

# Dissipation of Magnetohydrodynamic Waves on Energetic Particles: Impact on Interstellar Turbulence and Cosmic Ray Transport

Vladimir S. Ptuskin<sup>1</sup>

*Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation (IZMIRAN),  
Troitsk, Moscow region 142190, Russia*

vptuskin@izmiran.rssi.ru

Igor V. Moskalenko<sup>2</sup>

*Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305*

imos@stanford.edu

Frank C. Jones

*Exploration of the Universe Division, NASA/Goddard Space Flight Center, Code 661,  
Greenbelt, MD 20771*

Frank.C.Jones@gsfc.nasa.gov

Andrew W. Strong

*Max-Planck-Institut für extraterrestrische Physik, Postfach 1312, D-85741 Garching,  
Germany*

aws@mpe.mpg.de

and

Vladimir N. Zirakashvili<sup>3</sup>

*Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation (IZMIRAN),  
Troitsk, Moscow region 142190, Russia*

zirak@pimail.mpi-hd.mpg.de

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<sup>1</sup>Also Institute for Physical Science and Technology, University of Maryland, College Park, MD 20742

<sup>2</sup>Also Kavli Institute for Particle Astrophysics and Cosmology, Stanford University, Stanford, CA 94309

<sup>3</sup>Also Max-Planck-Institut für Kernphysik, D-69029, Heidelberg, Postfach 103980, Germany

## ABSTRACT

The physical processes involved in diffusion of Galactic cosmic rays in the interstellar medium are addressed. We study the possibility that the nonlinear MHD cascade sets the power-law spectrum of turbulence which scatters charged energetic particles. We find that the dissipation of waves due to the resonant interaction with cosmic ray particles may terminate the Kraichnan-type cascade below wavelengths  $10^{13}$  cm. The effect of this wave dissipation has been incorporated in the GALPROP numerical propagation code in order to assess the impact on measurable astrophysical data. The energy-dependence of the cosmic-ray diffusion coefficient found in the resulting self-consistent model may explain the peaks in the secondary to primary nuclei ratios observed at about 1 GeV/nucleon.

*Subject headings:* diffusion — MHD — elementary particles — turbulence — waves — cosmic rays

## 1. Introduction

The Galactic cosmic rays – the gas of relativistic charged particles with high energy density – cannot always be treated as test particles moving in given magnetic fields. In particular, the stochastic acceleration of cosmic rays by MHD waves is accompanied by the damping of the waves, since the wave energy is dissipated. The rate of wave damping on cosmic rays through the cyclotron resonance interaction was first estimated by Tidman (1966). If we exclude cold HI regions (where the waves are damped by collisions of ions with neutral atoms) and also perhaps regions of the interstellar gas with very high temperature  $T \sim 10^6$  K and weak magnetic field (where Landau damping on thermal particles is high), then one finds that this mechanism of dissipation could dominate over other known mechanisms in the interval of wavelengths  $10^{11}$  cm to  $10^{14}$  cm. The non-resonant interaction of diffusing cosmic rays with magnetosonic waves (Ptuskin 1981) is important only at large wavelengths and is not important for the present investigation. We also do not consider the large body of instabilities in cosmic rays that may arise because of the non-equilibrium distribution of charged energetic particles caused by possible strong anisotropy or large gradients and which may amplify waves in the background plasma – see e.g. Berezhinskii et al. (1990) and Diehl et al. (2001) for a review of such processes. Cyclotron wave damping on cosmic rays changes the wave spectrum in the interstellar medium, which in turn affects the particle transport since the cosmic ray diffusion coefficient is determined by the level of turbulence which scatters charged particles. Thus in principle the study of cosmic ray diffusion requires

a self-consistent approach. We shall see below that the effect of cosmic rays on interstellar turbulence should be taken into account at energies below a few GeV/nucleon, and this may result in a considerable increase in the particle diffusion coefficient. This picture will be also verified by its consistency with observations of interstellar turbulence. Consistency of this picture with observations of interstellar turbulence is also discussed

It is remarkable that the interpretation of cosmic ray data on secondary nuclei may require this effect. The secondary nuclei are produced in cosmic rays in the course of diffusion and nuclear interactions of primary nuclei with interstellar gas. The  $^2\text{H}$ ,  $^3\text{He}$ , Li, Be, B and many other isotopes and elements are almost pure secondaries. Their abundance in cosmic ray sources is negligible and they result from the fragmentation of heavier nuclei. Cosmic ray antiprotons and the major fraction of positrons are among the secondary species as well. The ratio of secondary to primary nuclei such as B/C has a peak at about 1 GeV/n and decreases both with increasing and decreasing energy, e.g. Engelmann et al. (1990). The high energy behavior is naturally explained by particle scattering on interstellar MHD turbulence with a power-law spectrum in wavelength, but the required sharp increase of the diffusion coefficient at small energies has usually been considered as improbable since it needs a drastic “physically unjustified” bending down of the wave spectrum at wave numbers  $k > 1/3 \times 10^{12} \text{ cm}^{-1}$ , see e.g. Jones et al. (2001), Moskalenko et al. (2002). Alternative explanations of the peaks in secondary to primary ratios without invoking peculiarities in the cosmic ray diffusion coefficient have been suggested. The two most popular of these involve diffusive-convective particle transport in the hypothetical Galactic wind (Jones 1979; Ptuskin et al. 1997) and the stochastic reacceleration of cosmic rays by interstellar turbulence (Simon et al. 1986; Seo & Ptuskin 1994). These processes are included in the numerical computations of cosmic ray transport in the Galaxy in the frameworks of the GALPROP code (Strong & Moskalenko 1998; Moskalenko et al. 2002). The situation with the interpretation of the energy dependence of secondaries in cosmic rays is still uncertain. The peaks in the secondary to primary nuclei ratios calculated in diffusion-convection models seem to be too wide. Models with reacceleration reproduce the shape of the peaks but the absolute flux of antiprotons turns out to be too low compared to observations (Moskalenko et al. 2002). This is why the effect considered in the present work is so important. We investigate a self-consistent model of cosmic-ray diffusion in interstellar turbulence where the wave damping on energetic particles is taken into account, and implement this effect in the GALPROP code. To make the first calculations reasonably tractable we keep only the most essential features of cosmic-ray diffusion in random magnetic fields and use the simplest description of the nonlinear wave cascade in the interstellar turbulence.

## 2. Equations for Cosmic Rays

The steady state transport equation that describes diffusion and convective transport of cosmic ray protons and nuclei in the interstellar medium is of the form (see e.g., Berezhinskii et al. 1990, for discussion):

$$-\nabla D \nabla \Psi + \nabla(\mathbf{u}\Psi) - \frac{\partial}{\partial p} \left[ p^2 K \frac{\partial}{\partial p} (p^{-2} \Psi) \right] - \frac{\partial}{\partial p} \left( p \frac{\nabla \mathbf{u}}{3} \Psi \right) + \frac{\partial}{\partial p} (\dot{p}_{\text{loss}} \Psi) + \frac{\Psi}{\tau} = q. \quad (1)$$

Here  $\Psi(p, \mathbf{r})$  is the particle distribution function in momentum  $p$  normalized on total cosmic ray number density as  $N_{\text{cr}} = \int dp \Psi$ ,  $D(p, \mathbf{r})$  is the spatial diffusion coefficient,  $K(p, \mathbf{r})$  is the diffusion coefficient in momentum,  $\mathbf{u}(\mathbf{r})$  is the velocity of large-scale motions of the interstellar medium (e.g. the velocity of a Galactic wind),  $\dot{p}_{\text{loss}} = dp/dt < 0$  is the momentum loss rate for the energetic charged particles moving through the interstellar medium,  $\tau(\mathbf{r})$  is the time scale for nuclear fragmentation. If needed, the supplementary term which describes radioactive decay can be added to eq. (1). The spatial boundary condition for  $\Psi$  is  $\Psi|_{\Sigma} = 0$ , corresponding to the free exit of cosmic rays from the Galaxy to intergalactic space where their density is negligible. The region of cosmic ray diffusion is a cylinder of radius 30 kpc and total width  $2H$  ( $H \approx 4$  kpc). The source term  $q(p, \mathbf{r})$  includes both the direct production of primary energetic particles accelerated from the thermal background in Galactic sources (e.g. supernova remnants) and the contribution to the nuclei considered via the processes of nuclear fragmentation and radioactive decay of heavier nuclei. The typical value of the diffusion coefficient found from the fit to cosmic ray and radioastronomical data is  $D \sim 3 \times 10^{28}$  cm<sup>2</sup>/s at energy  $\sim 1$  GeV/n, giving a diffusion mean free path  $l = 3D/v \sim 1$  pc ( $v \approx c$  is the particle velocity).

On the ‘‘microscopic level’’ the spatial and momentum diffusion of cosmic rays results from the particle scattering on random MHD waves and discontinuities. In the linear approximation, the Alfvén (with the dispersion relation  $\omega(\mathbf{k}) = k_{\parallel} V_a$ ), the fast magnetosonic ( $\omega(\mathbf{k}) = k V_a$ ), and the slow magnetosonic ( $\omega(\mathbf{k}) = k_{\parallel} V_s$ ) waves can propagate in a low  $\beta$  plasma,  $\beta = (V_s/V_a)^2 < 1$ , where  $V_a = B/\sqrt{4\pi\rho}$  is the Alfvén velocity and  $V_s = \sqrt{P_g/\rho}$  is the collisionless sound velocity (for MHD conditions it is a factor of a square root of the gas adiabatic index larger), where  $B$ ,  $P_g$  and  $\rho$  are the magnetic field strength, the pressure, and the mass density of the interstellar gas respectively. In addition to waves, the static entropy variations can exist in the interstellar medium. In a collisionless plasma, the slow magnetosonic waves are not heavily damped only if the plasma is non-isothermal and the electron temperature considerably exceeds the ion temperature. The Alfvén velocity is  $1.8 \times 10^5 B_{\mu\text{G}}/\sqrt{n}$  cm/s, where  $n$  is the number density of hydrogen atoms. The value of the sound velocity is  $7.7 \times 10^5 \sqrt{T_4}$  cm/s in neutral gas with temperature  $T = 10^4 T_4$  K;  $V_s$  is larger in the ionized gas because of the free electron contribution to gas pressure  $P_g$ . The

approximation of low  $\beta$  plasma is valid in the dominant part of the interstellar medium at  $B \approx 5 \mu\text{G}$  since typically  $n = 0.002 \text{ cm}^{-3}$  and  $T = 10^6 \text{ K}$  in hot HII regions,  $n = 0.2 \text{ cm}^{-3}$  and  $T = 8 \times 10^3 \text{ K}$  in warm intercloud gas,  $n = 30 \text{ cm}^{-3}$  and  $T = 100 \text{ K}$  in clouds of atomic hydrogen,  $n = 200 \text{ cm}^{-3}$  and  $T = 10 \text{ K}$  in molecular clouds and thus  $\beta \approx 0.1 - 0.3$  everywhere. The effective “collision integral” for energetic charged particles moving in small amplitude random fields  $\delta B \ll B$  can be taken from the standard quasi-linear theory of plasma turbulence, see e.g. Kennel & Engelmann (1966). The “collisional integral” averaged over fast gyro-rotation of particles about the magnetic field  $\mathbf{B}$  contains the spectral densities of the effective frequencies of particle collisions with plasmons in the form

$$\begin{aligned} \nu_\mu^\alpha(\mathbf{k}, s, p) = & \frac{4\pi^2 v}{B^2 |\mu| r_g^2} [(1 - \delta_{s0}) \delta(k_\parallel - s/(r_g |\mu|)) M_\perp^\alpha(\mathbf{k}) (J_{s+1}^2 + J_{s-1}^2) + \\ & \frac{\delta_{s0}}{2} \delta(k_\parallel - k_\perp V_a / (v |\mu|)) J_1^2 M_\parallel^\alpha(\mathbf{k}) k_\parallel^2 k_\perp^{-2}], \end{aligned} \quad (2)$$

$k_\parallel > 0$  (as presented by Ptuskin 1989). Here the waves are assumed symmetric about the magnetic field direction and to have zero average helicity. The index  $\alpha$  characterizes the type of waves including the direction of their propagation,  $\mu$  is the cosine of pitch angle, the energy densities of random magnetic fields for perpendicular and parallel to the average field  $\mathbf{B}$  components are  $M_\perp^\alpha(\mathbf{k})$  and  $M_\parallel^\alpha(\mathbf{k})$  respectively,  $J_m = J_m(k_\perp r_g \sqrt{1 - \mu^2})$  is the Bessel function. It is explicitly taken into account that the scattering at  $s = 0$  occurs only on the fast magnetosonic waves propagating almost perpendicular to the magnetic field. The Larmor radius is  $r_g = pc/(ZeB) = 3.3 \times 10^{12} R_{\text{GV}}/B_{\mu\text{G}} \text{ cm}$ , where the particle magnetic rigidity  $R = pc/Ze$  is measured in GV, and the average magnetic field is measured in  $\mu\text{G}$ . The wave-particle interaction is of resonant character so that an energetic particle is predominantly scattered by the irregularities of magnetic field  $\delta B$  that have the projection of wave vector on the magnetic field direction equal to  $k_\parallel = \pm s/(r_g \mu)$ . The integers  $s = 0, 1, 2, \dots$  correspond to the cyclotron resonances of different orders. The efficiency of particle scattering depends on the polarization of the waves and on their distribution in  $\mathbf{k}$ -space. The first-order resonance  $s = 1$  is the most important for the isotropic and also for the one-dimensional distribution of random MHD waves along the average magnetic field,  $\mathbf{k} \parallel \mathbf{B}$ . In some cases – for calculation of scattering at small  $\mu$  and for calculation of perpendicular diffusion – the broadening of resonances and magnetic mirroring effects should be taken into account.

The evolution of the particle distribution function on time-scales  $\Delta t \gg \nu^{-1}$  and distances  $\Delta z \gg v\nu^{-1}$  can be described in the diffusion approximation with the following expressions for the spatial diffusion coefficient along the magnetic field  $D_\parallel$  and the diffusion

coefficient in momentum  $D_{pp}$ :

$$D_{\parallel}(p) = \frac{v^2}{4} \int_{-1}^{+1} d\mu (1 - \mu^2) \left( \sum_{\alpha, s, \mathbf{k}} \nu_{\mu}^{\alpha}(\mathbf{k}, s, p) \right)^{-1}, \quad (3)$$

$$D_{pp}(p) = \frac{p^2}{4} \int_{-1}^{+1} d\mu (1 - \mu^2) \times \left[ \sum_{\alpha, s, \mathbf{k}} \nu_{\mu}^{\alpha}(\mathbf{k}, s, p) \frac{[V_f^{\alpha}(\mathbf{k})]^2}{v^2} - \left( \sum_{\alpha, s, \mathbf{k}} \nu_{\mu}^{\alpha}(\mathbf{k}, s, p) \frac{V_f^{\alpha}(\mathbf{k})}{v} \right)^2 \left( \sum_{\alpha, s, \mathbf{k}} \nu_{\mu}^{\alpha}(\mathbf{k}, s, p) \right)^{-1} \right], \quad (4)$$

see Berezhinskii et al. (1990) where the analogous equations were derived for one-dimensional turbulence with  $k_{\perp} = 0$ . Here  $V_f^{\alpha}(\mathbf{k}) = \omega^{\alpha}(\mathbf{k})/k_{\parallel}$ , the summations contain integrals over  $\mathbf{k}$ -space, and the terms which contain  $\nu_{\mu}^{\alpha}(\mathbf{k}, s = 0, p)$  should be corrected in eqs. (3), (4) compared to eq. (2): multiplied by  $(1 - \mu^2)^2$  in  $(\sum \nu_{\mu}^{\alpha})^{-1}$ , and multiplied by  $(1 - \mu^2)$  in other terms. One can check that the interaction of energetic particles with slow magnetosonic waves is relatively weak (as  $V_s^2/V_a^2 \ll 1$ ) and can be ignored.

Locally, the cosmic ray diffusion is anisotropic and occurs along the local magnetic field because the particles are strongly magnetized,  $r_g \ll l$ . The isotropization is accounted for by the presence of strong large-scale ( $\sim 100$  pc) fluctuations of the Galactic magnetic field. The problem is not trivial even in the case of relatively weak large scale random fields, since the field is almost static and the strictly one-dimensional diffusion along the magnetic field lines does not lead to non-zero diffusion perpendicular to  $\mathbf{B}$ , see Chuvilgin & Ptuskin (1993), Giacalone & Jokipii (1999), Casse et al. (2001).

The eqs. (3) and (4) are too cumbersome for our present application. Based on eq. (3) and with reference to the detailed treatment of cosmic ray diffusion by Toptygin (1985) and Berezhinskii et al. (1990), we use below the following simplified equation for the diffusion coefficient:

$$D = vr_g B^2 / [12\pi k_{\text{res}} W(k_{\text{res}})], \quad (5)$$

where  $k_{\text{res}} = 1/r_g$  is the resonant wave number, and  $W(k)$  is the spectral energy density of waves normalized as  $\int dk W(k) = \delta B^2 / 4\pi$ . The random field at the resonance scale is assumed to be weak,  $\delta B_{\text{res}} \ll B$ .

The cosmic ray diffusion coefficient is equal to

$$D = \frac{vr_g^a B^2}{3(1-a)k_L^{1-a} \delta B_L^2} \quad (6)$$

for particles with  $r_g < k_L^{-1}$  under the assumption of a power law spectrum of turbulence  $W(k) \propto 1/k^{2-a}$ ,  $k > k_L$ . We introduced here the principle wave number of the turbulence  $k_L$  and the amplitude of random field  $\delta B_L$  at this scale. The diffusion has scaling  $D \propto v(p/Z)^a$ .

The diffusion in momentum is described by the following equation which is a simple approximation of eq. (4):

$$K = p^2 V_a^2 / (9D). \quad (7)$$

Eqs. (5)-(7) give estimates of particle diffusion in position and momentum needed in eq. (1). They reflect the most essential features of cosmic-ray transport: the frequency of particle scattering on random magnetic field is determined by the energy density of this field at the resonance wave number  $k_{\text{res}}$ ; the acceleration is produced by the waves moving with typical velocity  $V_a$  and is stochastic in nature. Eq. (5) implies equal intensities of waves moving along the magnetic field in opposite directions, the imbalanced part of the total wave energy density should not be taken into account when calculating  $K(p)$ , see Berezhinskii et al. (1990) for details. Eqs. (5)-(7) are also valid for small-amplitude nonlinear waves including weak shocks. The equations can be used for the isotropic distribution of Alfvén and fast magnetosonic waves and they give correct order of magnitude estimates for the wave distribution concentrated around the direction of average magnetic field. It should be pointed out however that the isotropization of the diffusion tensor does not occur in the case of a pure parallel propagation of waves ( $\mathbf{k} \parallel \mathbf{B}$ ). Another special case is 2D turbulence with perpendicular propagation of waves ( $\mathbf{k} \perp \mathbf{B}$ ). In this case, the scattering occurs only for magnetosonic waves through the resonance  $s = 0$  which leads to a very large diffusion coefficient, about a factor  $(v/V_a)^2$  larger than given by eqs. (5), (6).

Eq. (6) shows that the level of interstellar turbulence that is needed to account for the diffusion of GeV cosmic rays is very small:  $\delta B_{\text{res}}^2/B^2 = (1-a)r_g/l \sim 10^{-6}$  at  $k_{\text{res}}^{-1} \sim 10^{12}$  cm (if  $a \sim 0.5$ ). An extension to smaller wave numbers gives  $\delta B^2(> k)/B^2 \sim 10^{-6}(10^{12}k)^{1-a}$  where  $k$  is in  $\text{cm}^{-1}$ .

### 3. Equations for Interstellar Turbulence

The description of MHD turbulence is a complicated and not completely solved problem even in the case of small-amplitude random fields. Comprehensive reviews of MHD turbulence have been given by Verma (2004), and by Zhou et al. (2004), Elmergreen & Scalo (2004), and Scalo & Elmergreen (2004) with application to interstellar turbulence.

The classic problem is the determination of the wave spectrum in the presence of sources at small wave numbers  $k \sim k_L$  and the strong dissipation at much larger wave numbers (in some cases the cascade is inverse). Note that the spectrum of interstellar MHD turbulence determines the transport coefficients eqs. (5), (7). According to the Kolmogorov-Obukhov hypothesis (Kolmogorov 1941; Obukhov 1941), the resulting spectrum at intermediate  $k$ , i. e.

in the inertial range, is characterized by a constant energy flux to higher wave numbers. The hypothesis was originally suggested for the description of developed hydrodynamic turbulence in incompressible fluids. The Kolmogorov spectrum is of the form  $W(k) \propto k^{-5/3}$ . The spectrum of weak acoustic turbulence  $W(k) \propto k^{-3/2}$  was found by Zakharov & Sagdeev (1970). This result was criticized by Kadomtsev & Petviashvili (1973). They argued that the developed shocks produce an additional dissipation of acoustic wave energy. The shock-dominated turbulence (also called the Burgers turbulence) is characterized by the spectrum  $k^{-2}$  irrespective of the nature of dissipation at the shock front.

The presence of magnetic field in MHD turbulence complicates the issue because the turbulence becomes anisotropic and new types of waves arise in a magnetized medium. Generally, all gradients are larger perpendicular to the field and all perturbations are elongated along the magnetic field direction even with isotropic excitation. Iroshnikov (1963) and Kraichnan (1965) gave the first phenomenological theory of MHD turbulence and obtained the spectrum  $W(k) \propto k^{-3/2}$ . They assumed that small-scale fluctuations are isotropic, which is contradictory because wave interactions break the isotropy in the presence of an external magnetic field.

Direct observations of MHD turbulence in the solar wind plasma where  $\beta \sim 1$  have shown the existence of a Kolmogorov-type turbulence spectrum that contains waves moving both along the magnetic field and in almost perpendicular directions (Saur & Bieber 1999). Such a spectrum was obtained over several decades of wave numbers in solar wind radio propagation studies (Woo & Armstrong 1979). Numerical simulations of incompressible ( $\beta \gg 1$ ) MHD turbulence favored the Kolmogorov spectrum (Verma et al. 1996).

Over the past decade, there has been a renewed interest in understanding of magneto-hydrodynamic turbulence as it applies to interstellar magnetic field and density fluctuations (Goldreich & Sridhar 1995, 1997; Ng & Bhattacharjee 1997; Galtier et al. 2000, see also earlier work by Shebalin et al. 1983). Goldreich & Sridhar (1995) exploited anisotropy in MHD turbulence and obtained Kolmogorov-like spectrum for the energy density of Alfvén waves. The “elementary interactions” between Alfvén waves satisfies the three-wave resonance conditions. However there is no exact relation between wave number and frequency in this case of strong turbulence. The main part of the energy density in this turbulence is concentrated perpendicular to the local magnetic field wave vectors  $k_{\perp} \approx k$ , while the parallel wave numbers are small:  $k_{\parallel} \sim [kW(k)/(B_0^2/4\pi)]^{1/2} k_{\perp}$ . The cascade is anisotropic with energy confined within the cone  $k_{\parallel} \propto k_{\perp}^{2/3}$ . Numerical simulations have confirmed this concept (Cho & Vishniac 2000).

Although the formalism has been developed for incompressible MHD turbulence, Lithwick & Goldreich (2001) argued that the compressibility does not essentially alter the results



on the Alfvén wave spectrum. The distribution of slow magnetosonic waves passively follows that of Alfvén waves. The fast magnetosonic waves have an independent nonlinear cascade which is isotropic and has a Kraichnan-type spectrum  $W(k) \propto k^{-3/2}$ . These conclusions were supported by numerical simulations by Cho & Lazarian (2002).

The description of weak MHD turbulence in low  $\beta$  plasma outlined above is probably not complete and needs further analysis before it is accepted as a standard model of interstellar turbulence. First, there is still the discrepancy between theoretical results of different authors. Thus considering the scattering of Alfvén waves and fast magnetosonic waves on slow magnetosonic waves, Kuznetsov (2001) found the Kraichnan-type spectra for all these types of waves with their preferentially parallel propagation, which disagrees with Lithwick & Goldreich (2001). Second, consideration of processes in turbulent collisionless plasmas at the kinetic level involves additional nonlinear processes of induced wave scattering on thermal ions (Livshits & Tsytovich 1970) that may change the spectra (Chashei & Shishov 1985). Third, the real turbulence can be strongly intermittent, imbalanced, etc. (e.g. Lithwick & Goldreich 2003), which may also affect the interstellar MHD spectrum.

Information on the extended interstellar turbulence spectrum has been obtained from radio scintillation and refraction observations (sensitive to fluctuations of thermal electron density), measurements of the differential Faraday rotation angles from distant sources (mainly produced by fluctuations in the interstellar magnetic field), and the observations of random motions in the interstellar gas. These data are consistent with the assumption that a single close-to-Kolmogorov spectrum extends from scales  $10^8$  to  $10^{20}$  cm (Lee & Jokipii 1976), see Armstrong et al. (1981, 1995) and references therein.

In the absence of an easily manageable and commonly accepted exact equation for the energy density  $W(k)$ , we employ below the simplest steady state phenomenological equation that represents the concept of a wave cascade in the inertial range of wave numbers:

$$\frac{\partial}{\partial k} \left( \frac{kW(k)}{T_{nl}} \right) = 0 \quad (8)$$

(Tu 1988; Norman & Ferrara 1996; this approach goes back to Chandrasekhar 1948 and Heisenberg 1948). Here the approximation of a characteristic time  $T_{nl}$  is used to account for the non-linear wave interactions which provide the transfer of energy in  $k$ -space<sup>1</sup>.

Formally, the Kolmogorov-type spectrum  $W(k) \propto k^{-5/3}$  follows from eq. (8) if  $T_{nl} =$

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<sup>1</sup>See also Cranmer & van Ballegoijen (2003) and references therein for possible modifications of eq. (8) when a diffusive flux of wave energy in  $k$ -space is assumed and when the equation is written for a power spectrum  $W(\mathbf{k})$  not averaged over direction (here  $d^3kW(\mathbf{k}) = dkW(k)$ ).

$T_A = \left[ C_A k \sqrt{k W(k) / (4\pi\rho)} \right]^{-1}$ , where the constant  $C_A \sim 0.3$  as can be estimated from the simulations of Verma et al (1996). The Kraichnan spectrum  $W(k) \propto k^{-5/3}$  is obtained from eq. (8) if  $T_{nl} = T_M = [C_M k^2 W(k) / (\rho V_a)]^{-1}$ , where  $C_M \sim 1$ .

The Kolmogorov-type nonlinear rate  $1/T_A$  is relatively high since it is proportional to the amplitude of weak random field whereas the Kraichnan rate  $1/T_M$  is proportional to the amplitude squared. Our preliminary estimates (Moskalenko et al. 2003a; Ptuskin et al. 2003a) showed that the MHD cascade with the Kolmogorov rate is not significantly affected by damping on cosmic rays even if the waves propagate along the magnetic field which makes the wave-particle interactions the most efficient. In addition, if the concept of Goldreich & Sridhar (1995) works for interstellar turbulence, then the Kolmogorov rate refers to the Alfvén waves which are distributed almost perpendicular to an external magnetic field and thus do not produce any significant scattering of cosmic rays, see Section 2. So, we do not consider the damping of cascades with the Kolmogorov rate in what follows.

For a cascade with the Kraichnan rate  $1/T_M$  which describes an isotropic or close to parallel distribution of MHD waves (according to Goldreich & Sridhar 1995, it refers only to fast magnetosonic waves but not to Alfvén waves), the equation for wave energy density can be written as follows:

$$\frac{\partial}{\partial k} \left( \frac{C_M}{\rho V_a} k^3 W^2(k) \right) = -2\Gamma(k)W(k) + S\delta(k - k_L), \quad (9)$$

$k \geq k_L$ . Here  $\Gamma = \Gamma_{cr} + \Gamma_{th}$  is the wave attenuation rate on cosmic-ray particles ( $\Gamma_{cr}$ ) and on thermal particles ( $\Gamma_{th}$ ),  $S$  characterizes the source strength. The main sources of interstellar turbulence are supernova explosions, winds of massive stars, superbubbles, and differential rotation of the Galactic disk. These sources produce strong random magnetic fields and probably initiate nonlinear wave cascades at the scale  $k_L^{-1} \sim 100$  pc.

Below we ignore the dissipation on thermal particles (see discussion below) and set  $\Gamma_{th} = 0$  to study purely the effect of damping on cosmic rays. We actually deal with a small part of the possible global wave spectrum at wave numbers  $k \gtrsim 10^{-14} \text{ cm}^{-1}$  which resonantly scatter cosmic rays with energies less than about 100 GeV/n. The existence of a single interstellar MHD spectrum over 12 orders of magnitude is an unsolved problem in itself and is beyond the scope of the present work. The quantities  $k_L$  and  $W(k_L)$  are used here only for purposes of normalization.

The equation for wave amplitude attenuation on cosmic rays is (Berezinskii et al. 1990):

$$\Gamma_{cr}(k) = \frac{\pi Z^2 e^2 V_a^2}{2k c^2} \int_{p_{res}(k)}^{\infty} \frac{dp}{p} \Psi(p), \quad (10)$$

where  $p_{res}(k) = ZeB/ck$ .

The solution of eqs. (9), (10) allows us to find the wave spectrum:

$$W(k) = k^{-3/2} \left[ k_L^{3/2} W(k_L) - \frac{Z^2 e^2 B^2 V_a}{8 C_M c^2} \int_{k_L}^k dk_1 k_1^{-5/2} \int_{p_{res}(k_1)}^{\infty} \frac{dp \Psi(p)}{p} \right], \quad (11)$$

$k > k_L$ . Here the spectral wave density at the principal scale is determined by the source strength:  $W(k_L) = \sqrt{\rho V_a S / C_M} k_L^{-3/2}$ . The second term in square brackets is increasing with  $k$ . The wave damping on cosmic rays decreases the wave energy density and can even terminate the cascade if the expression in square brackets reduces to zero at some  $k = k_*$ . The wave energy density should be set to zero at  $k > k_*$  in this case.

Now with the use of eqs. (5), (11) one can determine the cosmic ray diffusion coefficient, which is

$$D(p) = \frac{D_0(p)}{1 - g \int_p^{p_L} dp_2 p_2^{1/2} \int_{p_2}^{\infty} dp_1 p_1^{-1} \Psi(p_1)}, \quad (12)$$

$$g = \frac{3\pi V_a p^{1/2} D_0(p)}{2 C_M B^2 r_g v} = \frac{\sqrt{ZeB^2}}{16 \sqrt{\pi \rho c} C_M k_L^{3/2} W(k_L)},$$

$p < p_L$ ,  $p_L = p_{res}(k_L)$ . Here  $D_0(p) = vr_g^{1/2} B^2 / [12\pi k_L^{3/2} W(k_L)] \propto v(p/Z)^{1/2}$  is the diffusion coefficient calculated for the Kraichnan-type spectrum without considering wave damping. The second term in the denominator of eq. (12) describes the modification of the diffusion coefficient due to wave damping. The effect becomes stronger to smaller  $p$ . The diffusion coefficient should be formally set to infinity at  $p < p_*$  if the square bracket in eq. (11) goes to zero at some  $p = p_*$ . The constant  $g$  characterizes the strength of the effect for a given cosmic ray spectrum  $\Psi(p)$ .

As the most abundant species, the cosmic ray protons mainly determine the wave dissipation. Thus, with good precision only the proton component with  $Z = 1$  needs to be taken into account to calculate  $D(p)$  by the simultaneous solution of eqs. (1) and (12). The diffusion mean free path for other nuclei of charge  $Z$  is  $l(p/Z)$  if it is  $l(p)$  for protons.

Let us estimate the effect of wave damping. Assuming that the cosmic-ray energy density is about  $1 \text{ eV/cm}^3$ , the diffusion coefficient  $D = 3 \times 10^{28} \text{ cm}^2/\text{s}$  at  $1 \text{ GeV}$ ,  $V_a = 10 \text{ km/s}$ ,  $B = 5 \mu\text{G}$ , and  $C_M = 1$ , the second term in the denominator in eq. (12) is about unity at GeV energies and falls at higher energies. We conclude that the Kraichnan-type cascade is significantly affected by damping on cosmic rays, and this should lead to the modification of cosmic-ray transport at energies less than about  $10 \text{ GeV/n}$ .

#### 4. Simple Selfconsistent Model

To demonstrate the effect of wave damping, let us consider a simple case of one-dimensional diffusion with source distribution  $q = q_0(p)\delta(z)$  (corresponding to an infinitely thin disk of cosmic-ray sources located at the Galactic mid-plane  $z = 0$ ) and a flat cosmic-ray halo of height  $H$ , see Jones et al. (2001). The source spectrum at  $R < 40$  GV is  $q_0 \propto p^{-\gamma_s}$ , where  $\gamma_s = 2.5$  approximately. Considering energetic protons, we ignore energy losses and nuclear fragmentation ( $\dot{p}_{\text{loss}} = 0$ ,  $1/\tau = 0$ ) and assume that a Galactic wind is absent ( $u = 0$ ) in eq. (1). Let us also assume that stochastic reacceleration does not significantly change the energies of cosmic-ray particles during the time of their diffusive leakage from the Galaxy and set  $K = 0$  in eq. (1). The solution of eq. (1) in the Galactic disk is then

$$\Psi(p) = \frac{q_0(p)H}{2D(p)}. \quad (13)$$

The escape length (the “grammage”), which determines the production of secondaries during the cosmic-ray leakage from the Galaxy, is equal to  $X = \mu_g v H / (2D)$ , where  $\mu_g$  is the gas surface density of the Galactic disk.

The simultaneous solution of eqs. (12) and (13) allows us to find  $\Psi(p)$  and  $D(p)$ . Introducing the function  $\Psi_0(p) = q_0(p)H/[2D_0(p)]$ , which is the cosmic ray spectrum for an unmodified Kraichnan spectrum, and substituting  $\Psi$  from eq. (13) into eq. (12), one finds after two differentiations the following equation for the ratio  $\varphi = D_0(p)/D(p)$  as a function of  $x = p^{3/2}$ :

$$\frac{d^2\varphi(x)}{dx^2} = -\frac{4g\Psi_0[p(x)]}{9x}\varphi(x), \quad (14)$$

with the constraint  $\varphi(\infty) = 1$ . Note that the same equation is valid for the ratio  $\Psi(p)/\Psi_0(p)$ .

The function  $\Psi_0/x$  in r.h.s. of eq. (14) can be approximated by a power law function proportional to  $x^{-b}$ ,  $b = \text{const}$ . Thus for  $\gamma_s = 2.5$ , one has  $b = 3$  for ultrarelativistic ( $v = c$ ) and  $b = 11/3$  for nonrelativistic ( $v \ll c$ ) protons respectively. This allows us to find the solution of eq. (14) in an explicit form. The diffusion coefficient  $D(p) = D_0(p)/\varphi(p)$  is then:

$$D(p) = D_0(p) \frac{w^{1/(b-2)}(p)}{\Gamma[(b-1)/(b-2)] J_{1/(b-2)}[2w(p)]}, \quad w(p) = \frac{2}{3(b-2)} \sqrt{gp^{3/2}\Psi_0(p)}, \quad (15)$$

where  $\Gamma(z)$  is the gamma function,  $J_\nu(z)$  is the Bessel function of the first kind. With  $p$  decreasing from infinity and the corresponding increase of  $w(p)$  from zero, the Bessel function goes to zero at some  $w(p_*) = w_*$  (where  $w_* = 1.92$  at  $b = 3$ , and  $w_* = 1.64$  at  $b = 11/3$ ). This means that  $D(p)$  becomes infinite at  $p = p_*$  because of complete termination of the cascade;  $D(p) = \infty$  at  $p \leq p_*$ .

The expansion of the Bessel function for small and large arguments results in the following approximations for eq. (15):

$$D(p) \approx \frac{D_0(p)}{1 - \{\Gamma[(b-1)/(b-2)]/\Gamma[(2b-3)/(b-2)]\} w^2(p)} \text{ at } w(p) \ll 1, \quad (16)$$

and

$$D(p) \approx D_0(p) \frac{\sqrt{\pi} w^{b/[2(b-2)]}}{\Gamma[(b-1)/(b-2)] \sin\left[\frac{\pi}{4}(3b-4)/(b-2) - 2w(p)\right]} \text{ at } 1 \lesssim w(p) < w_*. \quad (17)$$

The last approximate expression for  $D(p)$  has a singularity at  $w(p) = w_* = (\pi/8)(3b - 4)/(b - 2)$ ;  $w_* = 1.96$  at  $b = 3$ , and  $w_* = 1.65$  at  $b = 11/3$  which is close to the values obtained from eq. (15). The corresponding  $p_*$  can be found from the equation  $p_*^{3/2} \Psi_0(p_*) = [3\pi(3b - 4)/16]^2 g^{-1}$ .

Eqs. (15)-(17) exhibit the rapid transition from unmodified to infinite diffusion. Fig. 1 demonstrates that eq. (15) derived in the considerably simplified model of cosmic-ray propagation (dash-dot) agrees well with the shape of the diffusion coefficient found in the numerical simulations (solid) described in the next Section. The results of this Section can be used for approximate estimates of cosmic ray diffusion coefficient without invoking the full-scale calculations based on the GALPROP code.

## 5. Numerical Solution with GALPROP Code

The numerical calculations have been made using the GALPROP code for cosmic-ray propagation. GALPROP is a flexible numerical code written in C++ which incorporates *as much realistic* astrophysical input *as possible* together with latest theoretical developments. The model is designed to perform cosmic-ray propagation calculations for nuclei (isotopes of H through Ni), antiprotons, electrons and positrons, and computes  $\gamma$ -rays and synchrotron emission in the same framework. The current version of the model includes substantial optimizations in comparison to the older model and a full 3-dimensional spatial grid for all cosmic-ray species. It explicitly solves the full nuclear reaction network on a spatially resolved grid. GALPROP models have been described in detail elsewhere (Strong & Moskalenko 1998; Moskalenko et al. 2002, 2003b). For the present calculation we use three-dimensional Galactic models with cylindrical symmetry:  $(R, z, p)$  – spatial variables plus momentum.

GALPROP solves the transport equation eq. (1) for all cosmic-ray species using a Crank-Nicholson implicit second-order scheme. The spatial boundary conditions assume free particle escape. The diffusion coefficient as a function of momentum and the reacceleration or

convection parameters are determined by boron-to-carbon (B/C) ratio data. The spatial diffusion coefficient is taken as  $D = \kappa\beta(R/R_0)^a$ ,  $\kappa = \text{const}$ ,  $\beta = v/c$ , if necessary with a break ( $a = a_1$  below rigidity  $R_0$ ,  $a = a_2$  above rigidity  $R_0$ ). The injection spectrum of nucleons is assumed to be a power law in momentum,  $q(p) \propto p^{-\gamma_s}$  and may also have breaks. To account for stochastic reacceleration in the interstellar medium, the momentum-space diffusion coefficient  $K$  is related to the spatial coefficient  $D$  via the Alfvén speed  $V_A$ , see eq. (7).

The interstellar hydrogen distribution (Moskalenko et al. 2002) uses H I and CO surveys and information on the ionized component; the helium fraction of the gas is taken as 0.11 by number. The H<sub>2</sub> and H I gas number densities in the Galactic plane are defined in the form of tables, which are interpolated linearly. The conversion factor is taken as  $X_{\text{CO}} \equiv N_{\text{H}_2}/W_{\text{CO}} = 1.9 \times 10^{20} \text{ mols. cm}^{-2}/(\text{K km s}^{-1})$  (Strong & Mattox 1996). The extension of the gas distribution to an arbitrary height above the plane is made using analytical approximations. The halo size is assumed to be  $H = 4$  kpc. The distribution of cosmic-ray sources is chosen to reproduce the cosmic-ray distribution determined by analysis of EGRET  $\gamma$ -ray data (Strong & Mattox 1996; Strong & Moskalenko 1998):

$$q(R, z) = q_0 \left( \frac{R}{R_\odot} \right)^\alpha \exp \left( -\beta \frac{R - R_\odot}{R_\odot} - \frac{|z|}{0.2 \text{ kpc}} \right), \quad (18)$$

where  $q_0$  is a normalization constant,  $R_\odot = 8.5$  kpc,  $\alpha = 0.5$  and  $\beta = 1.0$  are parameters. We note that the adapted source distribution<sup>2</sup> is flatter than the distribution of SNR (Case & Bhattacharya 1998) and pulsars (Lorimer 2004).

The code includes cross-section measurements and energy dependent fitting functions (Strong & Moskalenko 2001). The nuclear reaction network is built using the Nuclear Data Sheets. The isotopic cross section database is built using the extensive T16 Los Alamos compilation of the cross sections (Mashnik et al. 1998) and modern nuclear codes CEM2k and LAQGSM (Mashnik et al. 2004). The most important isotopic production cross sections (<sup>2</sup>H, <sup>3</sup>H, <sup>3</sup>He, Li, Be, B, Al, Cl, Sc, Ti, V, Mn) are calculated using our fits to major production channels (e.g., Moskalenko et al. 2001, 2003b; Moskalenko & Mashnik 2003). Other cross sections are calculated using phenomenological approximations by Webber et al. (1990) (code WNEWTR.FOR versions of 1993 and 2003) and/or Silberberg and Tsao (Silberberg et al. 1998) (code YIELDX\_011000.FOR version of 2000) renormalized to the

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<sup>2</sup>A recent analysis has shown that the apparent discrepancy between the radial gradient in the diffuse Galactic  $\gamma$ -ray emissivity and the distribution of SNR, believed to be the sources of cosmic rays, can be plausibly solved by adopting a conversion factor  $X_{\text{CO}}$  which increases with  $R$  (see a discussion in Strong et al. 2004).

data where it exists. For  $pA$  inelastic cross section we adapted the parametrization by Barashenkov & Polanski (code CROSEC, Barashenkov 1993; Barashenkov & Polanski 1994). The details of proton and antiproton cross sections are given in Moskalenko et al. (2002). Secondary positron and electron production is computed using the formalism described in Moskalenko & Strong (1998), which includes a reevaluation of the secondary  $\pi^\pm$ - and  $K^\pm$ -meson decay calculations. Energy losses for nucleons and leptons by ionization, Coulomb scattering, bremsstrahlung, inverse Compton scattering, and synchrotron are included in the calculations. The heliospheric modulation is treated using the force-field approximation (Gleeson & Axford 1968).

The reaction network is solved starting at the heaviest nuclei (i.e.,  $^{64}\text{Ni}$ ). The propagation equation is solved, computing all the resulting secondary source functions, and then proceeds to the nuclei with  $A - 1$ . The procedure is repeated down to  $A = 1$ . In this way all secondary, tertiary etc. reactions are automatically accounted for. To be completely accurate for all isotopes, e.g., for some rare cases of  $\beta^\pm$ -decay, the whole loop is repeated twice.

For the present work, a new iterative procedure which includes a self-consistent determination of the cosmic-ray diffusion coefficient was developed. In the first step, cosmic-ray propagation is calculated using the undisturbed diffusion coefficient  $D_0(p)$ . In the second, we use the propagated proton spectrum at every spatial grid point to re-calculate the modified diffusion coefficient according to eq. (12). In the third step, cosmic-ray propagation is calculated using the new diffusion coefficient. Steps 2 and 3 are repeated until convergence is obtained. The convergence is fast and the procedure requires only few iterations. In this way, using the secondary to primary ratio  $B/C$ , one can derive the damping constant  $g = 0.085$  which is considered as an adjustable parameter.

The new self-consistent model of diffusive reacceleration with damping (DRD model) is compared with two reference models which were earlier found to give good fits to cosmic ray data: the plain diffusion model (PD model) with an ad-hoc break in the diffusion coefficient and the model with distributed reacceleration (DR model) and power-law diffusion with no breaks (Moskalenko et al. 2002). The parameters of these models found from the fit to the observed spectra of primary and secondary nuclei are summarized in Table 1. Note that the parameters of the DRD model given in Table 1 are the “seed” values used in the iteration procedure. The diffusion coefficient after iteration calculations is shown in Fig. 1.

The diffusion coefficients in all three models are presented in Fig. 1. The DR model diffusion coefficient has a weak energy dependence with a single index  $1/3$  as dictated by the assumed Kolmogorov spectrum of interstellar turbulence. The PD model requires a break in the diffusion coefficient and an additional factor of  $\beta^{-3}$  to be able to match the  $B/C$  ratio at low energies. The DRD model diffusion coefficient lies between these at high energies

and has a sharp increase at 1 GV. Of course, the actual mean free path length can not be infinite. We put an upper limit of  $\sim 15$  pc based on the estimate of streaming instability effect that arises at low rigidities and leads to the generation of additional turbulence which limits cosmic ray escape from the Galaxy, see discussion below in Section 6. The selfconsistent spectrum of waves calculated in the DRD model is shown in Fig. 2. The spectrum bends downward at wavenumbers  $k > 6 \times 10^{-12} \text{ cm}^{-1}$  due to the wave damping on cosmic rays.

The B/C ratio and Carbon spectra (primary) after modulation look almost identical in all three models (Figs. 3, 4). The modulated proton spectrum (Fig. 5) matches the data in all three cases after some tuning, while the interstellar spectra are different. The most dramatic difference is the reduction of proton flux in the DRD model at low energies, consistent with the steep increase of the diffusion coefficient. The antiproton spectrum (Fig. 6) appears to be very sensitive to the model assumptions and might help to discriminate between the models. The DR model produces too few antiprotons, a well known effect (Moskalenko et al. 2002). The PD and DRD models are both consistent with antiproton measurements in the heliosphere, but predict very different spectra in the interstellar medium; the interstellar antiproton flux at low energies ( $< 600$  MeV) in DRD model is an order of magnitude lower than in two other models. In both reacceleration models (DR and DRD), the majority of low-energy antiprotons come from inelastic scattering (so-called “tertiary” antiprotons).

Figs. 7 and 8 show secondary positrons and primary plus secondary electrons as calculated in all three models. The spectra are similar in the PD and DR models, while DRD spectra exhibit lower intensities at low energies. This may be an observable effect since the models predict different synchrotron emission spectra (electrons).

## 6. Discussion

Damping on cosmic rays may terminate the slow Kraichnan-type cascade in the interstellar medium at  $k \sim 10^{-12} \text{ cm}^{-1}$ . Our estimates were made for the level of MHD turbulence which produces the empirical value of cosmic-ray diffusion coefficient. This finding suggests a possible explanation for the peaks in secondary/primary nuclei ratios at about 1 GeV/n observed in cosmic rays: the amplitude of short waves is small because of damping and thus the low energy particles rapidly exit the Galaxy without producing many secondaries. There is no other obvious reasons for a sharp cut off in the wave spectrum. If the concept of MHD turbulence by Goldreich & Sridhar (1995) works for interstellar turbulence, the MHD waves we are dealing with in this context are the fast magnetosonic waves. The Alfvén waves propagate predominantly perpendicular to the magnetic field and because of this they do not significantly scatter cosmic rays. It also explains why radio scintillation observations show



no sign of the termination of electron density fluctuations at wave numbers between  $10^{-14}$  to  $10^{-8}$   $\text{cm}^{-1}$ . According to Lithwick & Goldreich (2001) these fluctuations are produced by the slow magnetosonic waves with  $k_{\perp} \gg k_{\parallel}$  which are almost not damped on cosmic rays. An alternative explanation is that the wave damping on cosmic rays and the radio scintillations mainly occur in separate regions of the interstellar medium (see below in the discussion on the “sandwich” model of cosmic ray propagation in the Galaxy). Some minor contribution to the observed scintillations is possible from fast magnetosonic waves interacting with energetic particles, and in this respect it is of interest that the observations may need an enhancement in the power on large “refractive” scales  $10^{13} - 10^{14}$  cm relative to the power on small “diffractive” scales  $10^9 - 10^{10}$  cm (Lambert & Rickett 2000). This may indicate the cutoff of the spectrum of fast magnetosonic waves due to cosmic ray action.

While the mere fact of a wave spectrum steepening under the action of damping on cosmic rays can be described by a simple eq. (9) with some characteristic time for nonlinear wave interactions  $T_{nl}[W(k), k]$ , the exact form of the function  $W(k)$  at large  $k$  where the damping is significant depends critically on the form of the equation for waves. It involves in particular to the vanishing of  $W(k)$  at some  $k_*$  ( $\sim 10^{-12}$   $\text{cm}^{-1}$ ) and the corresponding singularity of  $D(p)$  at some  $p_*$  ( $\sim 1$  GV) found in our calculations. Less significant in this sense is our approximation of the resonant wave number in eq. (5), which does not include the particle pitch angle in an explicit form. (If it were included, the term with  $g$  in eq. [9] would have a third integration over the pitch angle.) We note however that this effect was included in the derivation of eq. [10] for the attenuation rate.

In the context of the approximations adopted in the present work, the problem of cosmic-ray transport at  $p < p_*$  arises. The free streaming of cosmic rays from the Galaxy leads to an instability and to the growth of waves which scatter particles and thus slow down the streaming, see e.g. Berezhinskii et al. (1990). We shall consider the processes at low energies in a separate work. Here we note only that the streaming instability develops above the Galactic disk at  $D(1 \text{ GV}) \gtrsim V_a H \sim 3 \times 10^{29}$   $\text{cm}^2/\text{s}$  (if the wave damping is absent) and leads to the diffusive-convective transport of cosmic rays. The given estimate of the diffusion coefficient at 1 GV follows from the condition of cosmic ray streaming instability  $U_{cr} > V_a$  where  $U_{cr}$  is the bulk velocity of cosmic ray gas. The bulk velocity is  $U_{cr} \approx \delta_{cr} c$  where the cosmic ray anisotropy perpendicular to galactic disk is  $\delta_{cr} \approx D/cH$ . The Alfvén velocity in galactic halo is about  $2 \times 10^7$   $\text{cm/s}$ . It is important that the magnetic rigidity 1 GV corresponds to a kinetic energy 0.43 GeV for protons, and 0.13 GeV/n for nuclei with charge to mass ratio  $Z/A = 1/2$ . The Galactic spectrum of cosmic-ray protons and nuclei at such low energies can not be derived from direct observations at the Earth because of strong modulation in the solar wind.

Dissipation other than on cosmic rays has been neglected in the present work, though it may completely destroy the MHD cascade or considerably change its angular distribution and thus affect  $D(p)$  in a large part of the interstellar medium, see McIvor (1977), Cesarsky (1980), Yan & Lazarian (2004). The region of the cosmic ray halo is the most “safe” in this sense (Yan & Lazarian 2004). In particular, dissipation on ion-neutral collisions may destroy MHD turbulence in the Galactic disk but not in the halo where neutrals are absent. We then come to the “sandwich” model for cosmic-ray propagation, with different diffusion coefficients in the Galactic disk,  $D_g$ , and in the halo,  $D_h$  (it is assumed here that some scattering is present in the Galactic disk and the diffusion approximation works). In this case, the mean matter thickness traversed by cosmic rays does not depend on diffusion inside the disk even at zero gas density in the halo:  $X = \frac{\mu_g v}{2} \left( \frac{h}{2D_g} + \frac{H}{D_h} \right) \approx \mu_g v H / (2D_h)$  (Ginzburg & Ptuskin 1976). The energy dependence of secondary to primary ratios in cosmic rays is determined by the diffusion coefficient in the cosmic ray halo where the model developed in the present paper is applied.

The estimate based on the empirical value of the diffusion coefficient for GeV particles (see Section 1) gives the level of turbulence at the principal scale  $\delta B_{\text{tot}}^2 / B^2 \sim 0.03$  for a Kraichnan-type spectrum  $W(k) \propto k^{-3/2}$ , and  $\delta B_{\text{tot}}^2 / B^2 \sim 1$  for a Kolmogorov-type spectrum  $W(k) \propto k^{-5/3}$ , if  $k_L = 10^{-21} \text{ cm}^{-1}$ . At the same time, the data on Faraday rotation angles favor the Kolmogorov spectrum with  $\delta B_{\text{tot}}^2 / B^2 \sim 1$  and  $k_L = 10^{-21} \text{ cm}^{-1}$ . The cascades of Alfvén waves (with the scaling  $k^{-5/3}$ ) and the fast magnetosonic waves ( $k^{-3/2}$ ) are independent in the Goldreich & Sridhar (1995) model of MHD turbulence, and the amplitude of Alfvén wave cascade may dominate at the principle scale. Also, in the “sandwich” model described in a previous paragraph, the turbulence, which determines the confinement of cosmic rays in the Galaxy, is distributed in the halo of size  $H \sim 4 \text{ kpc}$  whereas the observations of interstellar turbulence refer to the Galactic disk and the adjacent region, where it can be much stronger.

## 7. Conclusions

On the whole, the empirical diffusion model for cosmic rays with energies from  $10^8$  to  $10^{17} \text{ eV}$  implies the presence of random magnetic field with an extended power law spectrum of fluctuations  $W(k) \propto k^{-2+a}$ ,  $a \lesssim 0.5$  at wave numbers from  $3 \times 10^{-12}$  to  $10^{-20} \text{ cm}^{-1}$ . The existence of such a turbulence spectrum in the interstellar medium seems confirmed by various astronomical observations. It should be emphasized that this does not prove the existence of a spectral cascade or spectral transfer throughout this enormous wavenumber range, however tempting that conclusion may be. Among other puzzles, an oddity is the

absence of a spectral feature on spatial scales where ion-neutral collisional processes should be most pronounced. The two special cases of turbulence spectrum with  $a = 1/3$  and  $a = 1/2$  which correspond to the Kolmogorov and the Kraichnan spectra respectively are used in popular versions of the diffusion model as described in Section 5. In both cases, as the present work has demonstrated, one can get a satisfactory fit to the data on energy spectra of secondary and primary nuclei if account is taken for wave damping on cosmic rays for a slow Kraichnan cascade. The fit can be obtained either in the DR model with reacceleration on the Kolmogorov spectrum with no significant effect of cosmic ray damping (because the Kolmogorov cascade is fast) or in the DRD model with relatively weak reacceleration on the Kraichnan spectrum which is significantly modified by cosmic ray damping. Some problems still remain to be solved. The main difficulties with the Kolmogorov spectrum are, firstly, the contradiction with the leading theory of MHD turbulence where this spectrum is associated with perpendicular propagating Alfvén waves which almost do not scatter cosmic-ray particles; and secondly, the low flux of antiprotons characteristic of models with relatively strong reacceleration. A major problem of concern for diffusion on a Kraichnan spectrum is the relatively strong dependence of diffusion on energy which leads to an unacceptably large anisotropy of cosmic rays especially above  $10^{14}$  eV, see Jones et al. (2001), Ptuskin et al. (2003b).

Let us emphasize again that the models of cosmic ray propagation discussed in the present paper assume that the MHD turbulence required for cosmic-ray scattering is produced by some external sources. An alternative is the Galactic wind model by Zirakashvili et al. (1996) and Ptuskin et al. (1997). In this model, the cosmic-ray pressure drives a wind with a frozen-in regular magnetic field which is shaped into huge spirals (the radius of the Galactic wind cavity is about 300 Kpc). The MHD turbulence in the wind is created by the streaming instability of cosmic rays moving predominantly along the regular magnetic field lines outward from the Galactic disk. The level of turbulence is regulated by the non-linear Landau damping on thermal ions. The model explains well the cosmic-ray data up to ultra-high energies  $\sim 10^{17}$  eV with the exception of the observed low anisotropy (about the same difficulty with anisotropy as occurs in the diffusion model with a given Kraichnan spectrum). As regards interpretation of cosmic-ray observations, a preference cannot yet be given to any of the models of cosmic-ray transport in the Galaxy discussed above.

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Table 1. Propagation parameter sets.

Model	Injection index <sup>a</sup>		Break rigidity, GV	Diffusion coefficient @ 3 GV		Alfvén speed, $V_a$ , km s <sup>-1</sup>	galdef-file
	Nucleons, $\gamma_s$	Electrons, $\gamma_e$		$\kappa$ , cm <sup>2</sup> s <sup>-1</sup>	Index, $a$		
Plain Diffusion (PD)	2.30/2.15	2.40	40	$2.2 \times 10^{28}$	0.0/0.60 <sup>b</sup>	—	44_999726
Diffusive Reacceleration (DR)	1.80/2.40	1.60/2.50	4	$5.2 \times 10^{28}$	0.34	36	44_599278
Diffusive Reacceleration with Damping (DRD)	2.40/2.24	2.70	40	$2.9 \times 10^{28}$	0.50	22	44_999714kr

Note. — Adopted halo size  $H = 4$  kpc.

<sup>a</sup>Index below/above the break rigidity.

<sup>b</sup>Index below/above  $R_0 = 3$  GV;  $D = \beta^{-2}\kappa(R/R_0)^a$ .

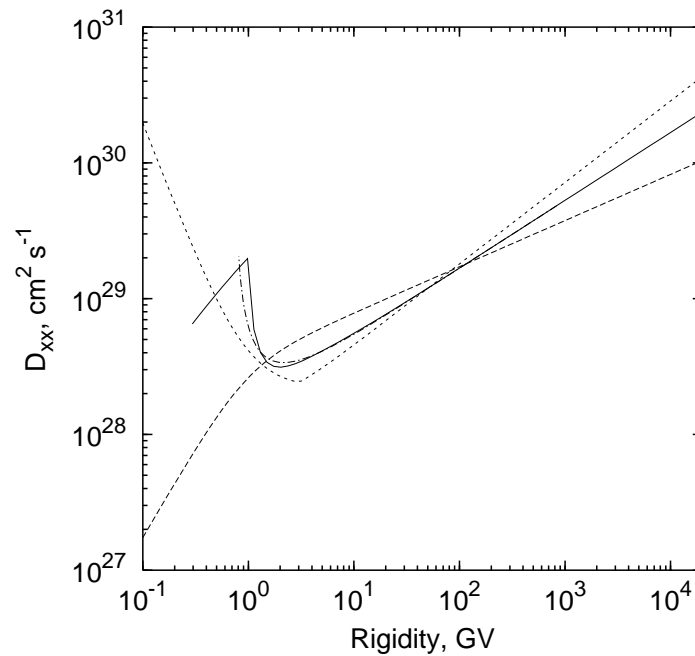


Fig. 1.— Diffusion coefficient  $D$  for the models being discussed in this paper: plain diffusion model (PD model) – dots, reacceleration model (RD model) – dashes, and diffusive reacceleration with damping model (DRD model) – solid, an approximate solution (eq. [15] with  $b = 3$ ) is shown by the dash-dot line.

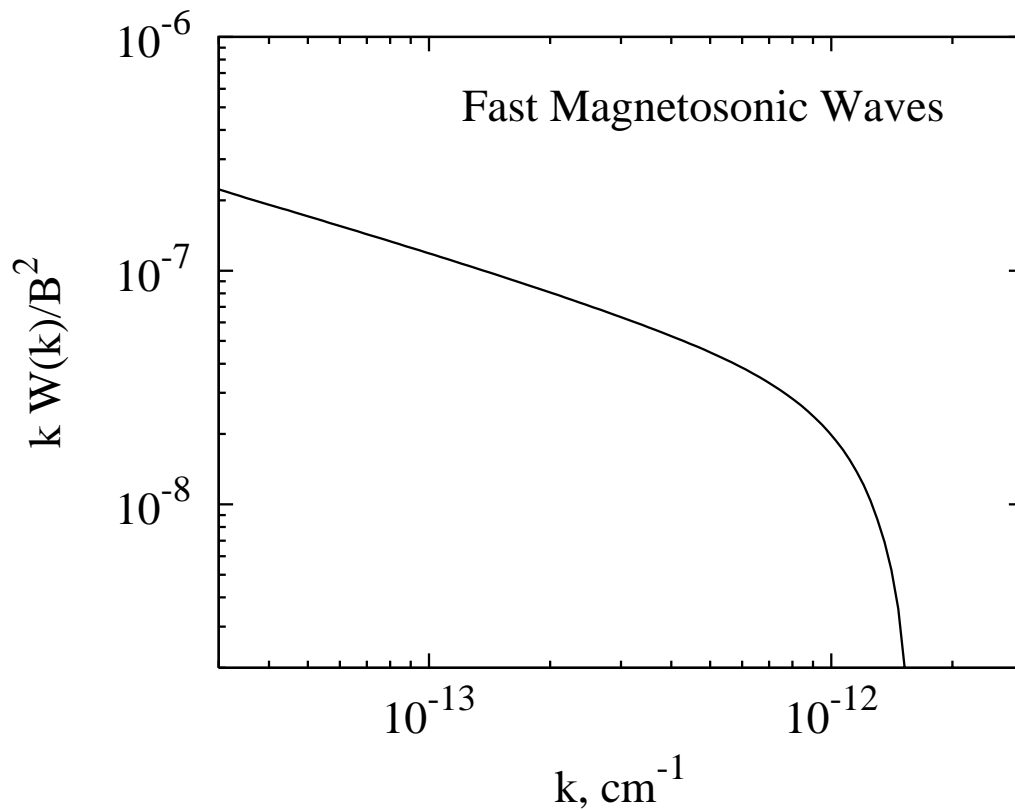


Fig. 2.— Dimensionless spectrum of fast magnetosonic waves  $kW(k)/B^2$  modified by interactions with cosmic rays in DRD model.

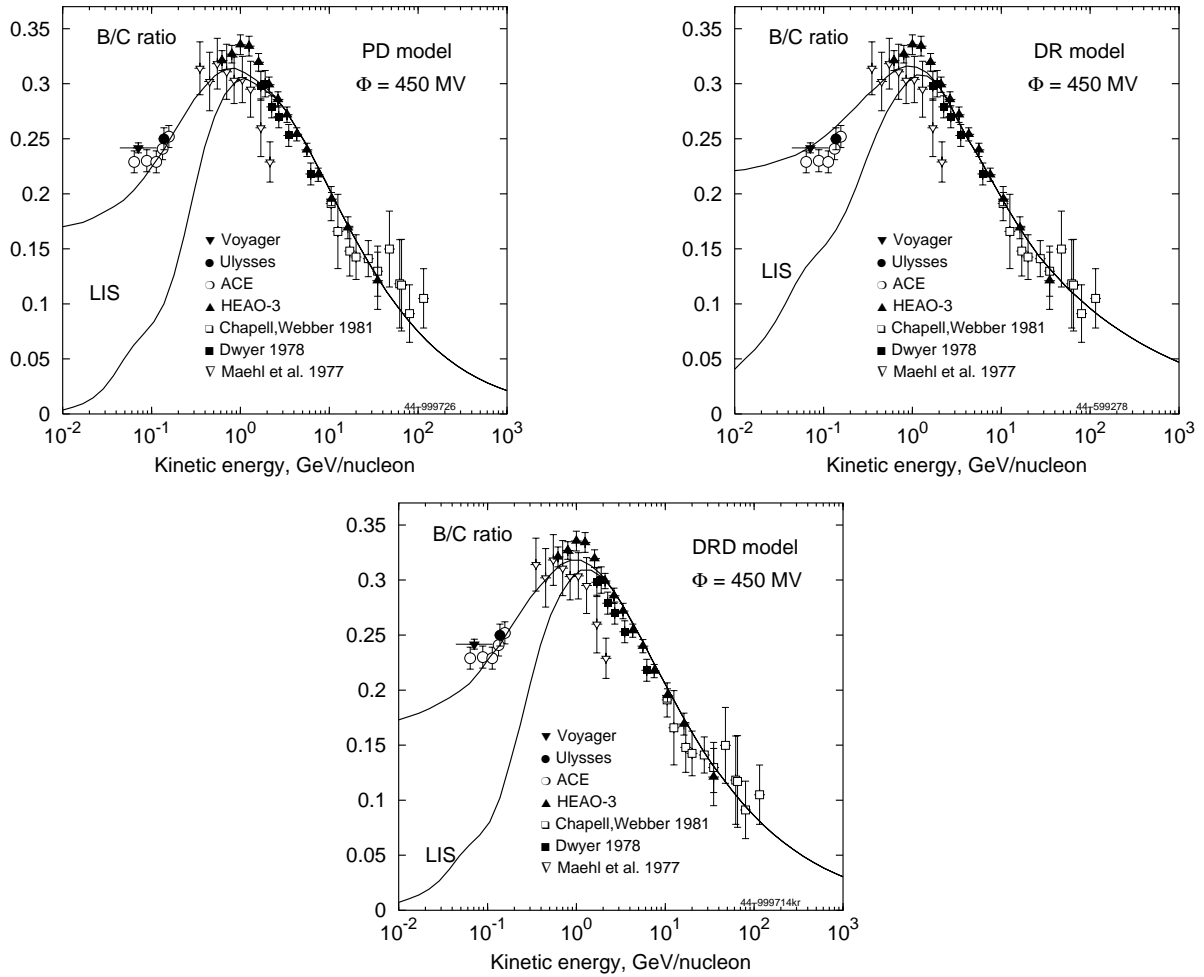


Fig. 3.— B/C ratio as calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). Lower curve – LIS, upper – modulated ( $\Phi = 450$  MV). Data below 200 MeV/nucleon: ACE (Davis et al. 2000), Ulysses (DuVernois, Simpson, & Thayer 1996), Voyager (Lukasiak, McDonald, & Webber 1999); high energy data: HEAO-3 (Engelmann et al. 1990), for other references see Stephens & Streitmatter (1998).

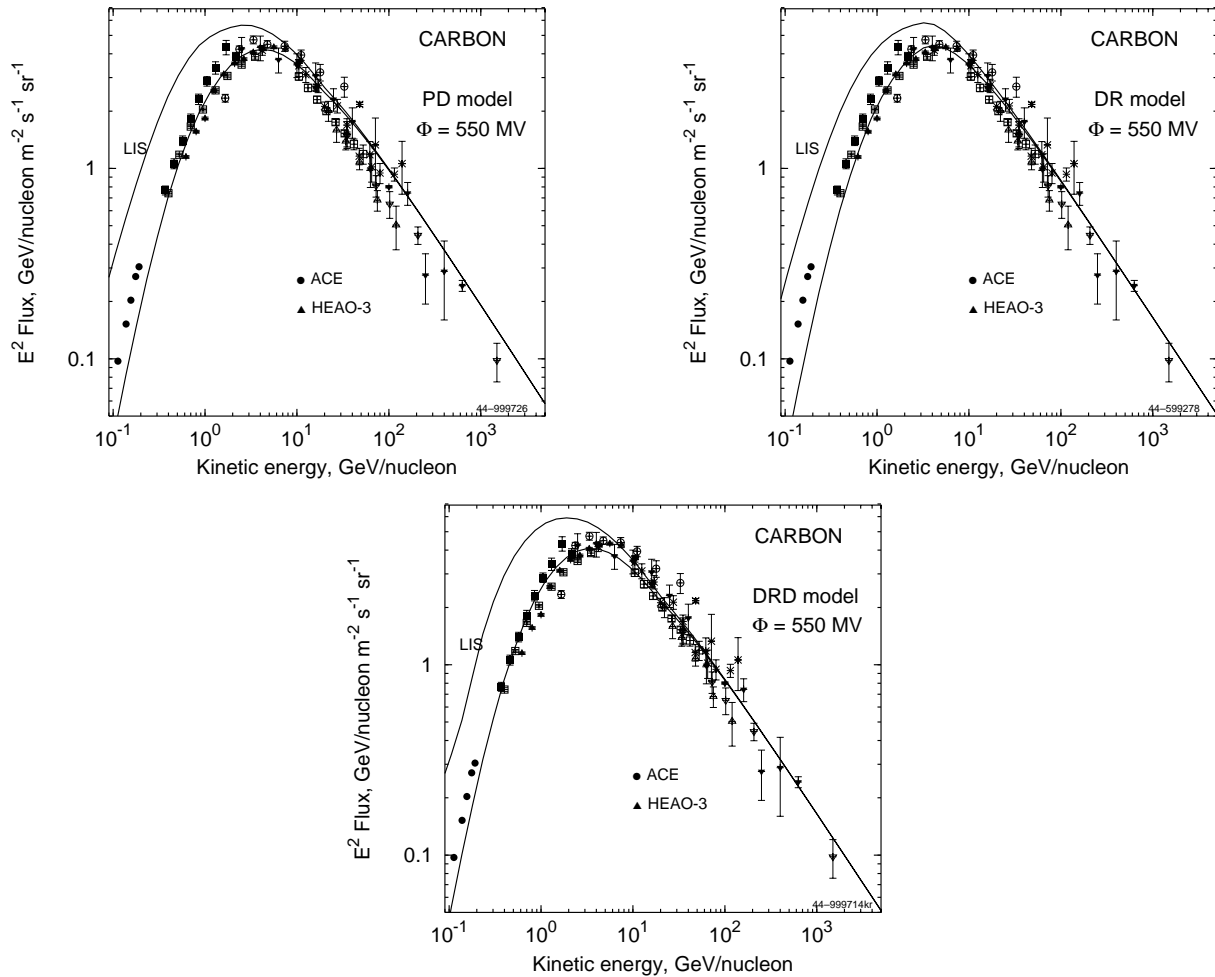


Fig. 4.— Spectrum of carbon calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). Upper curves – LIS, lower curves modulated using force field approximation ( $\Phi = 550 \text{ MV}$ ). Data: ACE (Davis et al. 2000, 2001), HEAO-3 (Engelmann et al. 1990), for other references see Stephens & Streitmatter (1998) (symbols are changed).

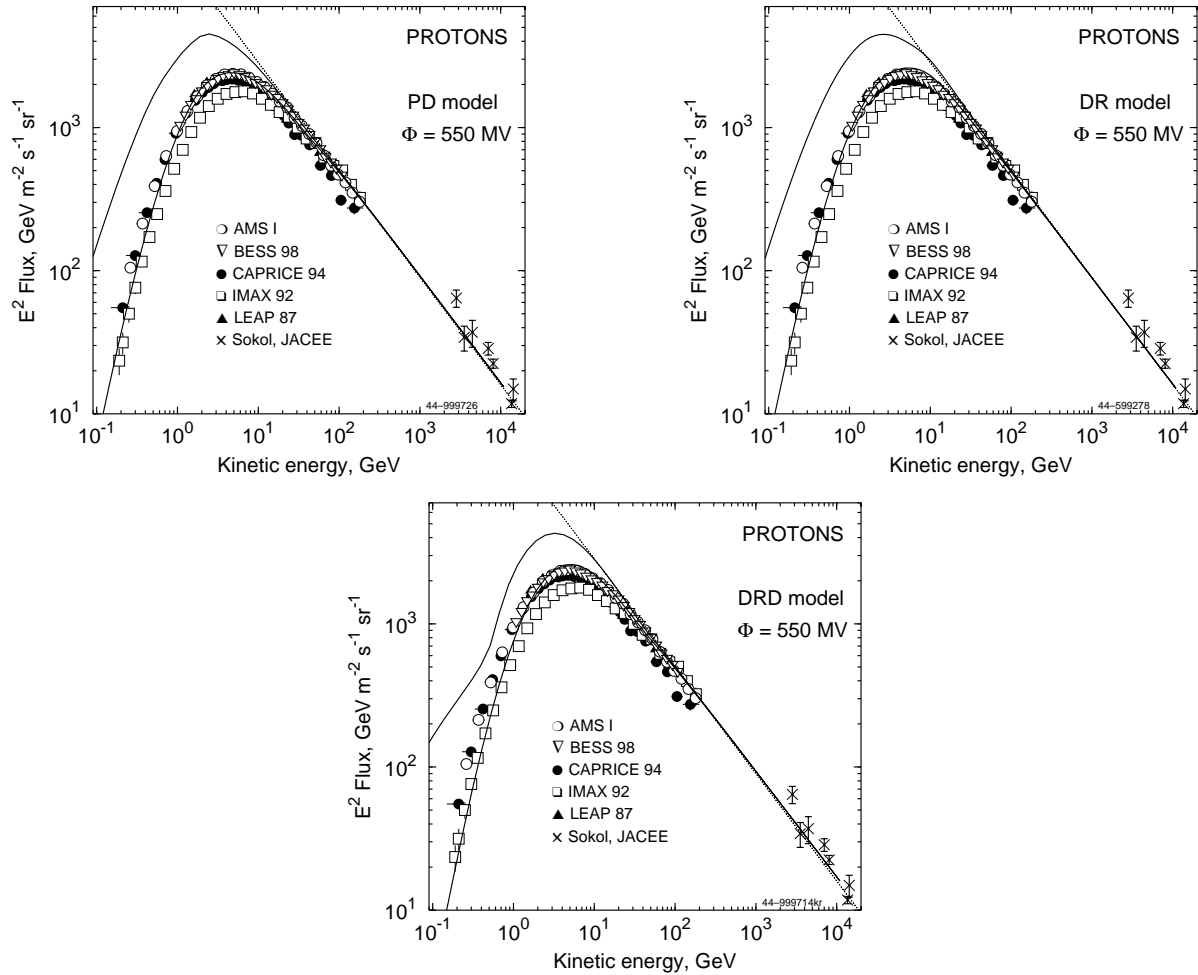


Fig. 5.— Proton spectra as calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). Upper curve – LIS, lower – modulated to 550 MV. Thin dotted line shows the LIS spectrum best fitted to the data above 20 GeV (Moskalenko et al. 2002). Data: AMS (Alcaraz et al. 2000b), BESS 98 (Sanuki et al. 2000), CAPRICE 94 (Boezio et al. 1999), IMAX 92 (Menn et al. 2000), LEAP 87 (Seo et al. 1991), Sokol (Ivanenko et al. 1993), JACEE (Asakimori et al. 1998).

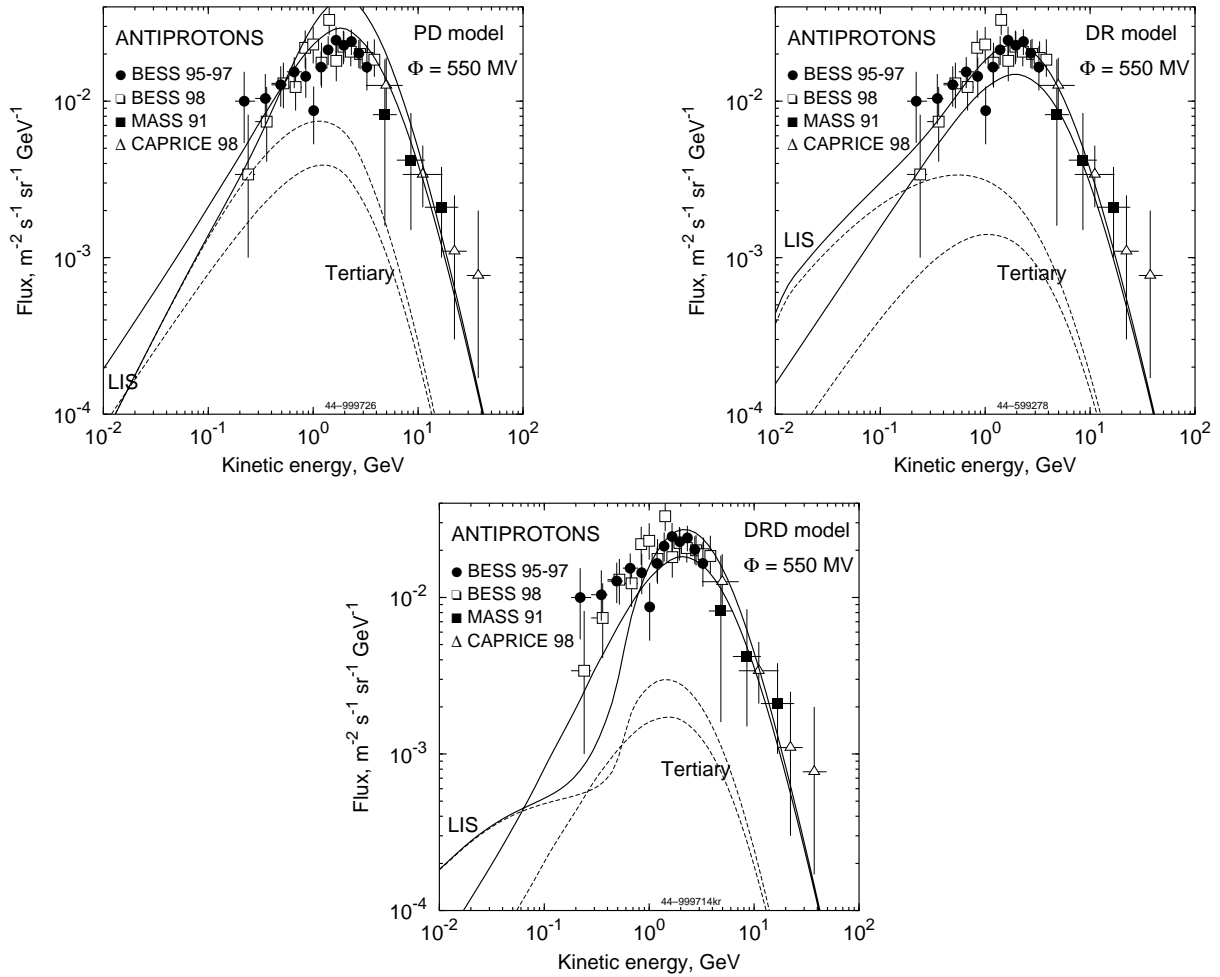


Fig. 6.— Antiproton flux as calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). Upper curve – LIS, lower – modulated to 550 MV. Data: BESS 95-97 (Orito et al. 2000), BESS 98 (Asaoka et al. 2002), MASS 91 (Basini et al. 1999), CAPRICE 98 (Boezio et al. 2001).

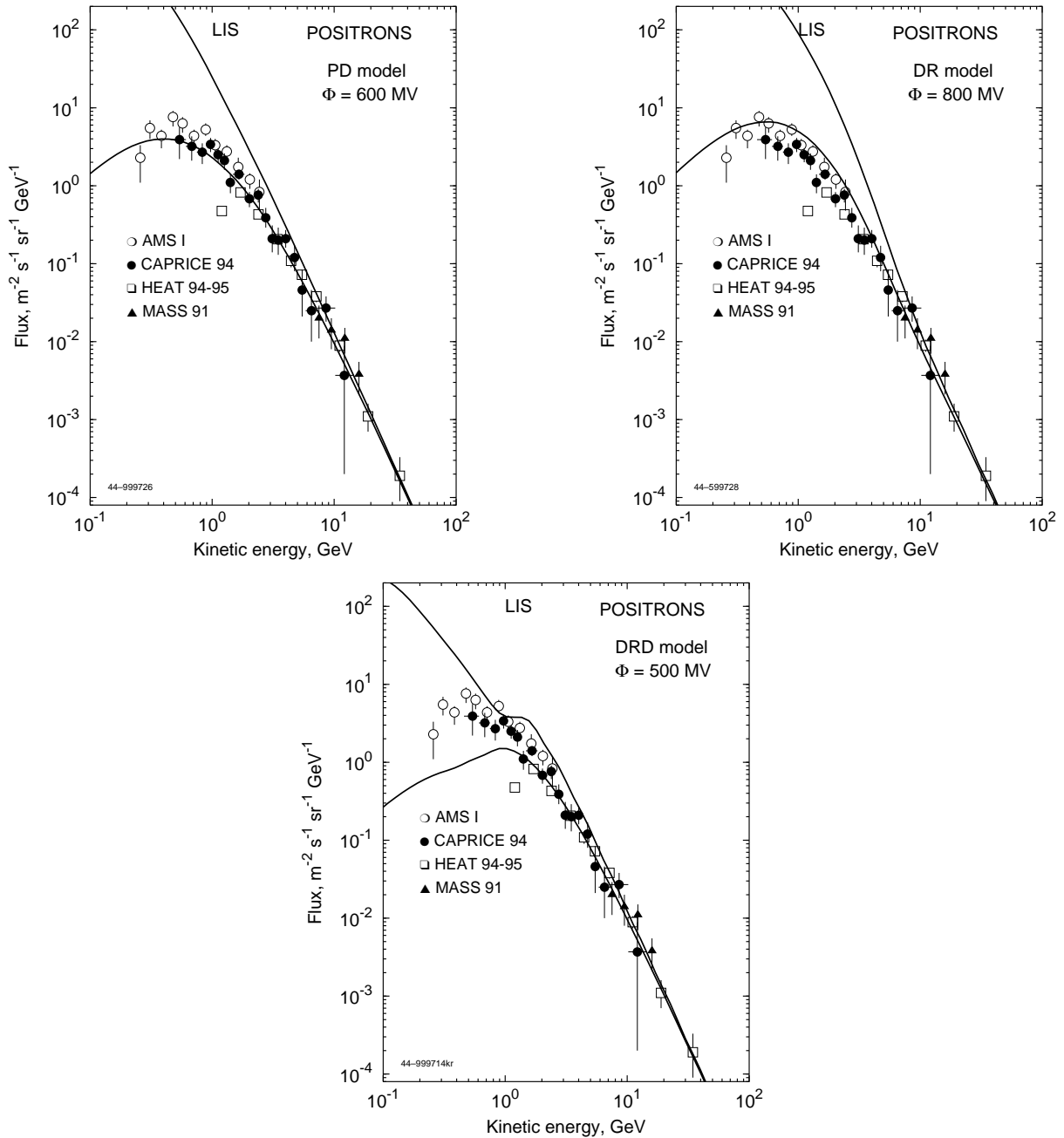


Fig. 7.— Positron flux as calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). Upper curve – LIS, lower – modulated. Data: AMS-I (Alcaraz et al. 2000a), CAPRICE 94 (Boezio et al. 2000), HEAT 94-95 (DuVernois et al. 2001), MASS 91 (Grimani et al. 2002).



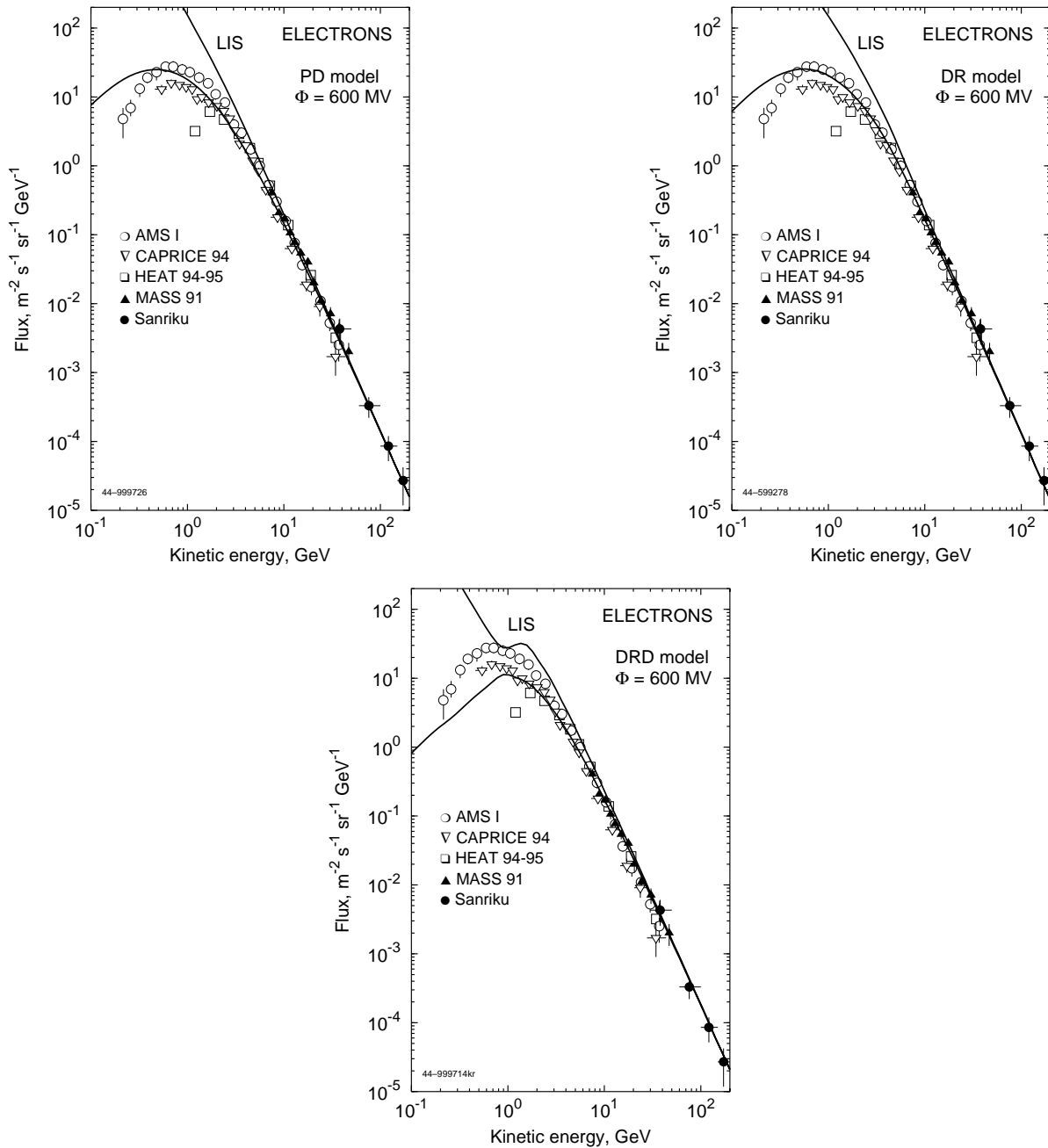


Fig. 8.— Electron flux (primary plus secondary) as calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). Upper curve – LIS, lower – modulated to 600 MV. Data: AMS-I (Alcaraz et al. 2000a), CAPRICE 94 (Boezio et al. 2000), HEAT 94-95 (DuVernois et al. 2001), MASS 91 (Grimani et al. 2002), Sanriku (Kobayashi et al. 1999).