Max-Planck-Institut für extraterrestrische Physik

Model-independent tests for scale-dependent non-Gaussianities in the CMB

C. Räth[(1)], A.J. Banday[(2)]
(1) Max-Planck-Institut für extraterrestrische Physik, 85748 Garching, Germany
(2) Max-Planck-Institut für Astrophysik, 85748 Garching, Germany

Abstract

The detection of signatures for primordial non-Gaussianity (NG) in the CMB could provide a powerful means to rule out or support various inflationary scenarios, where some of them predict scale dependent non-Gaussianities [1-10]. Thus proving the (non-)existence of such features in the CMB can help to constrain the classes of conceivable models for inflation.
In this contribution we propose and discuss novel model-independent tests for scale-dependent non-Gaussianities using the method of constrained randomisation [11-14]. The basic idea of this formalism is to compute statistics sensitive to higher order correlations (HOCs) for the original data set and for an ensemble of so-called surrogate data sets, which preserve the linear properties of the original data and scale-dependent parts of the HOCs while all other correlations are subject to randomisation The partly randomisation of the HOCs is achieved by shuffling the Fourier phases in the Fourier domain in a controlled manner. If the computed measure for the original data is significantly different from the values obtained for the set of surrogates, one can infer that the original data contains scale dependent HOCs.
Using simulated CMB maps containing well-specified HOCs and Minkowski functionals (MF) as test statistic we demonstrate that surrogate-based tests can detect scale-dependent non-Gaussianities with high significances.

Model-independent tests for non-Gaussianity

General Scheme:

CMB data
↓
Constrained Randomisation
↓
Surrogate CMB maps with the same power spectrum/ partly the same Fourier phases

→ Calculation of statistical measures M sensitiv to higher order correlations → Statistical comparison in terms of e.g. significances S, Confidence levels, etc.

M derived from e.g.:
- Bispectrum
- N-point corr. Funct.
- Minkowski-functionals
- Wavelets
- Scaling indices
- Etc......

$$S = \frac{M_{Original} - \langle M_{Surrogate} \rangle}{\sigma_{M_{Surrogate}}}$$

By applying methods of constrained randomisation one can develop model-independent tests for non-Gaussianities.

Probing scale-dependent non-Gaussianities using surrogates

Idea: Two-step surrogate test:

First step:
Generate a surrogate image, in which only the phases for a certain k-range are interchanged => higher order correlations on these scales are wiped out.

Second step:
Generate three classes of surrogates for the surrogate image, in which
a) all phases are interchanged
b) The phases of the complementary k-range are interchanged
c) The phases of the same k-range are interchanged
(as a check: shuffling of already shuffled phases shouldn't have an effect)

Minkowski functionals (MF)

two-dimensional CMB data => three Minkowski functionals:

Area: $M_0(\nu) = \int_{R(\nu)} dS$

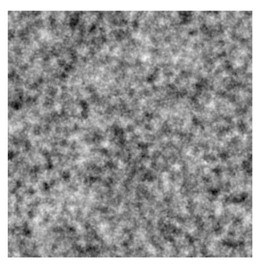
Circumference: $M_1(\nu) = \int_{\partial R(\nu)} dl$

Euler characteristic: $M_2(\nu) = \int_{\partial R(\nu)} \frac{dl}{r}$ of an excursion set $R(\nu)$

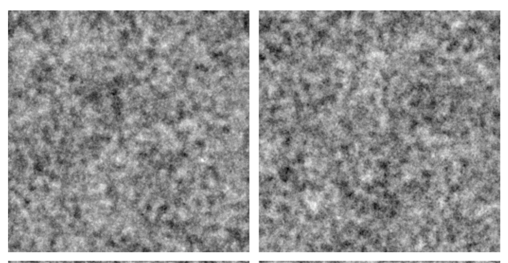
M_2 = # connected regions - # holes in the regions.

Information (of the sum) of *all* n-point correlation functions is contained in the MF

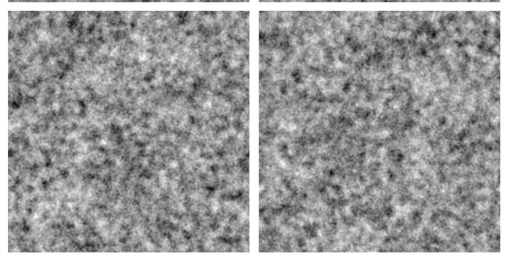
The data sets and their surrogates



Original simulated CMB map



First (left) and second (right) order surrogate for k_{cut} =20



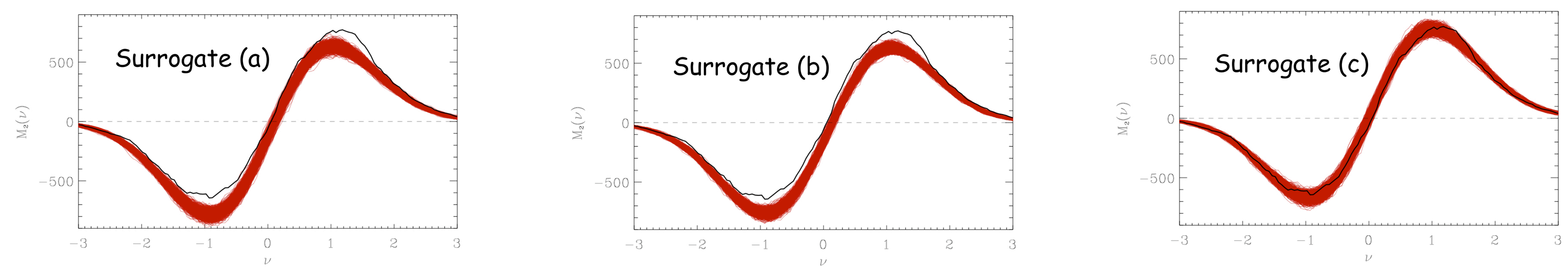
First (left) and second (right) order surrogate for k_{cut} =40

Non-Gaussian CMB-maps were simulated using the method proposed by Rocha et al. [15]. The intensities are rank-ordered remapped onto a Gaussian distribution, similarly the phases are remapped onto a set of uniformly distributed ones. For this preprocessed map several sets of first and second order surrogates are calculated:

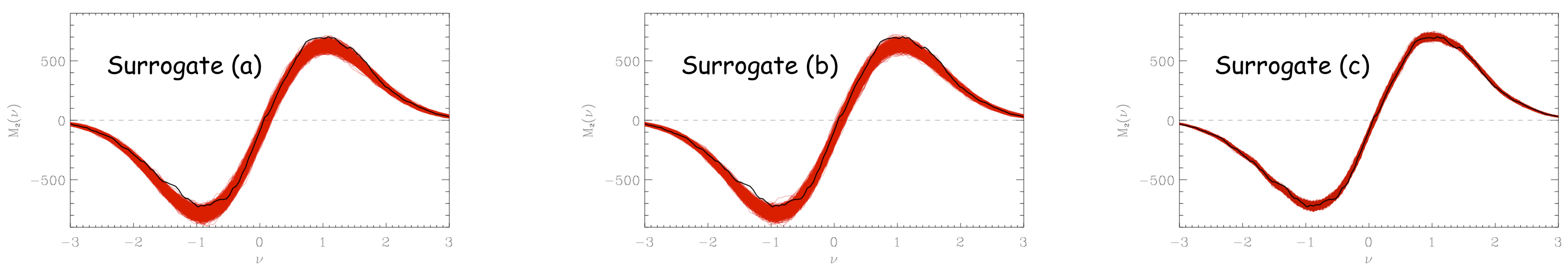
Original image: image size: 255 x 255
↓
First surrogate with partly randomised phases $\phi(k)$:
$k \in [0; 20], k \in [0; 40], \ldots, k \in [0; 120]$:
large scale HOCs destroyed => testing for small scale HOCs
$k \in [20;\infty], k \in [40;\infty], \ldots, k \in [120;\infty]$,
small scale HOCs destroyed => testing for large scale HOCs
↓
Surrogate (a) | Surrogate (b) | Surrogate (c)

Results (I.)

Euler characteristic for first and second order surrogates:



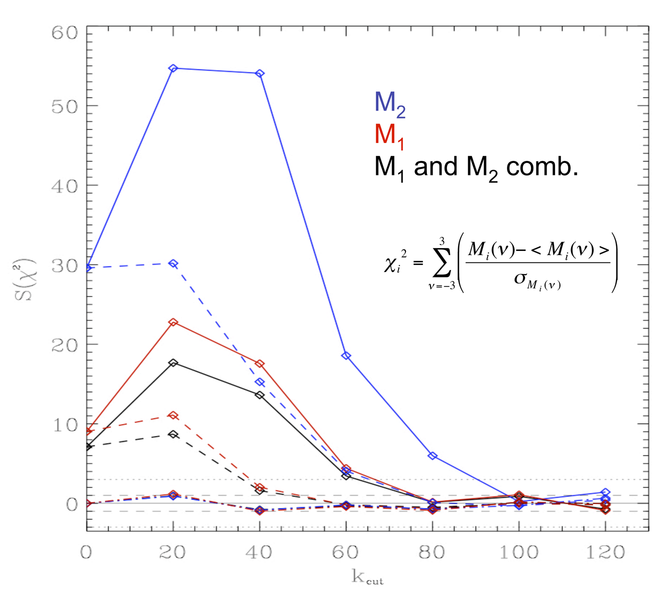
First order surrogate: $\phi(k)$, $k \in [0; 20]$ shuffled => Probing small scale NGs



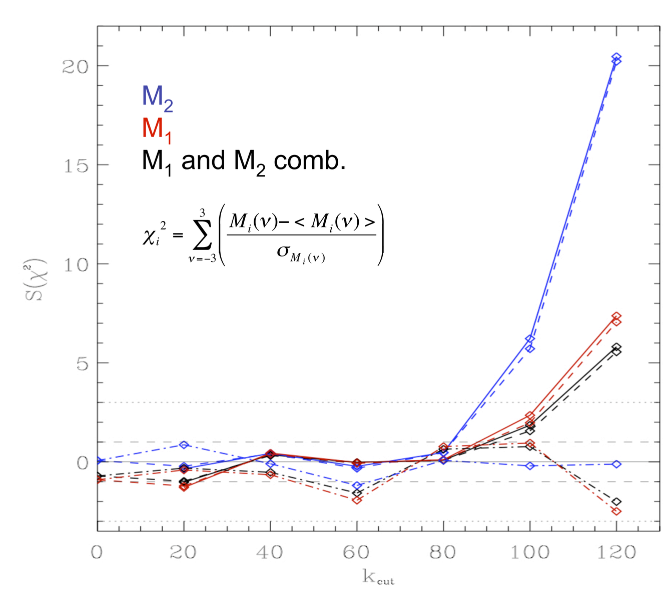
First order surrogate: $\phi(k)$, $k \in [100; \infty]$ shuffled => Probing large scale NGs

Results (II.)

Significances of diagonal χ^2-statistic derived from the MFs:



$\phi(k)$, $k \in [0; k_{cut}]$ shuffled



$\phi(k)$, $k \in [k_{cut}; \infty]$ shuffled

K-range sufficiently large: Significant detection of scale dependent NGs using MFs.

Conclusions - Outlook

- Surrogate data, which preserve the power spectrum and the intensity distribution with high accuracy *while partly randomising the phases* can be generated for 2D images using modified algorithms of constrained randomisation.
- Model-independent and highly sensitive tests for scale-dependent non-Gaussianities can be performed for both simulated and observational data.
- The combination of controlled non-Gaussian simulations and constrained randomisation allows for a systematic comparison of higher order statistics.

References

[1] A. H. Guth, Phys. Rev. D 23, 347 (1981).
[2] A. D. Linde, Phys. Lett. B 108, 389 (1982).
[3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).
[4] A. Linde and V. Mukhanov, Phys. Rev. D 56, 535 (1997).
[5] P. J. E. Peebles, Astrophys. J. Lett. 483, 1 (1997).
[6] F. Bernardeau and J.-P. Uzan, Phys. Rev. D 66, 103506 (2002).
[7] V. Acquaviva, N. Bartolo, S. Matarrese, and A. Riotto, Nucl. Phys. B 667, 119 (2003).
[8] C. Armendariz-Picon, T. Damour, and V. Mukhanov, Physics Letters B 458, 209 (1999).
[9] J. Garriga and V. F. Mukhanov, Physics Letters B 458, 219 (1999).
[10] M. Lo Verde, A. Miller, S. Shandera, and L. Verde, Journal of Cosmology and Astro-Particle Physics 4, 14 (2008).
[11] M. P. Pompilio, F. R. Bouchet, G. Murante, and A. Provenzale, Astrophys. J. 449, 1 (1995).
[12] C. Räth, W. Bunk, M. B. Huber, G. Morfill, J. Retzlaff, and P. Schuecker, Mon. Not. R. Astron. Soc. 337, 413 (2002).
[13] C. Räth and P. Schuecker, Mon. Not. R. Astron. Soc. 344, 115 (2003).
[14] J. Theiler, S. Eubank, A. Longtin, B. Galdrikian, and J. D. Farmer, Physica D 58, 77 (1992).
[15] G. Rocha, M. Hobson, S. Smith, P. Ferreira, A. Challinor, Mon. Not. R. Astron. Soc. 357, 1 (2005).