A fitting formula for the Convergence Power Spectrum Covariance in Weak Lensing

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Motivation

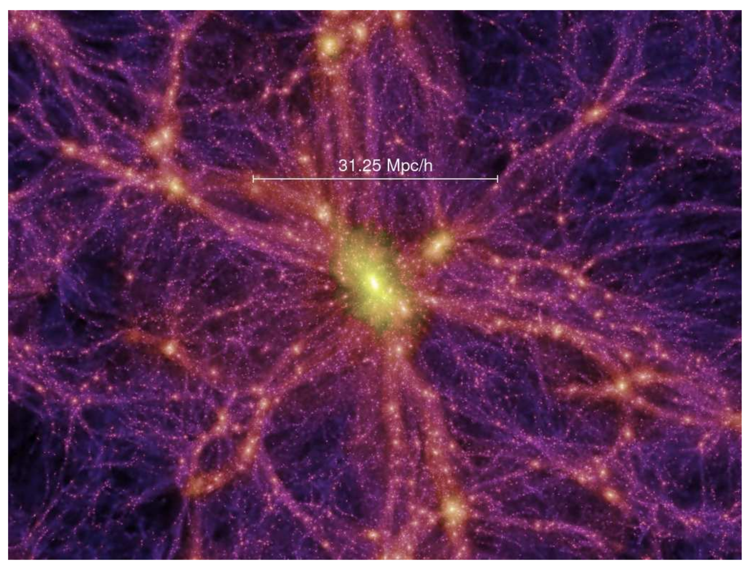

FIGURE 1: Distribution of Dark Matter at $z = 0$ according to the Millennium Run simulation (source: V. Springel et al. '05)

Short History of Structure Formation in the Universe:

- initially small and smooth primordial density perturbations are amplified through gravitational instability and form Large Scale Structure (LSS)
- primordial fluctuations were distributed according to homogeneous Gaussian random processes
- for Gaussian random fields all statistical information is encoded in the power spectrum!
- during most of the evolution inhomogeneities can be treated as linear perturbations (as for the CMB analysis)
- but: as perturbations grow and become non-linear, different modes of the density field become coupled
- this leads to non-Gaussian signatures in the matter density field
- since weak lensing probes low redshift regime and intermediate scales non-Gaussianities must be taken into account!

Why do we consider the Covariance?

Covariance of statistical quantity x is defined as:

$$C(x_i, x_j) \equiv \langle \langle x_i \rangle - x_i \rangle \langle \langle x_j \rangle - x_j \rangle \,, \tag{1}$$

where $\langle . \rangle$ denotes the ensemble average of x.

- gives error on the quantity x (diagonal elements) and amount of correlation between the different x_i (off-diagonal elements)
- generates in case of the convergence power spectrum estimator $\hat{P}_\kappa(l)$ non-linear, higher-order correlations
- is essential for the likelihood and Fisher matrix analysis of cosmological parameter estimation

Covariance Matrix for Weak Lensing

$$C(\hat{P}_\kappa(l_i), \hat{P}_\kappa(l_j)) = \frac{1}{A}\left[\frac{(2\pi)^2}{A_{\rm r}(l_i)} 2P_\kappa^2(l_i)\delta_{ij} + \bar{T}_\kappa(l_i, l_j)\right] \tag{2}$$

where A is the survey volume, $A_{\rm r}(l_i)$ the shell area and $\hat{P}_\kappa(l)$ an unbiased power spectrum estimator. The covariance is decomposed into a Gaussian and a non-Gaussian part. $P_\kappa(l)$ is the convergence power spectrum and $\bar{T}_\kappa(l_i, l_j)$ is the bin-averaged convergence trispectrum which are defined as

$$P_\kappa(l) \equiv \int_0^{w_{\rm H}} {\rm d}w\, G^2(w)\, P_\delta\left(\frac{l}{f_K(w)}, w\right) \,, \tag{3}$$

$$\bar{T}_\kappa(l_i, l_j) \equiv \int_{{\rm r},l_i} \frac{{\rm d}^2 l_1}{A_{\rm r}(l_i)} \int_{{\rm r},l_j} \frac{{\rm d}^2 l_2}{A_{\rm r}(l_j)} T_\kappa(\mathbf{l_1}, -\mathbf{l_1}, \mathbf{l_2}, -\mathbf{l_2}) \,, \tag{4}$$

where the weight function $G(w)$ sets the geometry of the background sources (see [1, 2, 5] for more details).

The Halo Model (HM)

Motivation:

- need to model non-linear, higher-order correlation functions
- Perturbation Theory description of gravitational clustering breaks down around $l \simeq 100$
- N-Body simulations of LSS are computationally very costly
- HM provides simple description for semi-analytic computation of the power- and trispectrum (see Cooray & Sheth [3] for more details)

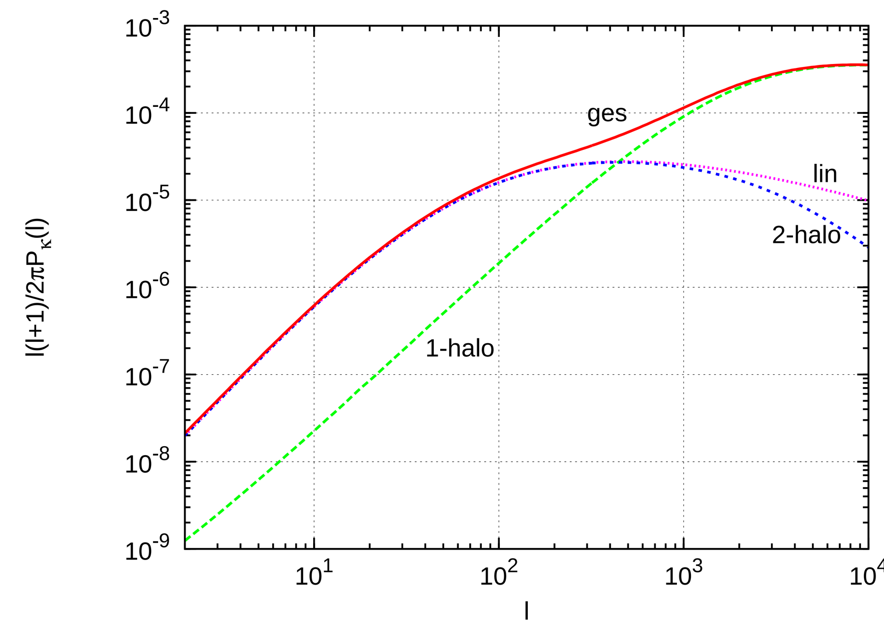

FIGURE 2: Dimensionless convergence power spectrum $\Delta_\kappa(l)$ as predicted by the HM. Perturbation theory breaks down around $l \simeq 100$. Typical splitting into two regimes: the 1-halo term is dominant on small scales and the 2-halo term on large scales.

Ingredients:

- idea: Dark Matter is distributed in spherically symmetric halos
- physics is split into two regimes:
  - small scales: spherical collapse model → halo profile
  - large scales: Perturbation Theory → spatial distribution of halos
- halo abundance (Sheth and Tormen mass function)
- halo clustering (Peak-Background-Split → halo bias)
- density profile of the halo according to universal profile (NFW)

Comparison with Simulations

For our work we use the N-body simulations of the VIRGO-collaboration published by Jenkins et al. (see [4]). The set of cosmological parameters used for the comparison with the halo model and the most important parameters for the setup of the simulation are:

Ω_m	Ω_Λ	h	Γ	σ_8	z_s	$L_{\rm box}/h^{-1}$ Mpc	$N_{\rm par}$	$m_{\rm par}/M_\odot$
0.30	0.70	0.7	0.21	0.9	1.0	141.3	256^3	1.4×10^{10}

From the simulations we use 200 convergence maps with a field view of $0.5° \times 0.5°$ and consider 20 bins of width $\Delta l \simeq 720$ starting at $l \simeq 720$.

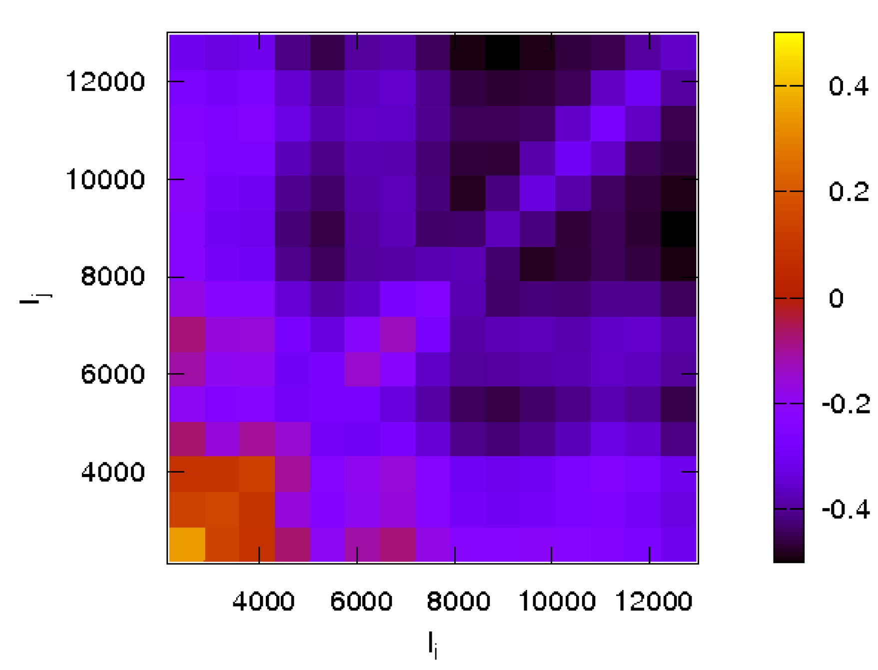

FIGURE 3: Relative deviation of halo model to simulation power spectrum covariance.

Fitting formula for the Covariance

A suitable quantity for a fitting formula is

$$\bar{\beta}(l_i, l_j) \equiv \frac{\bar{T}_\kappa(l_i, l_j)}{[P_\kappa(l_i) P_\kappa(l_j)]^{3/2}} \tag{5}$$

since it is well behaved on scales smaller than $l < 1000$ and is independent of the binning scheme chosen. The Gaussian contribution can be easily added on top in order to obtain the full covariance.

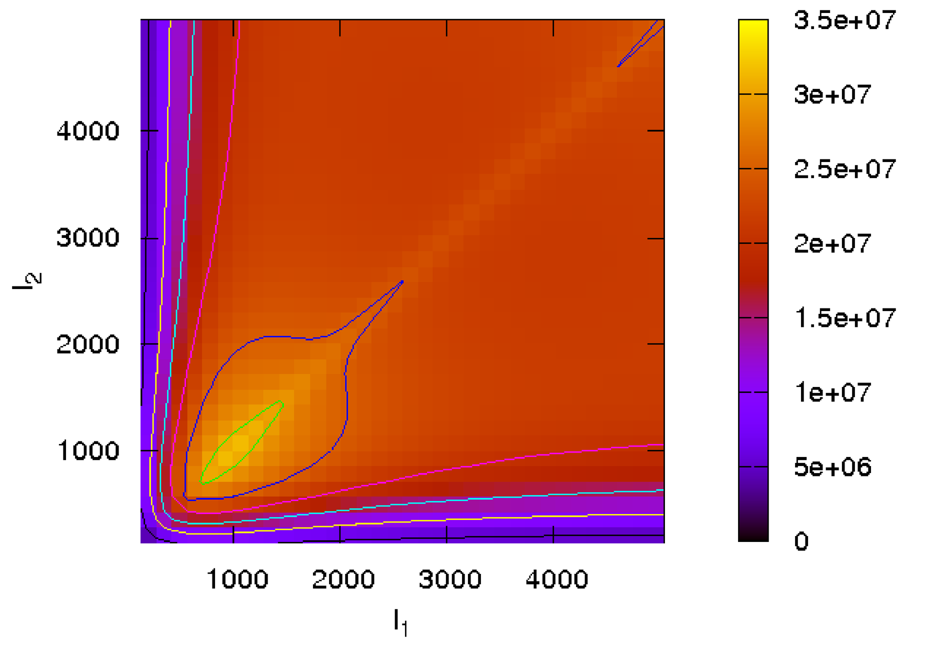

FIGURE 4: Fitting quantity $\bar{\beta}$ against wave-numbers (l_i, l_j).

As fitting function we choose a second order polynom in the dimensionless convergence power spectrum, such that

$$\frac{\bar{\beta}(l_i, l_j)}{2\pi^2} = a_0 + a_1\Delta_{\rm min} + a_2\Delta_{\rm max} + a_3\Delta_{\rm min}^2 + a_4\Delta_{\rm min}\Delta_{\rm max} + a_5\Delta_{\rm max}^2 \,, \tag{6}$$

where $\Delta_{\rm max} \equiv \max(\Delta(l_i), \Delta(l_j))$ and $\Delta_{\rm min} \equiv \min(\Delta(l_i), \Delta(l_j))$ denote dimensionless convergence power spectra. In order to apply the fitting formula to different cosmologies, we treat the parameters a_k as dependent on $\Omega_{\rm m}$ and σ_8. In this way we introduce 12 more parameters which we obtain by fitting different cosmological models to one fiducial.

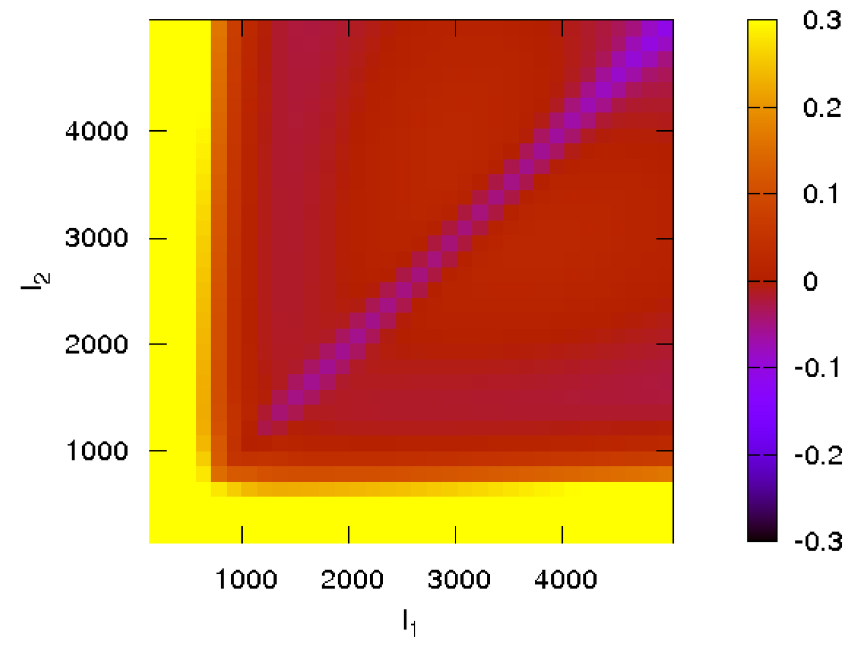

FIGURE 5: Relative deviation of fitting formula to HM.

Results

- on scales $l \lesssim 5000$ the HM differs around 20% from the simulations; on smaller scales the deviation can amount 50% or more
- on scales between $1000 < l < 5000$ the fitting formula deviates 10% or less from the HM; along the diagonal the accuracy is 15% or better

References

[1] Bartelmann, M. & Schneider, P. 2001, Phys. Rep., 340, 291

[2] Cooray, A. & Hu, W. 2001, ApJ, 554, 56

[3] Cooray, A. & Sheth, R. 2002, Phys. Rep., 372, 1

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[5] Scoccimarro, R., Zaldarriaga, M., & Hui, L. 1999, ApJ, 527, 1