“The $w < -1$ side of quintessence”

in preparation with

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Overview and motivations

• quintessence: <u>one degree of freedom</u> dark energy, one field $\phi(t)$

◦ quintessence is necessarily perturbed: interacts gravitationally with DM, which sources its perturbations

• general study of perturbations: derive <u>theoretical constraints</u> on quintessence perturbations and clustering properties

◦ if $c_s^2 = 0$ quintessence clusters, if $c_s^2 = 1$ it doesn't; clustering properties can distinguish between different models

• <u>the ***w*** **< – 1** side of quintessence</u>

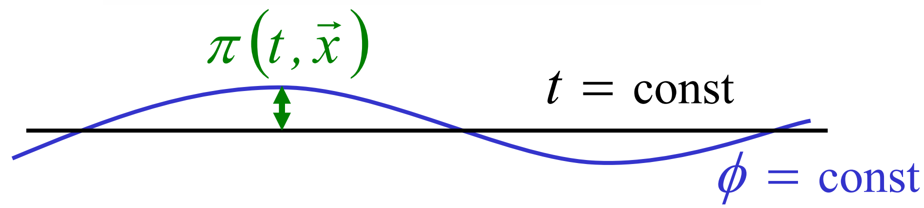
◦ can phantom ($w < -1$) be sable?
<u>stability through higher derivative operators</u>

◦ can we cross the phantom divide $w = -1$?
<u>yes, many times</u>

Ordinary scalar field

- action: $$S = \int d^4x \sqrt{-g}\left[\pm\frac{1}{2}X - V(\phi)\right], \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$
- rewrite it using π: $$\phi(t,\vec{x}) = \phi_0(t+\pi(t,\vec{x}))$$

[Cheung, Creminelli, Fitzpatrik, Kaplan, Senatore '08]



- expand at quadratic order and rewrite it in terms of SE tensor:

$$\rho_Q = \pm\frac{1}{2}X_0 + V(\phi_0)\,, \quad p_Q = \pm\frac{1}{2}X_0 - V(\phi_0)$$

$$S = \int d^4x\, a^3\left[\frac{1}{2}(\rho_Q + p_Q)\,\dot{\pi}^2 - \frac{1}{2}(\rho_Q+p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q+p_Q)\pi^2\right]$$

***w* < – 1 (<u>phantom</u>) ⇒ negative kinetic energy: <u>ghost</u>** [Caldwell '02]

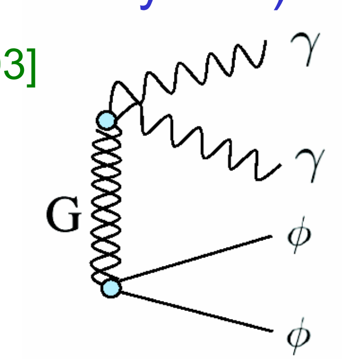
No ghost

• negative kinetic energy $\Rightarrow$ Hamiltonian unbounded from below: healthy excitations can be amplified by negative energy one [Vikman '05]

• classically: not a problem if perturbations are small and linear

• quantum mechanically: vacuum unstable (UV divergent decay rate)

[Cline, Jeon, Moore '03]



conservative approach: forbid these excitations

k-essence perturbations

[Amendariz-Picon, Damour, Mukhanov, ’99; Amendariz-Picon, Mukhanov, Steinhardt, ’00]

- action: $$S = \int \mathrm{d}^4 x \sqrt{-g}\, P(\phi, X) \,, \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- expand at quadratic order [Garriga, Mukhanov, ’99]

$$\rho_Q = 2X_0 P_X - P_0 \,, \qquad p_Q = P_0 \qquad M^4 \equiv P_{XX} X_0^2$$

$$S = \int \mathrm{d}^4 x\, a^3 \left[\frac{1}{2} \underbrace{\left(\rho_Q + p_Q + 4M^4\right)}_{>0} \dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 \right]$$

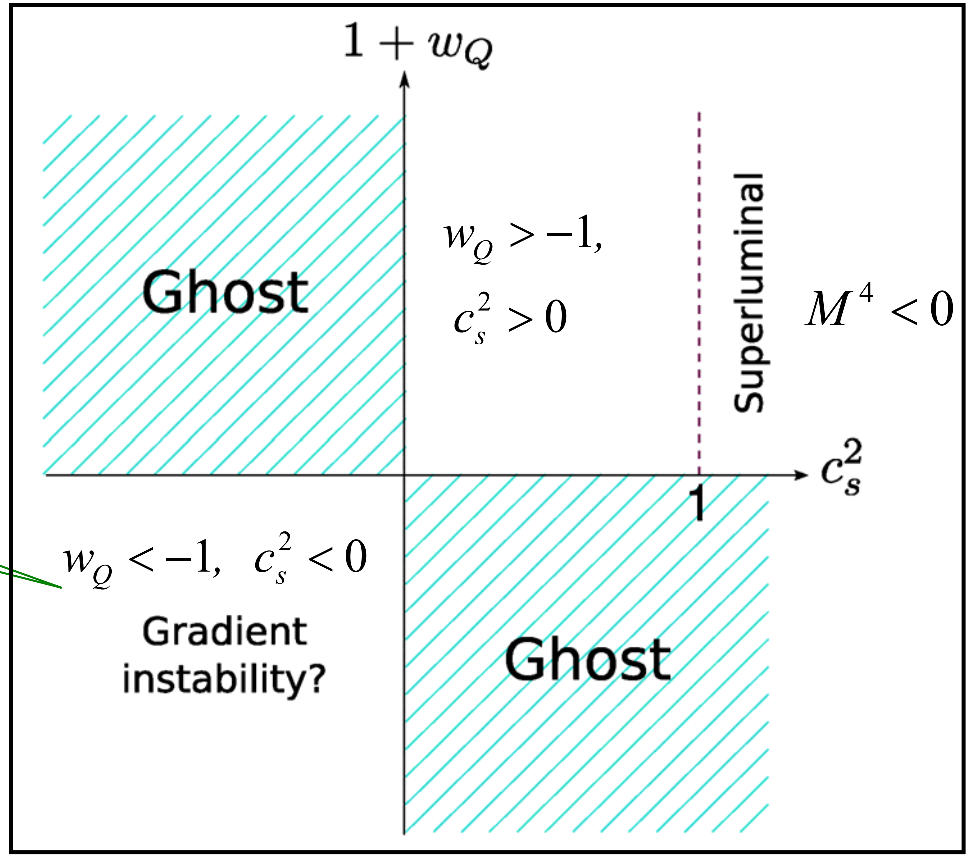
even for $w < -1$

- speed of sound of fluctuations: $$c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

Speed of sound

- speed of sound of fluctuations: $$c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

- quintessential plane:



I can have a phantom without a ghost provided I can "cure" instabilities...

- very small speed of sound?

$$c_s^2 \to 0$$

$$M^4 \gg |\rho_Q + p_Q|$$

Higher derivative terms

$$S = \int \mathrm{d}^4 x \sqrt{-g}\, P(\phi, X) - \frac{\bar{M}^2}{2} (\Box \phi)^2 \quad , \qquad X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

alters the
background evolution!

$$P(X,\phi) = \frac{1}{2} \left(\rho_Q + p_Q + 4M^4\right) \dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} + \frac{3}{2} \dot{H} (\rho_Q + p_Q) \pi^2$$

$$\downarrow$$

- given background evolution, always possible to construct an action

$$P(X,\phi) = \frac{1}{2}(p_Q - \rho_Q)(\phi) + \frac{1}{2}(\rho_Q + p_Q)(\phi) X + \frac{1}{2} M^4(\phi)(X-1)^2$$

such that $\phi = t$ is the background solution

Higher derivative terms

$$S = \int d^4 x \sqrt{-g}\, P(\phi, X) - \frac{\bar{M}^2}{2}(\Box\phi + 3H(\phi))^2, \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

does not alter the background evolution

- expand at quadratic order:

$$S = \int d^4x\, a^3 \left[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2}\right.$$
$$\left. - \frac{\bar{M}^2}{2}\left(\cancel{\ddot{\pi}} + 3H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2}\right)^2 + \ldots\right.$$

- <u>effective field theory</u> with cutoff $M \sim \bar{M}$: $\quad \left(10^{-3}\,\mathrm{eV} < M < 100\,\mathrm{GeV}\right)$

$\Rightarrow$ higher derivatives have to be treated <u>perturbatively</u>:

[Arkani-Hamed et al ‘03, Simon ’91, Weinberg ‘08]

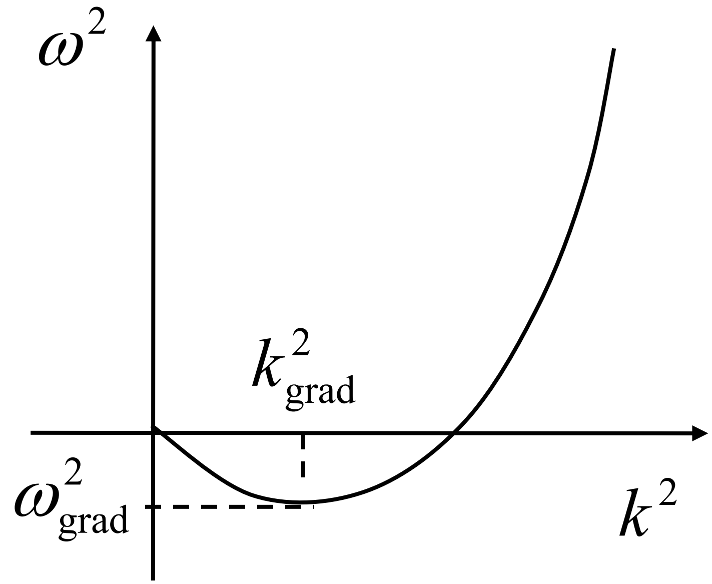
$$\omega << M \sim \bar{M} \quad \Rightarrow \quad M^4\dot{\pi}^2 >> \bar{M}^2\ddot{\pi}^2$$

- **non-standard dispersion relation:**

$$4M^4\omega^2 - (\rho_Q + p_Q)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0$$

Taming the instabilities

$$4M^4\omega^2 - (\rho_Q + p_Q)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0$$



- instability rate:

$$\omega^2_{\rm grad} \simeq -\frac{(\rho_Q + p_Q)^2}{\bar{M}^2 M^4} \lesssim H^2$$

- stability window:

$$-(1 + w_Q)\Omega_Q \lesssim \frac{\bar{M} M^2}{H M^2_{\rm Pl}} \lesssim 1$$

gradient instab. Jeans instab.

Stability region for *w* < –1

$$4M^4\omega^2 - (\rho_Q + p_Q)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0$$

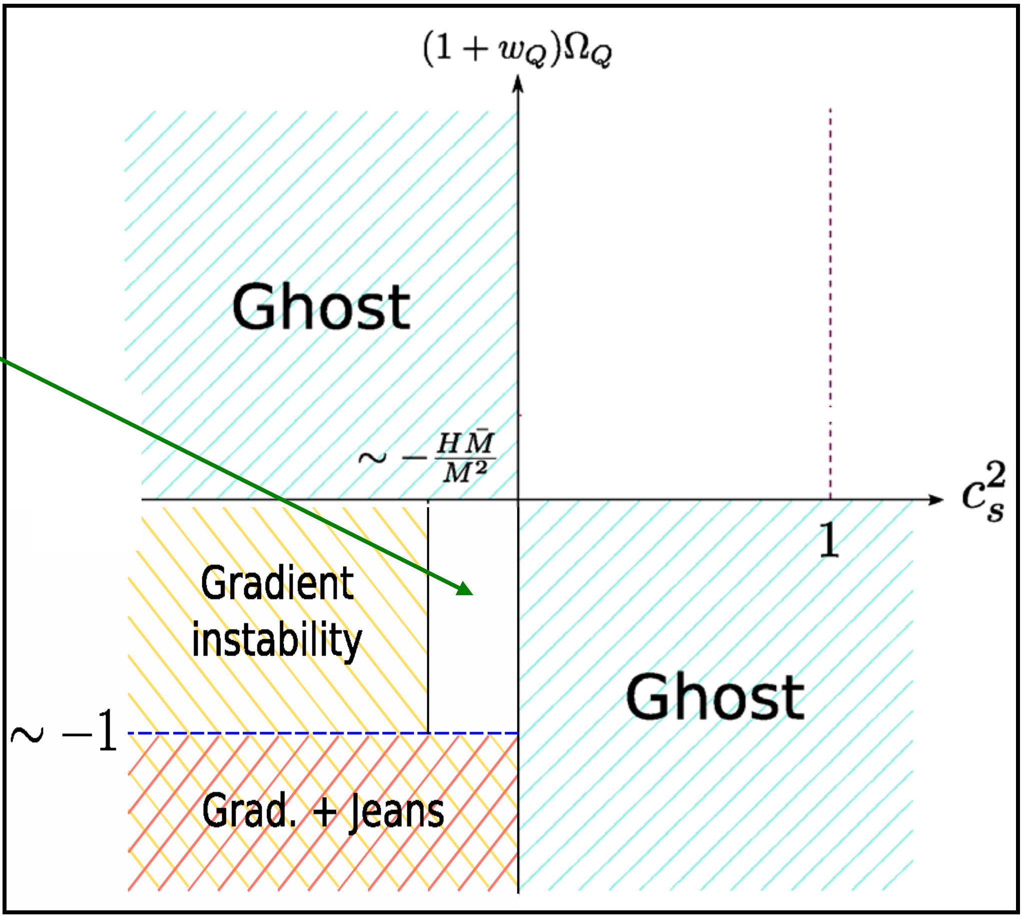
- stability window:

$$-(1+w_Q)\Omega_Q \lesssim \frac{\bar{M}M^2}{HM_{\rm Pl}^2} \lesssim 1$$

extremely small sound speed:

$$-c_s^2 \lesssim \left(\frac{H_0}{M_{\rm Pl}}\right)^{1/2} \sim 10^{-30}$$

$\Rightarrow$ for all practical purposes $c_s^2 = 0$



No fine-tuning: the GC limit

- for $\rho_Q + p_Q = 0$ we obtain:

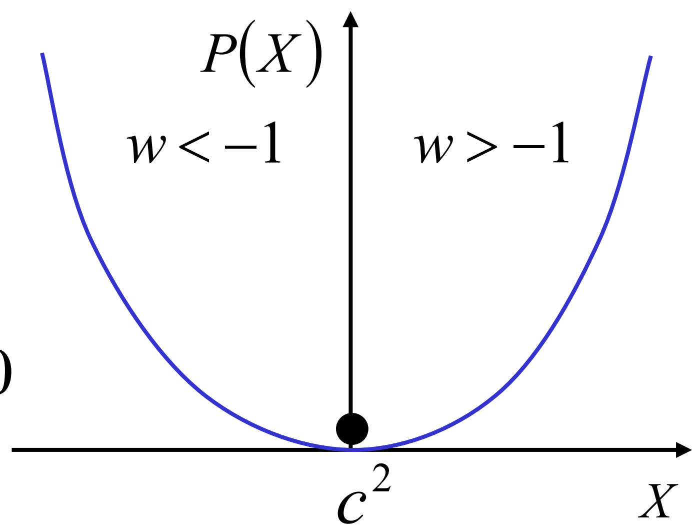
$$S = \int \mathrm{d}^4x \left[2M^4\dot{\pi}^2 - \frac{\bar{M}^2}{2} \left(\nabla^2\pi\right)^2 \right]$$

Ghost Condensate theory: [Arkani-Hamed et al '03]

$$S = \int d^3x dt \sqrt{-g} P(X) + \mathrm{h.\,d.\,ops.}$$

shift symmetry $\phi \to \phi + \lambda$

$$\frac{d}{dt}\left(a^3 P_{,X}\dot{\phi}\right) = 0 \;\Rightarrow\; \phi = ct \quad \text{and} \quad P_{,X}\left(c^2\right) = 0$$



Beyond the GC limit

$$4M^4\omega^2 - (\rho_Q + p_Q)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0$$

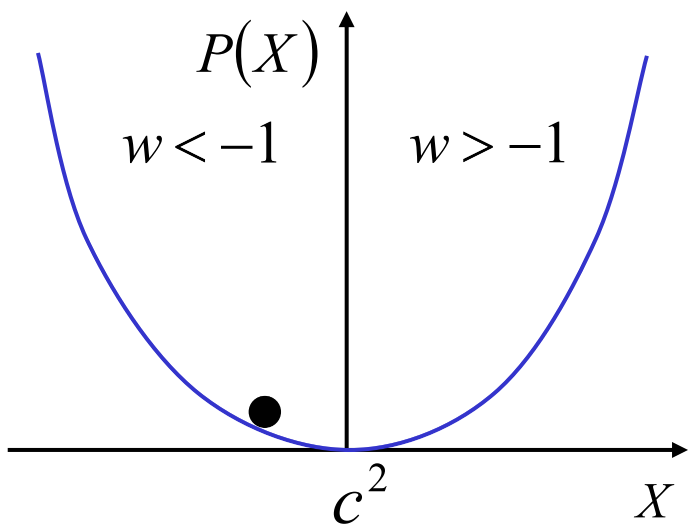
• on smaller scales $\rho_Q + p_Q << \overline{M}^2 k^2/a^2$ the *w* < –1 side of quintessence is a small deviation from the GC limit:

$$S = \int d^3x dt \sqrt{-g}\, P(X) + \mathrm{h.\,d.\,ops.}$$

~~shift symmetry~~ $\cancel{\phi \to \phi + \lambda}$

$\Rightarrow$ stable violation of NEC!

[Creminelli, Luty, Nicolis, Senatore '06]



• on larger scales $M^4 >> \rho_Q + p_Q >> \overline{M}^2 k^2/a^2$ gradients are negligible and we are left with *k*-essence with $c_s^2 = 0$!

Summary

$$\left(\rho_Q + p_Q + 4M^4\right)\omega^2 - (\rho_Q + p_Q)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0$$

On cosmological scales $k \sim aH$:

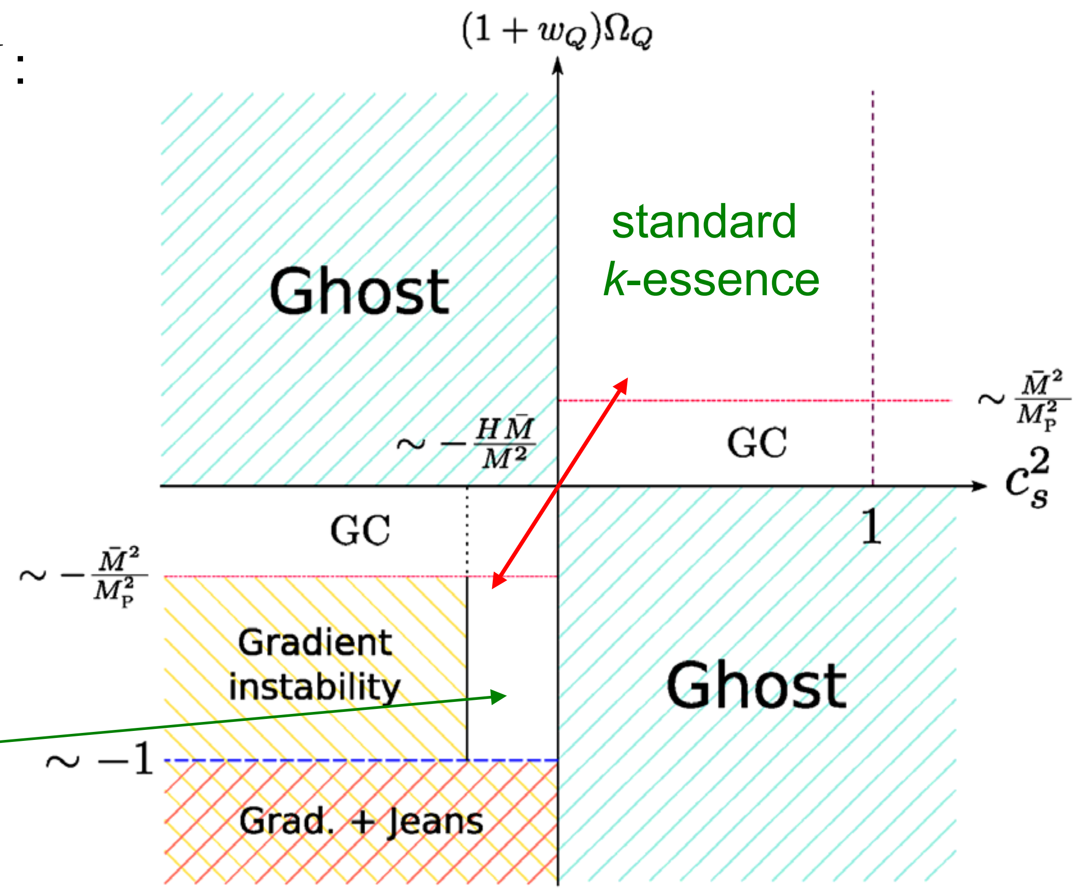
Ghost Condenstate window:

$$-\frac{\bar{M}^2}{M_{\rm Pl}^2} \lesssim (1+w_Q)\Omega_Q \lesssim \frac{\bar{M}^2}{M_{\rm Pl}^2}$$

$\Rightarrow$ w~-1, very close to CC value:

$$|1+w_Q|\Omega_Q \lesssim 10^{-34}$$

k-essence
with $c_s^2 = 0$



$\Rightarrow$ **no problem in crossing the phantom divide!**

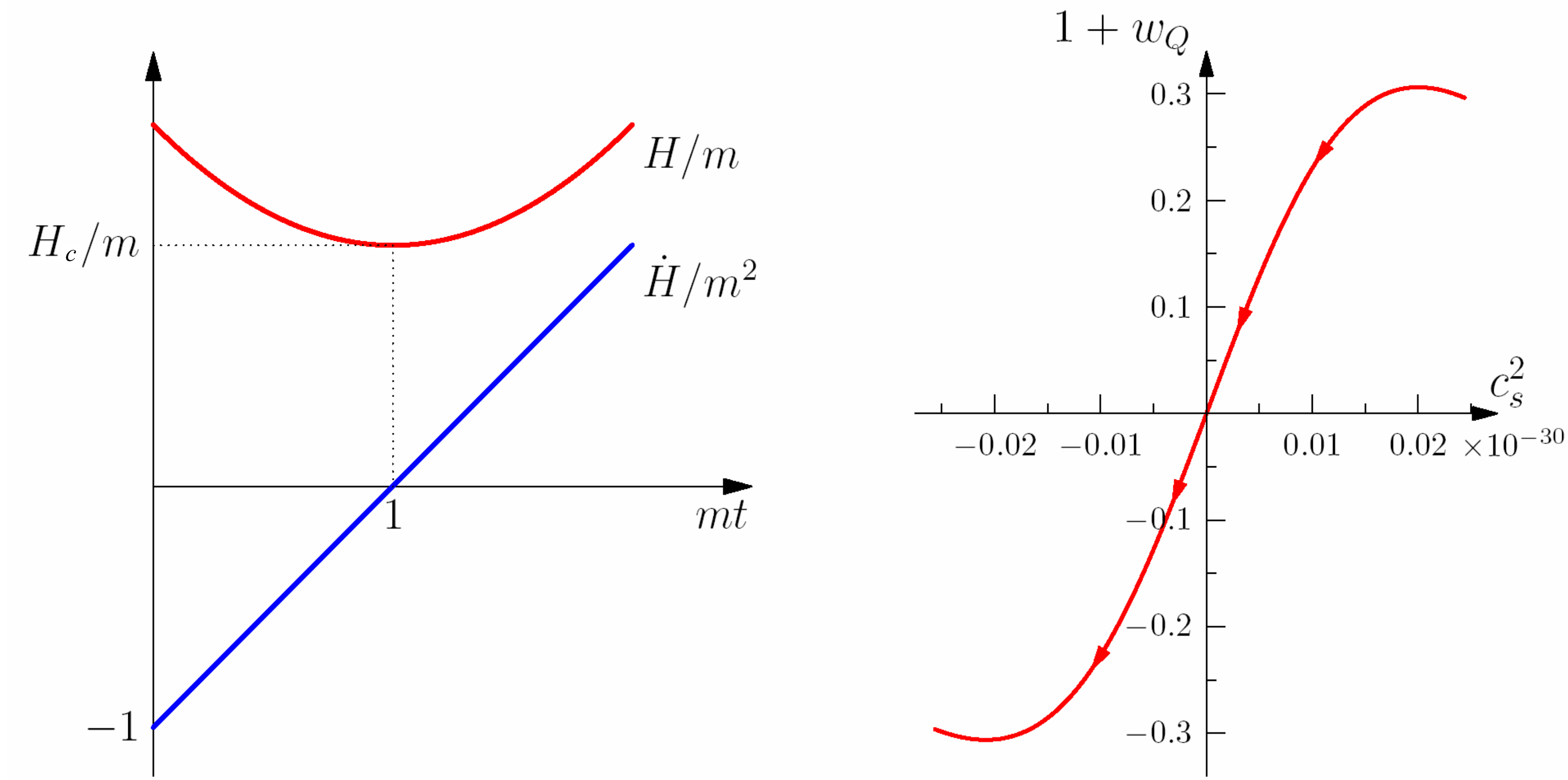
Crossing the phantom divide [Hu '04]

[Vikman '05, Caldwell, Doran '05]

- given background evolution, always possible to construct an action

$$P(X,\phi) = -3M_{\rm Pl}^2 H^2(\phi) - M_{\rm Pl}^2 \dot{H}(\phi)(X+1) + \frac{1}{2}M^4(\phi)(X-1)^2$$

such that $\phi = t$ is the background solution



Phenomenology with dark matter

- quintessence is coupled to DM through the metric (assume M.D.):
- in the GC window $w \sim -1$ the phenomenology is the same as a CC Λ:

$$\ddot{\pi} + 3H\dot{\pi} \simeq -\frac{\bar{M}^2}{12 M^4 M_{\rm Pl}^2}\frac{\nabla^2 \delta\rho_m}{a^2 H} \qquad \Rightarrow \qquad \delta\rho_Q \sim \frac{\bar{M}^2}{M_{\rm Pl}^2}\delta\rho_m \lesssim 10^{-42}\delta\rho_m$$

- for $w < -1$ quintessence behave as a *k*-essence fluid with $c_s^2 = 0$:

$$\delta_Q = \frac{1+w_Q}{1-3w_Q}\delta_m$$ smaller than DM fluctuations

$\Rightarrow$ strange fluid: anti-clusters (escapes from potential wells) [Weller, Lewis '03]

model not ruled out:

- **can we distinguish $c_s^2 = 0$ from $c_s^2 = 1$?**
- **until what value of 1 + *w* ?**

Conclusions

• study of quintessence perturbations in a general framework of effective field theory

• higher order operators stabilize the $w < -1$ side of quintessence: standard *k*-essence fluid with $c_s^2 = 0$ (no fine tuning: GC theory)

explore the region $w < -1$ using $c_s^2 = 0$

• important to distinguish $c_s^2 = 0$ from $c_s^2 = 1$ in future surveys

• phantom divide can be crossed

the end of the phantom psychosis?