatible with the possibility that we live in a 'small universe' having the shape of a flat equilateral 3-torus with side length $L \approx 4L_H$ corresponding to a volume of $\approx 5 \times 10^3 \text{Gpc}^3$.

Large scale temperature correlations of the WMAP data compared with the concordance model

2-point temperature correlation function:

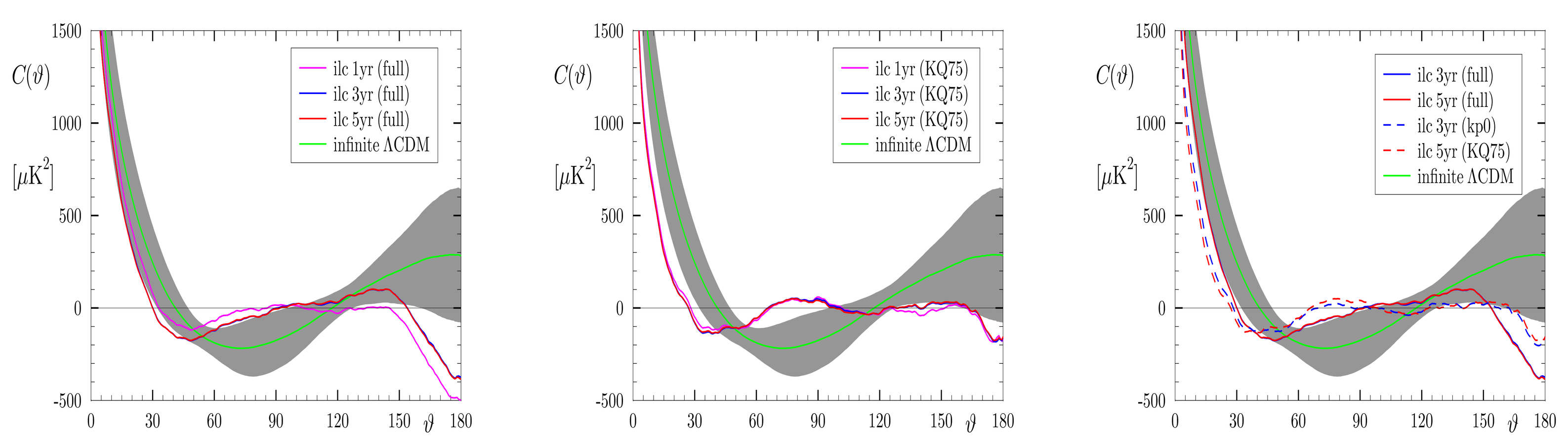
$$C(\vartheta) := \langle \delta T(\hat{n}) \delta T(\hat{n}') \rangle \quad \text{with } \hat{n} \cdot \hat{n}' = \cos\vartheta$$

Data: $\langle \ldots \rangle$ denotes the average over all directions separated by an angle ϑ in the admitted sky area.

Model: $\langle \ldots \rangle$ denotes the ensemble average over the primordial fluctuations. (If the model is statistically anisotropic in addition an average over all directions is done.)
$\Longrightarrow \quad C^{\text{model}}(\vartheta) = \sum_l \frac{2l+1}{4\pi} C_l^{\text{model}} P_l(\cos\vartheta)$
C_l^{model}: multipole moment of the model in respect of the ensemble average.

WMAP data vs. concordance model

The correlation functions of the WMAP ILC 1yr, 3yr and 5yr data [5] are compared with the concordance model [4]. The 1σ cosmic variance of the model is displayed as a grey band.



The correlation function of the full ILC map of the 1yr data is different from that of the 3yr and 5yr data (modifications of the method). With the KQ75 mask there are only small differences between the three correlation functions (middle figure).

The infinite ΛCDM model (concordance model) does not give a good description of the full ILC map on large scales (left/right figure) and is a good deal worse in the case of the KQ75 mask except of the largest scales (middle/right figure).

Copi et.al [2] infer from their investigations that only 0.025% of realizations of the concordance model can describe the low correlations on separation scales greater than 60° in the WMAP data admitted by the KQ75 mask (middle figure).

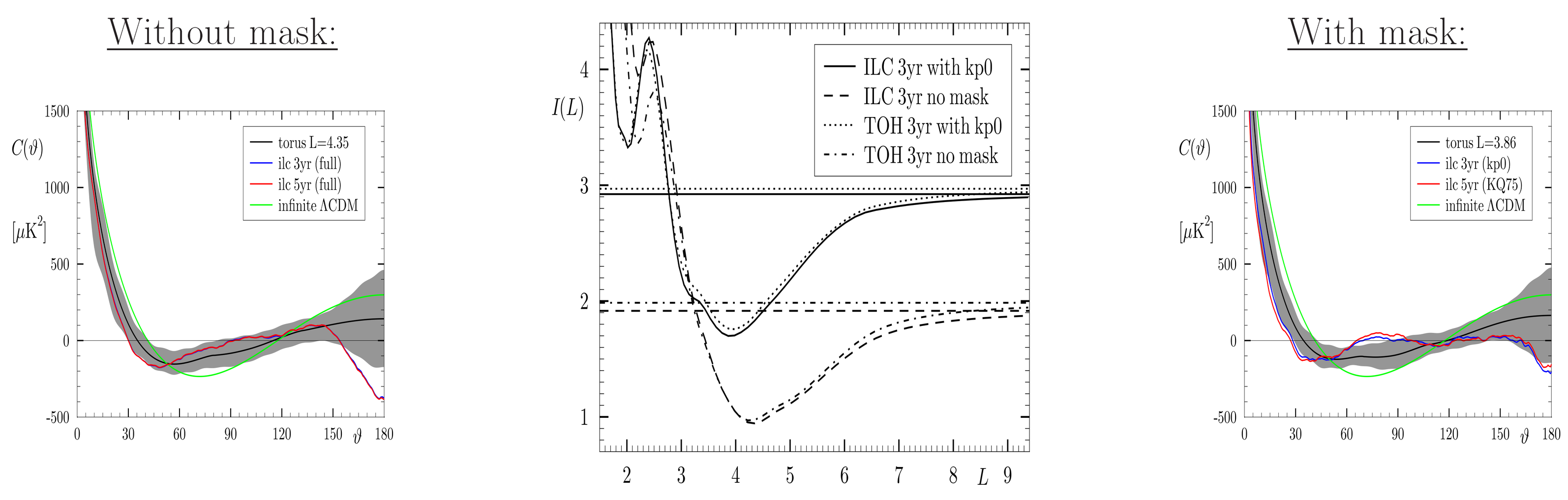
Does a finite torus model describe the data on large scales better than the concordance model?

To answer this question we have introduced

the integrated weighted temperature correlation difference [1]:

$$I := \int_{-1}^{1} d(\cos\vartheta) \frac{(C^{\text{model}}(\vartheta) - C^{\text{obs}}(\vartheta))^2}{Var(C^{\text{model}}(\vartheta))}$$

With this quantity we have compared the 3yr data with the infinite ΛCDM model (horizontal lines in the following middle figure) and the finite equilateral torus model for side lengths $L \approx 1.5 - 9.0 L_H$, $L_H := \frac{c}{H_0}$ (in the figures $L_H \equiv 1$). The ILC map from the WMAP team (ILC) and from Tegmark et al. (TOH) [6] are investigated with kp0 mask and without mask.



Without mask: the torus model in the vicinity of $L = 4.35L_H$ describes the data better than the concordance model (left figure).

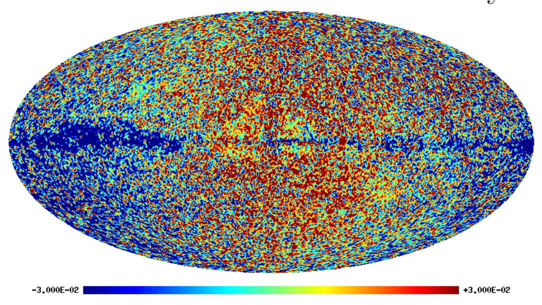
With mask: the torus model in the vicinity of $L = 3.86L_H$ describes the data better than the concordance model (right figure).

Error estimate of the methods used to reduce the foreground contamination

In the follwing we investigate the quality of the measured CMB maps. This is important for the so-called 'circle-in-the-sky' search.

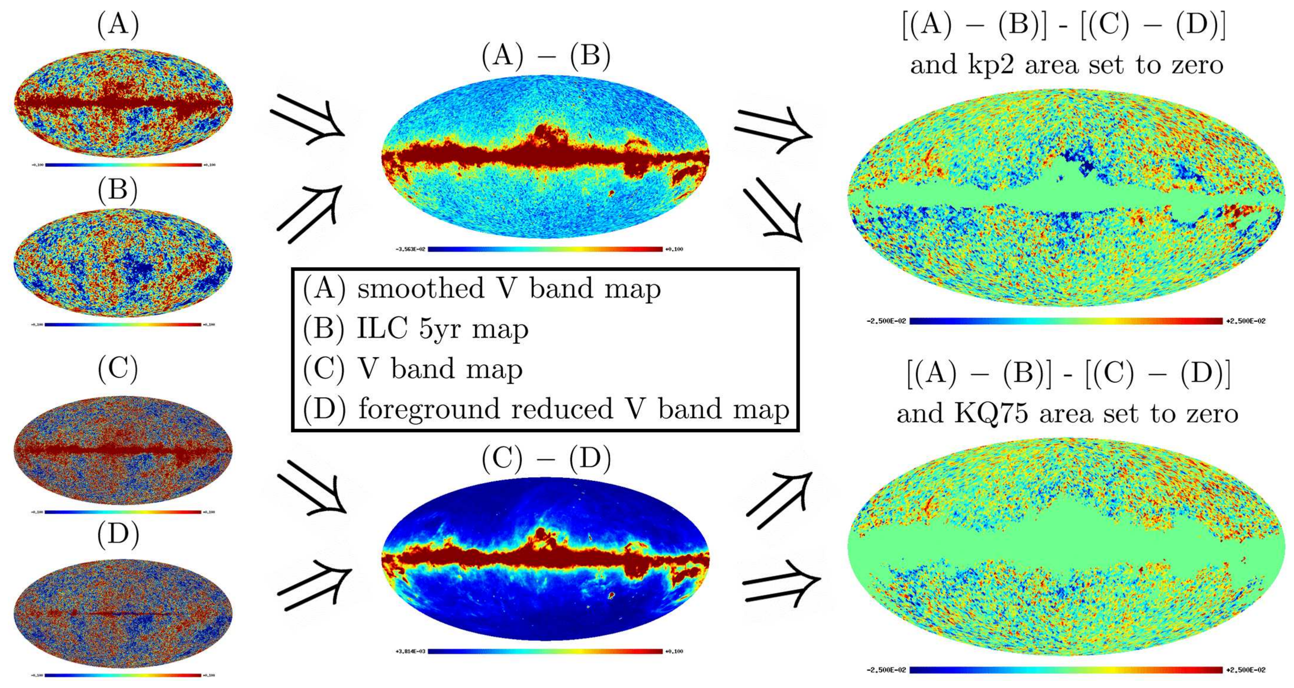
Maps to illustrate the uncertainty of the methods reducing the foreground contamination:

i) Difference of the WMAP ILC 5yr map and 1yr map



In these maps there are differences up to 30μK and more (probably because of the change of the method and less noise in the 5yr data).

ii) Comparing the methods reducing the foreground contamination (template and ILC):



There are different results from the methods up to 25μK outside the KQ75 mask and even larger differences outside the kp2 mask. Which method is correct? Can the ILC method reduce the signal of our galaxy and give the correct CMB signal within the KQ85 (kp2) mask?

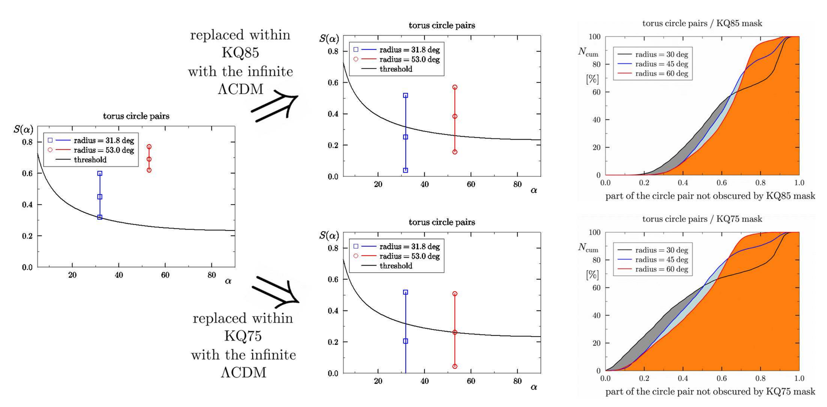
We have no definite answer to these questions!

Is the torus model excluded by the circle-in-the-sky search?

A torus model of side length $L = 3.86 - 4.35L_H$ predicts 3 circle pairs with radius $54° - 49°$ and 6 circle pairs with radius $35° - 22°$ (which are back-to-back and without shift) in the CMB map due to the periodicity condition of the torus.
Expanding the temperature $\delta T_i(\phi)$ along a circle in a Fourier series $\delta T_i(\phi) = \sum_m T_{im} e^{im\phi}$, $0 \le \phi \le 2\pi$, allows one to define the m-weighted circle signature for two circles i and j having a radius α as [3]

$$S_{ij}(\alpha, \beta) := \frac{2\sum_m m T_{im} T_{jm}^* e^{im\beta}}{\sum_m m \left(|T_{im}|^2 + |T_{jm}|^2 \right)} \quad (\beta\text{: shift parameter of the circles})$$

Cornish et al. [3] have done such a search in the WMAP 1yr data using a superposition of the foreground reduced Q, V and W band maps outside the kp2 mask and the ILC map inside the kp2 mask. They have concluded that models with back-to-back circle pairs with radius greater than 25° are excluded.
$\Longrightarrow$ This would exclude the torus model.
Base of this conclusion is a simulation of a CMB map for a torus model with detector noise. This map is investigated for circle pairs and used to compare with the measured data. The conclusion of Cornish et al. depends on the assumption that the errors caused by the methods used to reduce the foreground have no effect on their result.
To give an idea what can happen we have replaced in 38 torus models the temperature fluctuations inside the KQ75 or the KQ85 mask by that of the concordance model. The threshold of false positive circle pairs [3], minimum, mean and maximum value of 228 circle pairs with radius 31.8° and 114 circle pairs with radius 53.0° of the torus model are displayed in the figures (orientation fixed). Further we have illustrated the effect of the mask on the circle pairs for all orientations (right figures).



KQ85 mask: To miss circle pairs there is a chance of about 60% in the case of the radius 32° and 30% in the case of the radius 53°.

KQ75 mask: To miss circle pairs there is a chance of about 60% in the case of the radius 32° and 50% in the case of the radius 53°.

The results and the conclusion depend on the fraction of the circle pairs which is affected by the mask.

Conclusion

The result of the analysis of the correlation function $C(\vartheta)$ prefers the finite equilateral torus model to the concordance model.
Although no circle pairs have been found, the torus model is nevertheless **not definitely excluded**. To exclude multiconnected universes the errors caused by the methods reducing the foreground contamination have to be estimated realistically since the errors propagate into the search for the circle pairs. This has not been done yet.

References

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