

# Neutron Stars in the Braneworld

Mike Georg Bernhardt

Ruprecht-Karls-Universität Heidelberg  
Zentrum für Astronomie, Landessternwarte

24 April 2009

# Outline

## Introduction

Why bother with Extra Dimensions?

Braneworlds

Why Neutron stars?

## Neutron Stars in 4D

TOV equations

Solutions of the TOV equations

## Neutron Stars in 5D

Brane-TOV equations

Interior Solutions

Exterior Solutions

## Conclusions

## References

# Outline

## Introduction

Why bother with Extra Dimensions?

Braneworlds

Why Neutron stars?

## Neutron Stars in 4D

TOV equations

Solutions of the TOV equations

## Neutron Stars in 5D

Brane-TOV equations

Interior Solutions

Exterior Solutions

## Conclusions

## References



## Why bother with Extra Dimensions?

- The quest for **unification**
  - **Minkowski** (1909), Electromagnetism, 4D
  - **Einstein** (1915), General Relativity, 4D
  - **Kaluza** (1919) and **Klein** (1926)  
Unification of **Electromagnetism and Gravity** in 5D?
  - Supergravity, 11D or less
  - String theory (only bosons), 26D
  - 5 superstring theories (bosons and fermions), 10D
  - M-Theory, 11D
- Solving the **Hierarchy Problem**
  - **Arkani-Hamed, Dimopoulos** and **Dvali** (1998), 6D
  - **Randall and Sundrum** (1999), **5D braneworld**

## Braneworlds

- The braneworld paradigm is based on 3 assumptions:
  1. 4D spacetime is a hypersurface, called brane, of a 5D manifold.
  2. Gravity is a 5D phenomenon, described by the field equations

$$G_{AB} = -\Lambda_5 g_{AB} + \kappa_5 T_{AB}.$$

3. Matter and the fundamental interactions apart from gravity are confined to the brane,

$$T_{AB} = \delta(\chi) [ -\lambda \hat{g}_{AB} + \hat{T}_{AB} ].$$

- Due to the negative cosmological constant  $\Lambda_5$ , gravity is effectively localised near the brane.
- The 5<sup>th</sup> dimension is not compactified, but can be infinitely large.

## Braneworlds

- On the brane an **effective 4D gravity** is induced by the 5D field equations:

$$G_{\alpha\beta} = -\Lambda g_{\alpha\beta} + \kappa T_{\alpha\beta} + \frac{6\kappa}{\lambda} S_{\alpha\beta} - E_{\alpha\beta}$$

with

$$\Lambda := \frac{1}{2} (\Lambda_5 + \frac{1}{6} \kappa_5^2 \lambda^2),$$

$$\kappa := \frac{1}{6} \kappa_5^2 \lambda,$$

$$S_{\alpha\beta} := \frac{1}{12} T T_{\alpha\beta} - \frac{1}{4} T_{\alpha\mu} T^{\mu}_{\beta} + \frac{1}{24} g_{\alpha\beta} (3 T_{\mu\nu} T^{\mu\nu} - T^2).$$

- In the limit

$$\lambda \rightarrow \infty, \quad \kappa_5 \rightarrow 0, \quad E_{\alpha\beta} \rightarrow 0$$

(with finite  $\kappa$ ) gravity is localised on the brane and General Relativity is recovered.

## Why Neutron stars?

- Strong gravitational field  
→ Test bed for general relativity and alternative gravity theories
- Are observable properties of NS influenced by 5D effects?
- Observing NS → observing extra dimensions?
- Can braneworld models be excluded by observing NS?

# Outline

## Introduction

Why bother with Extra Dimensions?

Braneworlds

Why Neutron stars?

## Neutron Stars in 4D

TOV equations

Solutions of the TOV equations

## Neutron Stars in 5D

Brane-TOV equations

Interior Solutions

Exterior Solutions

## Conclusions

## References

# Tolman-Oppenheimer-Volkoff equations

- Our goal is to solve the Einstein equations

$$G_{\alpha\beta} := R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = \kappa T_{\alpha\beta}$$

for a **static** and **spherical** star.

- Ansatz for the line element

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2).$$

- Stellar matter modelled as a **perfect fluid**

$$T_{\alpha}^{\beta} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

## Tolman-Oppenheimer-Volkoff equations

- This yields the **TOV equations** of stellar structure

$$m' = 4\pi r^2 \varrho,$$

$$p' = -(\varrho + p)\Phi',$$

$$\Phi' = \frac{Gm + 4\pi Gr^3 p}{r(r - 2Gm)}$$

with the line element

$$ds^2 = -e^{2\Phi(r)} dt^2 + \left(1 - \frac{2Gm(r)}{r}\right)^{-1} dr^2 + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2).$$

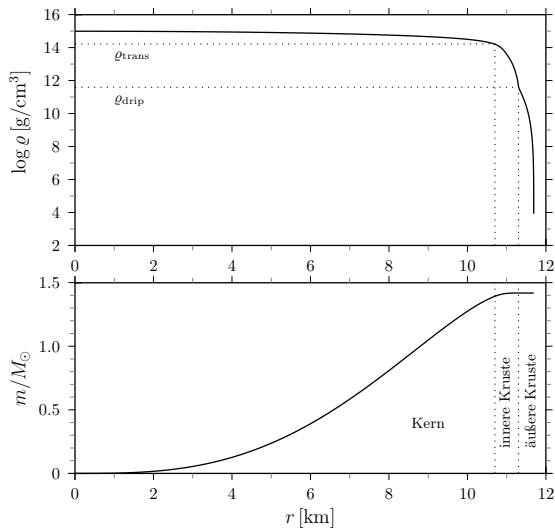
- 3 equations, 4 unknown functions  $m(r)$ ,  $\varrho(r)$ ,  $p(r)$  and  $\Phi(r)$ .
- In order to close the system, we need an **equation of state**

$$p = p(\varrho).$$



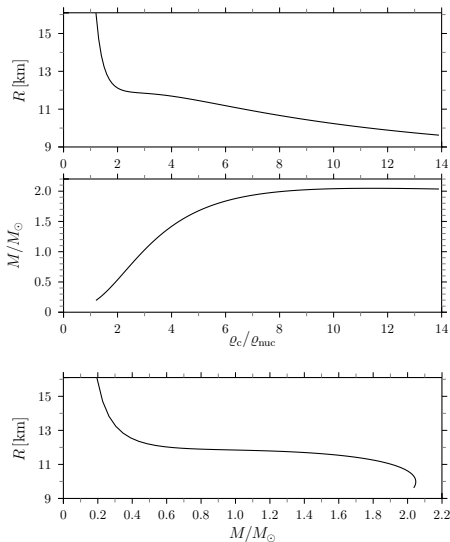


# Numerical Interior Solutions





# Numerical Interior Solutions



# Outline

## Introduction

Why bother with Extra Dimensions?

Braneworlds

Why Neutron stars?

## Neutron Stars in 4D

TOV equations

Solutions of the TOV equations

## Neutron Stars in 5D

Brane-TOV equations

Interior Solutions

Exterior Solutions

## Conclusions

## References

## Brane-TOV equations

- Now, we want to solve the modified Einstein equations

$$G_{\alpha\beta} = \kappa T_{\alpha\beta}^{\text{eff}}, \quad T_{\alpha\beta}^{\text{eff}} := T_{\alpha\beta} + \frac{6}{\lambda} S_{\alpha\beta} - \frac{1}{\kappa} E_{\alpha\beta}$$

for a **static** and **spherical** star on the brane.

- Ansatz for the line element

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2).$$

- The brane metric, and also the Einstein tensor  $G_{\alpha\beta}$  are the same as in 4D.

## Brane-TOV equations

- Stellar matter modelled as a **perfect fluid**

$$T_{\alpha}^{\beta} = \begin{pmatrix} -\varrho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}.$$

- This implies the **high-energy correction**

$$S_{\alpha}^{\beta} = \frac{1}{12} \begin{pmatrix} -\varrho^2 & 0 & 0 & 0 \\ 0 & (\varrho^2 + 2\varrho p) & 0 & 0 \\ 0 & 0 & (\varrho^2 + 2\varrho p) & 0 \\ 0 & 0 & 0 & (\varrho^2 + 2\varrho p) \end{pmatrix}.$$

- The static and spherically symmetric **projected Weyl tensor** can be written as

$$-\frac{1}{\kappa} E_{\alpha}^{\beta} = \begin{pmatrix} -\mathcal{U} & 0 & 0 & 0 \\ 0 & \frac{1}{3}(\mathcal{U} + 2\mathcal{P}) & 0 & 0 \\ 0 & 0 & \frac{1}{3}(\mathcal{U} - \mathcal{P}) & 0 \\ 0 & 0 & 0 & \frac{1}{3}(\mathcal{U} - \mathcal{P}) \end{pmatrix}.$$

## Brane-TOV equations

- We end up with the **effective energy-momentum tensor**

$$T_{\alpha}^{\beta \text{eff}} = \begin{pmatrix} -\varrho^{\text{eff}} & 0 & 0 & 0 \\ 0 & p_{\perp}^{\text{eff}} & 0 & 0 \\ 0 & 0 & p_{\parallel}^{\text{eff}} & 0 \\ 0 & 0 & 0 & p_{\parallel}^{\text{eff}} \end{pmatrix},$$

with

$$\begin{aligned} \varrho^{\text{eff}} &:= \varrho + \frac{\varrho^2}{2\lambda} + \mathcal{U}, \\ p_{\perp}^{\text{eff}} &:= p + \frac{p\varrho}{\lambda} + \frac{\varrho^2}{2\lambda} + \frac{\mathcal{U} + 2\mathcal{P}}{3}, \\ p_{\parallel}^{\text{eff}} &:= p + \frac{p\varrho}{\lambda} + \frac{\varrho^2}{2\lambda} + \frac{\mathcal{U} - \mathcal{P}}{3}. \end{aligned}$$

## Brane-TOV equations

- The **brane-TOV equations** follow from  $G_{\alpha\beta} = \kappa T_{\alpha\beta}^{\text{eff}}$ :

$$\tilde{m}' = 4\pi r^2 \left( \varrho + \frac{\varrho^2}{2\lambda} + \mathcal{U} \right),$$

$$p' = -(\varrho + p)\Phi',$$

$$\Phi' = \frac{G\tilde{m} + 4\pi Gr^3 \left[ p + \frac{p\varrho}{\lambda} + \frac{\varrho^2}{2\lambda} + \frac{1}{3}(\mathcal{U} + 2\mathcal{P}) \right]}{r(r - 2G\tilde{m})},$$

$$\mathcal{U}' = -2\mathcal{P}' - \frac{6\mathcal{P}}{r} - (4\mathcal{U} + 2\mathcal{P})\Phi' - 3\frac{\varrho + p}{\lambda}\varrho'$$

with the line element

$$ds^2 = -e^{2\Phi(r)} dt^2 + \left( 1 - \frac{2G\tilde{m}(r)}{r} \right)^{-1} dr^2 + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2).$$

## Brane-TOV equations

- 4 equations, 6 unknown functions  $\tilde{m}(r)$ ,  $\varrho(r)$ ,  $p(r)$ ,  $\Phi(r)$  and the Weyl terms  $\mathcal{U}$  and  $\mathcal{P}$ .
- In order to close the system, we need an **equation of state**  $p = p(\varrho)$  plus an **additional relation** between  $\mathcal{U}$  and/or  $\mathcal{P}$ .
- I assume, that the Weyl terms obey an equation-of-state like relation  $\mathcal{P} = w\mathcal{U}$ .
- Then, the brane-TOV equations read

$$\tilde{m}' = 4\pi r^2 \left( \varrho + \frac{\varrho^2}{2\lambda} + \mathcal{U} \right),$$

$$p' = -(\varrho + p)\Phi',$$

$$\Phi' = \frac{G\tilde{m} + 4\pi Gr^3 \left[ p + \frac{p\varrho}{\lambda} + \frac{\varrho^2}{2\lambda} + \frac{1+2w}{3}\mathcal{U} \right]}{r(r - 2G\tilde{m})},$$

$$\mathcal{U}' = -\frac{3}{1+2w} \left[ 2w\mathcal{U}r^{-1} + \frac{2}{3}(2+w)\mathcal{U}\Phi' + \frac{1}{\lambda}(\varrho + p)\varrho' \right].$$



## Interior Solutions

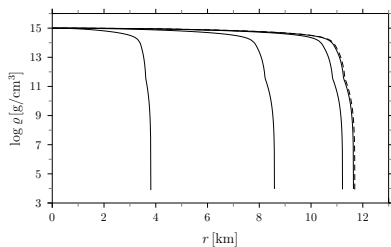
- As in the relativistic case, the brane-TOV equations can be **solved numerically**.
- I used the same equation of state (SLy) as in the relativistic case, taking into account the **inhomogeneous energy density** inside the neutron star.



# Numerical Interior Solutions

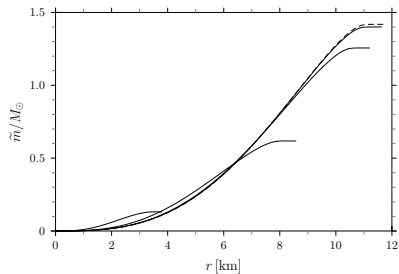
$$w = -100$$

$$\lambda \text{ [dyn/cm}^2\text{]} = 10^{35}, 10^{36}, 10^{37}, 10^{38}$$



$$w = -100$$

$$\lambda \text{ [dyn/cm}^2\text{]} = 10^{35}, 10^{36}, 10^{37}, 10^{38}$$

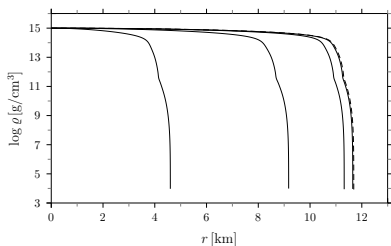




# Numerical Interior Solutions

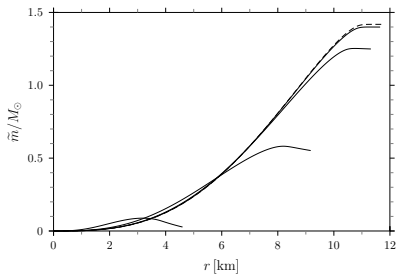
$$w = -2$$

$$\lambda \text{ [dyn/cm}^2\text{]} = 10^{35}, 10^{36}, 10^{37}, 10^{38}$$



$$w = -2$$

$$\lambda \text{ [dyn/cm}^2\text{]} = 10^{35}, 10^{36}, 10^{37}, 10^{38}$$



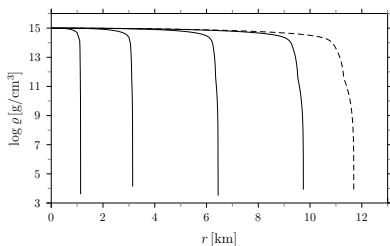




# Numerical Interior Solutions

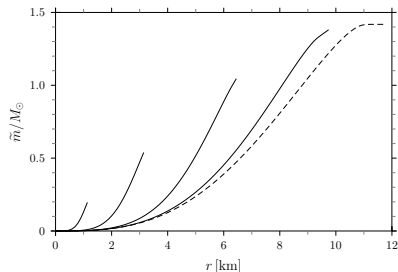
$$w = -0,2$$

$$\lambda [\text{dyn/cm}^2] = 10^{35}, 10^{36}, 10^{37}, 10^{38}$$



$$w = -0,2$$

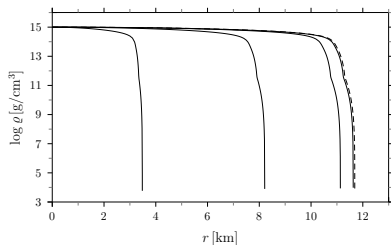
$$\lambda [\text{dyn/cm}^2] = 10^{35}, 10^{36}, 10^{37}, 10^{38}$$



# Numerical Interior Solutions

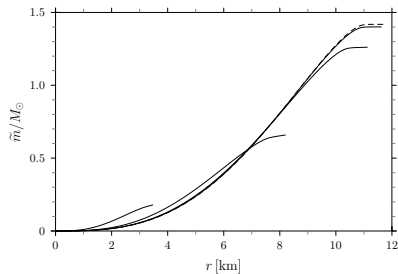
$$w = 2$$

$$\lambda \text{ [dyn/cm}^2\text{]} = 10^{35}, 10^{36}, 10^{37}, 10^{38}$$



$$w = 2$$

$$\lambda \text{ [dyn/cm}^2\text{]} = 10^{35}, 10^{36}, 10^{37}, 10^{38}$$

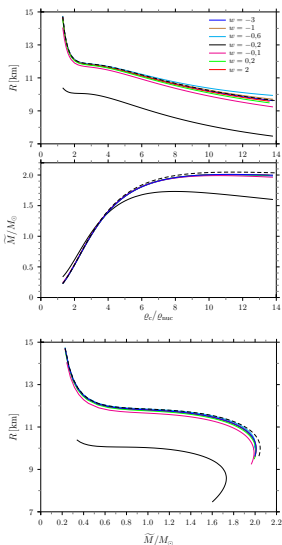




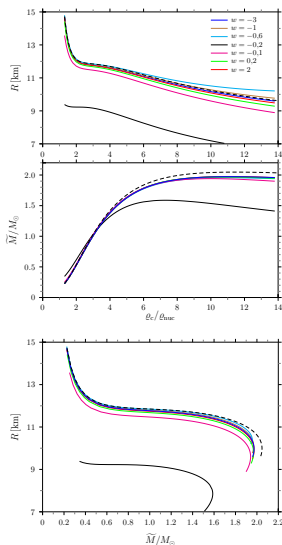


# Numerical Interior Solutions

$$\lambda \text{ [dyn/cm}^2\text{]} = 10^{38}$$

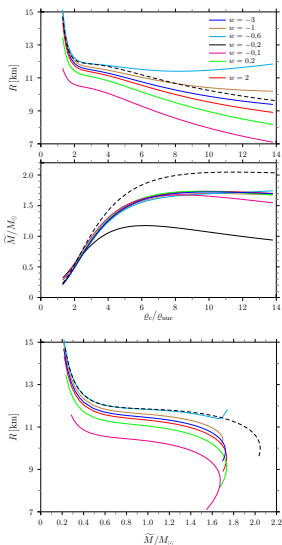
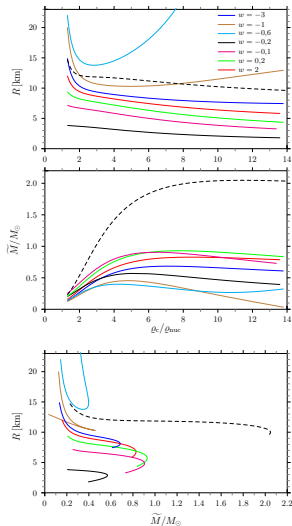


$$\lambda \text{ [dyn/cm}^2\text{]} = 5 \cdot 10^{37}$$





# Numerical Interior Solutions

 $\lambda \text{ [dyn/cm}^2\text{]} = 10^{37}$ 

 $\lambda \text{ [dyn/cm}^2\text{]} = 10^{36}$ 


## Exterior Solutions

- In the case  $w = -2$  the exterior brane-TOV equations can be solved **analytically**.
- Exterior:  $\varrho = 0, p = 0$  and

$$\begin{aligned} \tilde{m}' &= 4\pi r^2 \mathcal{U}, & \tilde{m}(r) &= \tilde{m}(R) + 4\pi \mathcal{U}(R) R^4 \left( \frac{1}{R} - \frac{1}{r} \right) \\ \Phi' &= \frac{G\tilde{m} - 4\pi G r^3 \mathcal{U}}{r(r - 2G\tilde{m})}, & \curvearrowright & e^{2\Phi} = 1 - \frac{2G\tilde{m}(r)}{r}, \\ \mathcal{U}' &= -\frac{4\mathcal{U}}{r}, & \mathcal{U}(r) &= \mathcal{U}(R) \left( \frac{R}{r} \right)^4. \end{aligned}$$

- If  $\mathcal{U}(R) \neq 0$ , then  $\tilde{m}(r) \neq \text{const}$  in the exterior!

## Exterior Solutions

- This leads to a **Reissner-Nordström like solution**

$$ds^2 = - \left( 1 - \frac{2G\tilde{m}_\infty}{r} + \frac{Q}{r^2} \right) dt^2 + \left( 1 - \frac{2G\tilde{m}_\infty}{r} + \frac{Q}{r^2} \right)^{-1} dr^2 \\ + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2)$$

with

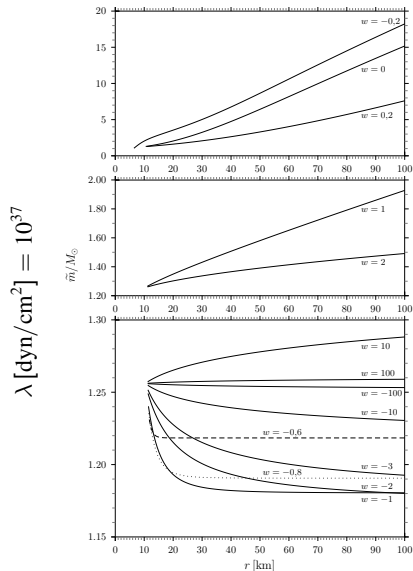
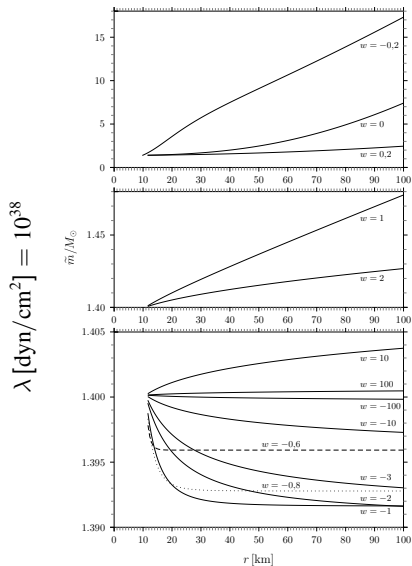
$$\tilde{m}_\infty := \tilde{m}(R) + 4\pi\mathcal{U}(R)R^3, \quad Q := \kappa\mathcal{U}(R)R^4.$$

- A numerical calculation shows, that for  $w = -2$  we have in general  $\mathcal{U}(R) < 0$ .
- That means, the effective mass tends to  $\tilde{m}_\infty < \tilde{m}(R)$ .





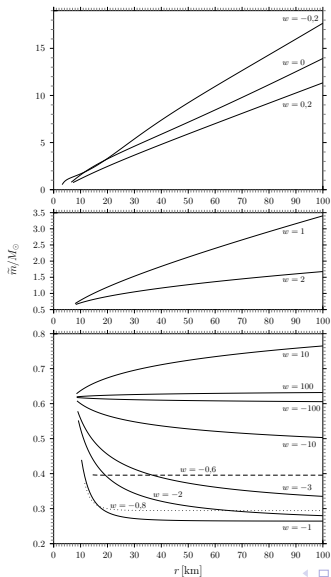
# Numerical Exterior Solutions





# Numerical Exterior Solutions

$$\lambda \text{ [dyn/cm}^2\text{]} = 10^{36}$$





# Outline

## Introduction

Why bother with Extra Dimensions?

Braneworlds

Why Neutron stars?

## Neutron Stars in 4D

TOV equations

Solutions of the TOV equations

## Neutron Stars in 5D

Brane-TOV equations

Interior Solutions

Exterior Solutions

## Conclusions

## References



## Conclusions

- Neutron stars on the brane are in general **more compact**: They have **smaller radii** (except for  $-1 \lesssim w < -0,5$ ) and are **less massive** (except for  $-0,5 < w \lesssim -0,1$  and  $\rho_c/\rho_{\text{nuc}} \lesssim 4$ ).
- The relative corrections of mass and density of a neutron star are bigger than  $10^{-3}$  if  $\lambda \lesssim 10^{39} \text{ dyn/cm}^2$ .
- A brane tension  $\lambda < 5 \cdot 10^{36} \text{ dyn/cm}^2$  is in contradiction with observed neutron star masses.
- A generic property of the brane-TOV equations are exterior solutions with  $\tilde{m} \neq \text{const.}$

# Outline

## Introduction

Why bother with Extra Dimensions?

Braneworlds

Why Neutron stars?

## Neutron Stars in 4D

TOV equations

Solutions of the TOV equations

## Neutron Stars in 5D

Brane-TOV equations

Interior Solutions

Exterior Solutions

## Conclusions

## References

# References



**M. G. Bernhardt**

*Kompakte Sterne in der Branenwelt*

Diplomarbeit, Ruprecht-Karls-Universität Heidelberg (2009)



**R. Maartens**

*Brane-World Gravity*

Living Rev. Relativity, **7**, (2004), 7,

<http://www.livingreviews.org/lrr-2004-7>



**C. Germani, R. Maartens**

*Stars in the braneworld*

Phys. Rev. D **64**, 124010 (2001), arXiv:hep-th/0107011v3



**N. Deruelle**

*Stars on branes: the view from the brane*

arXiv:gr-qc/0111065v1 (2001)