Probabilisty, propensity and probability of propensity

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"Probability is the very guide of life" (Cicero's *thought summarized*)

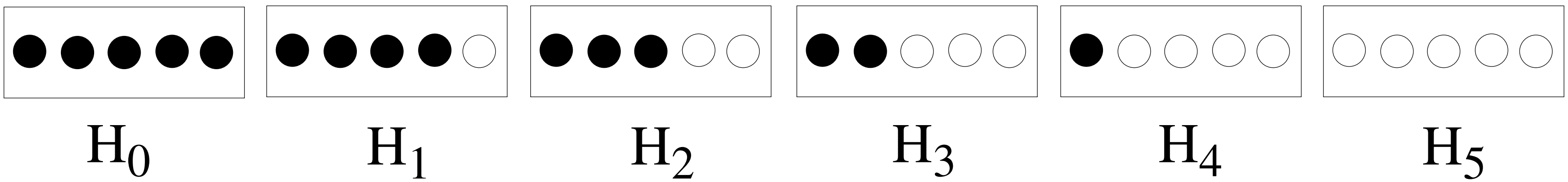
"Probability is good sense reduced to a calculus" (Laplace)

Preamble

" "I am a Bayesian in data analysis,
I am a frequentist in Physics"

(A Rome PhD student, 2011)

Which box? Which ball?



Let us take randomly one of these 6 boxes

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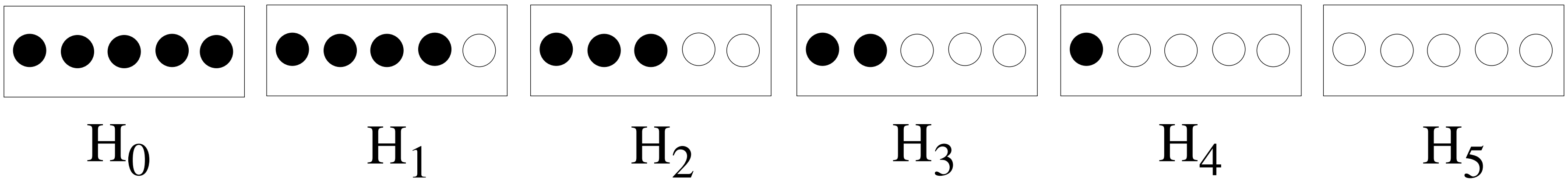

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We are in a state of uncertainty concerning several *events*, the most important of which correspond to the following questions:

(a) Which box have we chosen, $H_0, H_1, \ldots, H_5$?

(b) If we extract randomly a ball from the chosen box, will we observe a white ($E_W \equiv E_1$) or black ($E_B \equiv E_2$) ball?

Our certainties:

$$\begin{aligned} \cup_{j=0}^{5} H_j &= \Omega \\ \cup_{i=1}^{2} E_i &= \Omega\,. \end{aligned}$$

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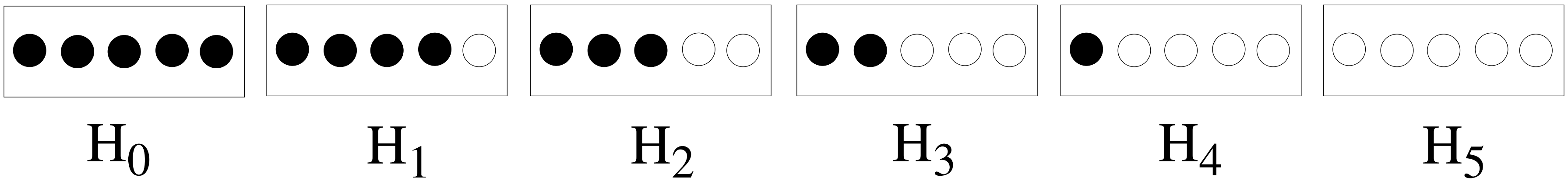

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- who feels more confident on either color?

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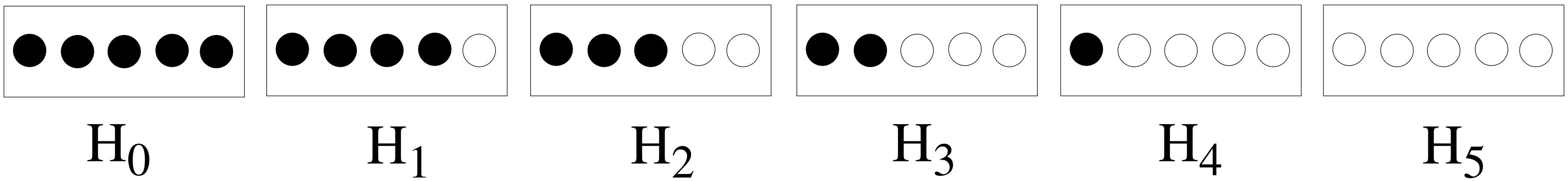

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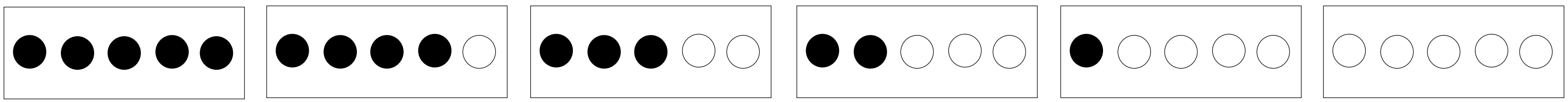

H_0 H_1 H_2 H_3 H_4 H_5

In particular:

- who feels more confident on either color?
- if you were going to receive a rich prize, would you bet on white or black?
- would you prefer to bet on white in this game or tails tossing a coin?

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- most people choose B_{5-5} ...
- ...and, mostly surprising, they continue to stick to B_{5-5} even in the second question!

Sequences from 2 extractions

Let us change the winning condition(s)

- You can make two extractions with reintroduction

⇒ You can choose one of the following 4 sequences from either box.

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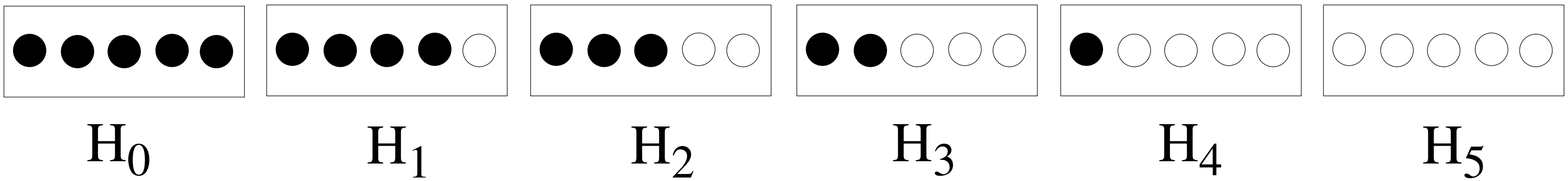
Learning from observations



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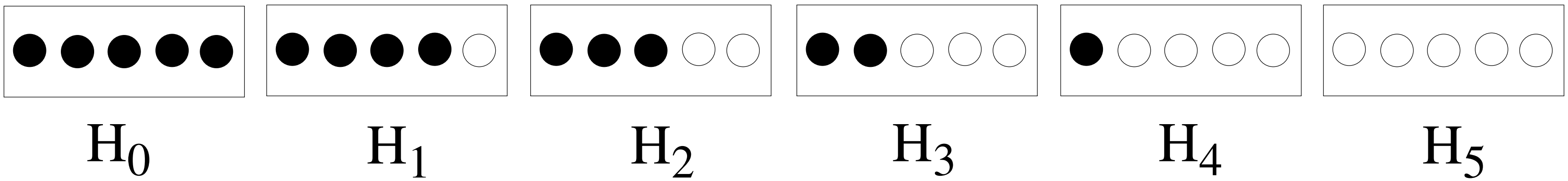

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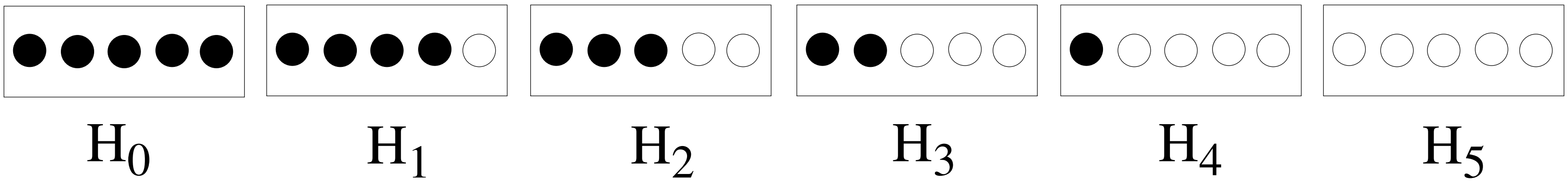

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  - Intuitively feel *how to* roughly *change* our opinion about
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  - Can we do it *quantitatively*, in an ‘objective way’?
- And after a sequence of extractions?

The toy inferential experiment

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The rule of the game is that we are not allowed to watch inside the box! (As we cannot open and electron and read its properties, unlike we read the MAC address of a PC interface.)

Our tool

Playing with Hugin Expert

- Interactive game $\longrightarrow$

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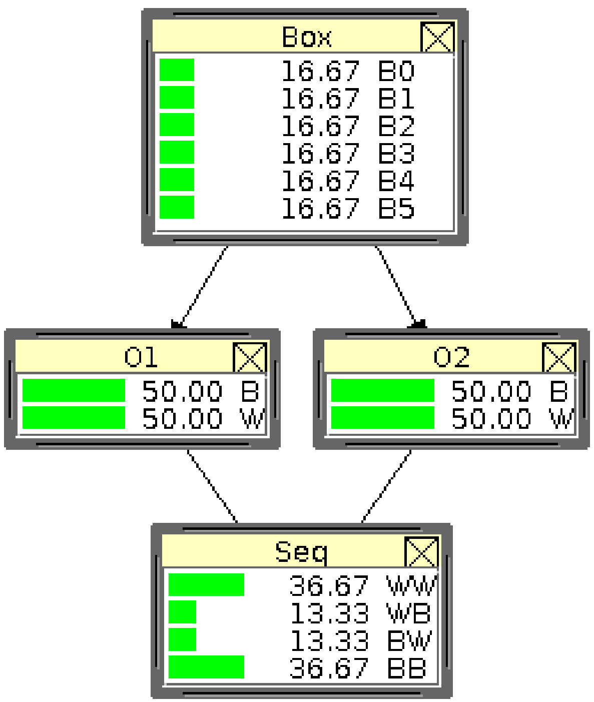

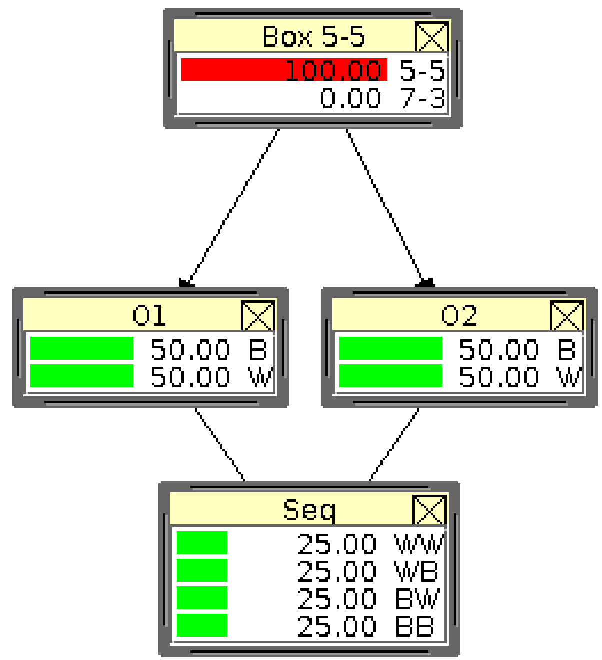

$\left(0.3667 \;\rightarrow\; \frac{11}{30}\right)$

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Where is the probability?

Certainly not *in* the box!

Subjective nature of probability

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Probability depends on the status of information of the *subject* who evaluates it.

Probability is always conditional probability

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$\Rightarrow$ Three box game

(Box with white ball wins)

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$\Rightarrow$ How much we believe something

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$\rightarrow$ 'Degree of belief' $\leftarrow$

Playing with R

1. Analysis of real data
2. Simulations of 100 extractions

   → Probability of future observations
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     Impossible! since in the frequentistic approach statements concerning the probabilities of the causes are simply not allowed!
     But – and what is the WORST – frequentists do not simply refuse to make statements about causes
     $\Rightarrow$ they do it, using terms that do not mean probabilities, but sound and are interpreted as such ('significance', 'CL', 'confidence intervale', 'p-values')

Playing with R

1. Analysis of real data
2. Simulations of 100 extractions
3. Complicating the model:
   - Estraction mediated by a Reporter (machine/human) which might err or lie
   - Doubt concerning the box preparation

Bayes' billiard

This is the original problem in the theory of chances solved by Thomas Bayes in late '700:

- imagine you roll a ball at random on a billiard;
- you mark the relative position of the ball along the billiard's length (l/L) and remove the ball
- then you roll at random other balls
  - write down if it stopped left or right of the first ball;
  - remove it and go on with n balls.
- Somebody has to guess the position of the first ball knowing only how mane balls stopped left and how many stoppe right

Bayes' billiard and Bernoulli trials

It is easy to recongnize the analogy:

- Left/Right $\rightarrow$ Success/Failure
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\ldots & \quad \ldots \\
f(p\,|\,\#S,\#F) &\propto p^{\#S}(1-p)^{\#F} = p^{\#S}(1-p)^{(1-\#s)} \\
f(p\,|\,x,n) &\propto p^x(1-p)^{(n-x)} \qquad [x = \#S]
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- How much we belief something (‘degree of belief’ – original meaning of probability from its Latin root$^{(*)}$).
- A property of a physical system to behave in a certain way (‘chance’ $\rightarrow$ ‘propensity’).

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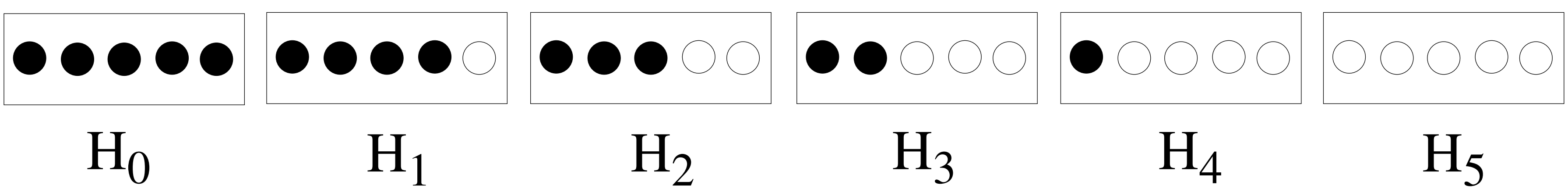
- A property of a physical system to behave in a certain way (‘chance’ $\rightarrow$ ‘propensity’).

The six box model can help to make the question clear.



Belief Vs ‘propension’

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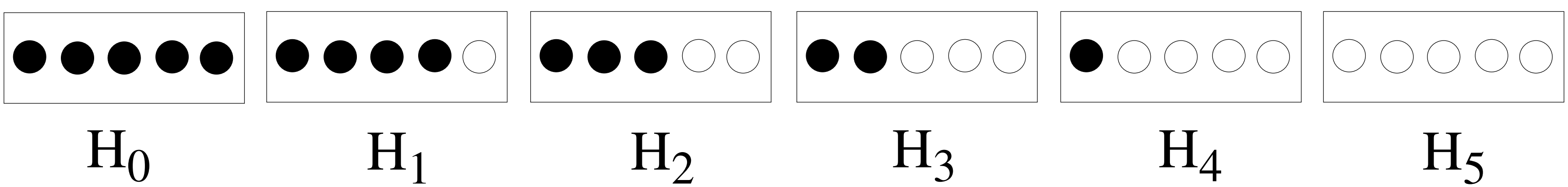

(*) For the record, a “grep -i probabil” in all files of `www.thelatinlibrary.com` reports 540 entries (97 by Cicero)

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  There is no need to adhere to the frequentistic ideology to say this

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Instead, "probability is the limit of frequency for $n \rightarrow \infty$" is not more than an empty statement.

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⇒ Simple result of probability theory:

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Probability theory (in Laplage's sense) allows to attach probabilities to whatever we feel uncertain about!

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- Other important parameters are related to background, systematics, 'etc.' [arguments not covere here]

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  (Diffidate chi vi promette di far germogliar zecchini nel Campo dei Miracoli!)

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- Trying to use different nouns for the two meanings would avoid confusion and misunderstanding, although I am perfectly aware that it is a ‘battle lost since the very beginning’
- . . . but at least being aware of the two meanings (NOT ‘interpretations’ – this is something I really dislike!) would already be usefull, since we are used, in all languages, that the same word can have a meaning depending on the context.

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