

The rotational broadening and the mass of the donor star of GRS 1915+105

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Abstract. The binary parameters of the microquasar GRS 1915+105 have been determined by the detection of Doppler-shifted ¹²CO and ¹³CO lines in its K-band spectrum (Greiner et al. 2001b). Here, we present further analysis of the same K-band VLT spectra and we derive a rotational broadening of the donor star of $\nu \sin i = 26 \pm 3 \text{ km s}^{-1}$ from the ¹²CO/¹³CO lines. Assuming that the K-type star is tidally locked to the black hole and is filling its Roche-lobe surface, then the implied mass ratio is $q = \frac{M_d}{M_x} = 0.058 \pm 0.033$. This result, combined with $(P, K, i) = (33.5 \text{ d}, 140 \text{ km s}^{-1}, 66^\circ)$ gives a more refined mass estimate for the black hole, $M_x = 14.0 \pm 4.4 M_\odot$, than previously estimated, using an inclination of $i = 66^\circ \pm 2^\circ$ (Fender et al. 1999) as derived from the orientation of the radio jets and a more accurate distance. The mass for the early K-type giant star is $M_d = 0.81 \pm 0.53 M_\odot$, consistent with a more evolved stripped-giant donor star in GRS 1915+105 than, for example, the donor star of the prototype black-hole X-ray transient, V404 Cyg which has the longest binary period after GRS 1915+105.

Key words. stars: infrared – stars: X-ray – black hole physics – stars: binaries – stars: individual: GRS 1915+105

1. Introduction

The microquasar GRS 1915+105 remains one of the most exotic and variable objects in the X-ray, IR and radio spectrum bands since its discovery (Castro-Tirado et al. 1994). A major accomplishment has been the implied connection between the jet and the accretion disk as interpreted through modelling of its X-ray variability (Belloni et al. 1997; Klein-Wolt et al. 2002). Thereafter, it is evident that crucial insight into the accretion physics of jet formation can be derived through intense theoretical and observational work. Fundamental for the modelling but also for the evolutionary history of GRS 1915+105 are the binary system parameters.

The binary model of the microquasar GRS 1915+105 was elusive, until recently, despite many observational efforts to search for the donor star (Castro-Tirado et al. 1996; Eikenberry et al. 1998; Mirabel et al. 1997; Martí et al. 2000; Harlaftis et al. 2001; Greiner et al. 2001a). Greiner et al. (2001b) were able to obtain phase-resolved IR spectra using the VLT-Antu with the ISAAC IR spectrograph and derive the orbital period of the binary system from the detected Doppler-shifted ¹²CO and ¹³CO absorption bands. The deduction of the masses of the binary, using the mass function equation,

$$f(M_x) = \frac{P K_d^3}{2\pi G} = \frac{M_x \sin^3 i}{(1+q)^2},$$

requires the orbital period P , the semi-amplitude of the radial velocity curve of the donor star around the black-hole K_d , the binary inclination i and the mass ratio $q = M_d/M_x$, where M_d is the mass of the donor star and M_x the mass of the black-hole. Indeed, the orbital period and the semi-amplitude of the radial velocity curve of the donor star around the black-hole have been determined by Greiner et al. (2001b; $P = 33.5 \pm 1.5$ days and $Kw_d = 140 \pm 15 \text{ km s}^{-1}$). The remaining unknown parameters are the binary inclination and the mass ratio. In fact, it is generally the binary inclination which dominates the uncertainty in the mass determination of the black hole via $\sim f(M_x) \sin^{-3} i$. The general method in deriving the inclination of X-ray binaries and cataclysmic variables is to model the ellipsoidal modulations of the companion star at infrared wavelengths. Luckily, the orbital inclination of GRS 1915+105 is deduced with unprecedented accuracy from the orientation of the jets ($i = 70^\circ \pm 2^\circ$; Mirabel & Rodriguez 1994; $i = 66^\circ \pm 2^\circ$ by Fender et al. 1999). The final parameter that is still undetermined for GRS 1915+105 is its mass ratio. The mass ratio of the binary system is related to the rotational broadening of the donor star using the formula

$$\frac{\nu \sin i}{K_d} = 0.46 [(1+q)^2 q]^{1/3}$$

where $\nu \sin i$ is the rotational broadening in the line-of-sight, assuming Roche lobe mass-transfer, spherical geometry for the star and tidal locking to the primary star (Gies & Bolton 1986; Horne et al. 1986). Here, we further analysed the same IR

spectra from VLT (Greiner et al. 2001b) in order to deduce the rotational broadening of the donor star from the width of the photospheric absorption lines (mainly ^{12}CO , ^{13}CO ; see Greiner et al. 2001a).

2. Observations, numerical technique and deduction of mass ratio

The GRS 1915+105 spectra, together with a KIII spectrum (HD 138185), were obtained between April and August 2000 with the ISAAC spectrograph on the Antu-VLT covering the range 2.2896–2.4131 μm , at a pixel resolution of 15.4 km s^{-1} (Greiner et al. 2001b). In order to measure the width of the photospheric absorption lines, we apply a numerical technique according to which we subtract from the Doppler-corrected spectrum of GRS 1915+105 a KIII template spectrum. In detail

- we produce the average spectrum after the individual spectra are Doppler-corrected by applying the radial velocity solution (Greiner et al. 2001b);
- we apply to the KIII template spectrum a rotational profile with a limb darkening coefficient of 0.5 (Gray 1992) and a veiling factor f which scales the depth of the absorption lines in order to simulate the GRS 1915+105 absorption lines;
- the difference spectrum represents mainly the accretion disk spectrum with all the photospheric absorption bands arising from the KIII donor star of GRS 1915+105 having been subtracted;
- we perform a χ^2 -test between the difference spectrum and its smoothed version (by applying a Gaussian profile of a large $FWHM = 450 \text{ km s}^{-1}$ to make it flat) which is a measure of how good the absorption lines of the donor star have been subtracted for each rotational profile and scaling factor $0 < f < 1$ applied;
- we iterate by changing the rotational profile width and the depth of the absorption lines – using a scaling factor f representing the the donor star line flux – until we reach χ^2 values moving along a parabolic surface.

The minimum χ^2 on the parabola ($v \sin i$, χ^2) gives the corresponding width of the rotational profile. We deduce a $v \sin i = 26 \pm 3 \text{ km s}^{-1}$ and $f = 14 \pm 3\%$, at the 90% significance level, where f is the light contribution from the donor star as estimated from its $^{12}\text{CO}/^{13}\text{CO}$ absorption bands. As a validation test, we performed the same iterations but now for the individual spectra rather than the average Doppler-corrected spectrum and took the average $v \sin i$ – equal to $26 \pm 5 \text{ km s}^{-1}$ – from the individual estimates. The mass ratio deduced is, only weakly sensitive, on the spectral type and veiling factor f . For more detailed discussion of the technique see Harlaftis et al. (1999 and references therein). Figure 1 shows, from bottom to top, the KIII template (with lines multiplied by the veiling factor of 0.14), the average Doppler-corrected spectrum of GRS 1915+105 (16 spectra), the difference spectrum (thus, representing the disk spectrum) and the difference spectrum between the template and the average Doppler-corrected spectrum of GRS 1915+105 at binary phase ~ 0.75 (same average as in Greiner et al. 2001b). The KIII spectrum has a ripple in

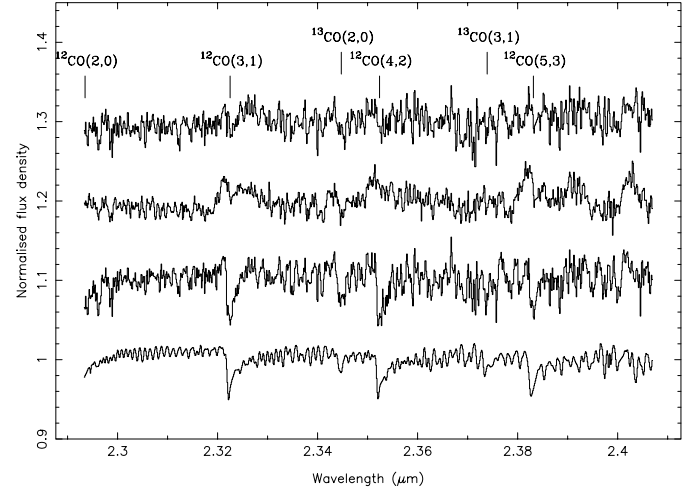


Fig. 1. The spectra show, from bottom to top, the KIII template (with lines multiplied by 0.14), the average Doppler-corrected spectrum of GRS 1915+105 (16 spectra), the difference spectrum (thus, representing the disk spectrum) and the difference spectrum between the template and the average Doppler-corrected spectrum of GRS1915+105 at binary phase ~ 0.75 (the same average spectrum as in Greiner et al. 2001b).

the continuum arising from variations in the star brightness between integrations (i.e. at different detector positions). There is a residual blue-shifted CO-band or other unidentified emission in the disk spectrum from phases other than 0.75 as well as two unidentified absorption features around 2.2962 and 2.2988 μm .

3. Discussion

We discuss now the various assumptions involved in the derivation of the mass ratio from the rotational broadening. The absorption vibration-rotation bands ^{12}CO and ^{13}CO consist of ~ 4500 lines at 3500 K (Kunde 1977). However the total damping width is of the order of $1 \times 10^{-4} \text{ \AA}$, the thermal width does not exceed 0.04 \AA at 4000 K, and the macroturbulence width is $\sim 5 \text{ km s}^{-1}$ in giant stars (Gray 1992). Other possible broadening factors other than rotation is pressure-broadening which is not significant in giant stars. Thus, the fact that there is a measurable velocity dispersion in the line profiles indicates that it is dominated by the rotational broadening profile. Shahbaz (2003) has developed a new, more accurate method involving synthetic spectra from the Roche lobe (thus avoiding assumptions about a spherical symmetric star with a limb darkening coefficient). However, simulations show that the standard method compared to his more accurate treatment is still compatible within 3σ . The assumption of a spherical star in the ($v \sin i, q$) relation has been shown to introduce an error in the evaluation of mass ratio q of $< 5\%$ (Marsh et al. 1994) which is much less than the uncertainty of our measurement ($\sim 50\%$), hence with a negligible effect. The assumption of synchronous rotation ($P_{\text{orbit}} = P_{\text{rot}}$) has been found to be valid for binaries as wide as 80 days even though giant stars may show a weaker synchronism (10% difference from the orbital period) than late-type stars (Giuricin et al. 1984).

The criterion for tidal locking is $R_d/\alpha > 0.125$, where R_d is the radius of the donor star and α the binary separation, assuming dynamical tide alone (Zahn 1977, 1992) which indeed holds for GRS 1915+105 ($R_d/\alpha = 0.176$). Involving other dissipative processes such as hydrodynamical meridional flow on the star lowers the limit for the fractional star radius, thus yielding much longer periods (Tassoul 1987; Tassoul & Tassoul 1992; see also for tidal interaction models Witte & Savonije 2002; Claret & Cunha 1997; Verbunt & Phinney 1995). The synchronisation timescale $t \approx (R_d/\alpha)^{-6} q^{-2} = 4.6$ million years (Giuricin et al. 1984) which is well within the nuclear evolution timescale of the donor star. Synchronisation is more rapid than circularisation of the orbit (Hall 1986; see also Lecar et al. 1976), and a χ^2_ν -test on the radial velocity data (Greiner et al. 2001b) does not justify an eccentric orbit ($e < 0.006$). Therefore, we conclude that the orbit is circular.

The constraint on the eccentricity of the radial velocity fit also indicates that there is no line irradiation affecting the measurement of the K semi-amplitude of the radial velocity of GRS 1915+105. For example, a 10% correction is applied in the value of the K semi-amplitude (reduced) from the NaI doublet absorption lines of the donor star in the eclipsing dwarf nova IP Peg from a measured eccentricity of the radial velocity fit of ($e = 0.089$; Martin et al. 1989). There has been no evidence for irradiation affecting the photospheric absorption lines in any quiescent black-hole X-ray transients with extreme mass ratios (e.g. Harlaftis et al. 1996). However, outbursts can irradiate the donor star and affect significantly the radial velocity amplitude, as for example with the FIII-IV sub-giant donor star in GRO J1655-40 (Shahbaz et al. 2000) which has a binary period of 2.6 days and a large mass ratio of 0.42 (Shahbaz 2003). The spectra of GRS 1915+105 were obtained during relatively quiet states (no outburst or flare) and in addition its binary orbit is long with a small cross section of the donor star for any irradiation to affect it. In fact, using the relation (2) from Phillips et al. (1999) we find the effect to be only 2.5% compared to the 11% uncertainty in K_d for GRS 1915+105.

The mass ratio thus derived from the rotational broadening width is $q = 0.058 \pm 0.033$ (see second relation in the Introduction). The masses can now be determined from $(P, K, i, q) = (33.5 \pm 1.5 \text{ d}, 140 \pm 15 \text{ km s}^{-1}, 66^\circ \pm 2^\circ, 0.058 \pm 0.033)$ giving $M_x = 14.0 \pm 4.4 M_\odot$ and $M_d = 0.81 \pm 0.53 M_\odot$ (see first relation in the Introduction). The large black-hole mass uncertainty is dominated by the uncertainty in the semi-amplitude radial velocity of 15 km s^{-1} . The contributions from the period uncertainty and mass ratio are about equal but only a few % whereas that of the inclination uncertainty is practically zero. The large mass ratio uncertainty of 50% is due to the large uncertainty in both the rotational broadening and the semi-amplitude of the radial velocity ($\sim 10\%$ for each parameter). Having made the above points for the assumptions involved we now discuss the ambiguity that may reside in the inclination value. The jet axis may not be perpendicular to the binary plane though a constraint is placed to be less than 25° in GRS 1915+105 (Maccarone 2002). Indeed, in one of the microquasars, namely GRO J1655-40, the binary inclination is 70° (Greene et al. 2001) whereas the jet inclination is found at 85° (Hjellming & Rupen 1995). The proper motions

of the plasmoids ejected from GRS 1915+105 have been found to vary in apparent velocity and this was attributed to a change in the jet velocity (Rodriguez & Mirabel 1999; Fender et al. 1999). However, it must be noted that the alternative explanation is that the jet axis may be precessing. Indeed, Rodriguez & Mirabel (1999) mention a change by $\sim 10^\circ$ of the ejection axis within one month as well as a 10–20% difference in the proper motion measured by Fender et al. (1999). However, Maccarone (2002) concludes, on the basis of close agreement between the dynamical estimate and the resonance interpretation of the observed X-ray QPO frequencies for the black hole mass, that a large misalignment between the jet axis and the disk axis is ruled out in GRS 1915+105.

The above discussion must be taken under consideration when using the adapted inclination of $i = 66^\circ \pm 2^\circ$ (Fender et al. 1999) for a distance of $11.2 \pm 0.8 \text{ kpc}$. Assuming a $\sim 10^\circ$ uncertainty in the inclination due to possible jet axis precession, the masses become $M_x = 11.6 \pm 3.3 M_\odot$ and $M_d = 0.68 \pm 0.43 M_\odot$ for an inclination of $i = 76^\circ$ ($M_x = 16.9 \pm 5.9 M_\odot$ and $M_d = 0.98 \pm 0.65 M_\odot$ for $i = 56^\circ$). However, extensive radio monitoring of GRS 1915+105 over the last 10 years indicates no measurable constant precession, thus giving ground to the interpretation of varying jet ejection rather than a jet axis precession. For direct comparison on the improvement of the mass estimate using the mass ratio determined here, we deduce a $M_x = 12.9 \pm 4.04 M_\odot$ compared to $M_x = 14.0 \pm 4.0 M_\odot$ for $i = 70^\circ \pm 2^\circ$ as adapted by Greiner et al. (2001a).

Single KIII stars have negligible $v \sin i$. Indeed, a KIII and a K5III giant star have a mass of 2.3, 2.2 M_\odot , a radius of 11, 28 R_\odot and a $< v \sin i >$ of 2.5, and less than 1.5 km s^{-1} , respectively (Gray 1992). Greiner et al. (2001a) identified the luminosity class of the donor star in GRS 1915+105 as a giant star mainly based on the equivalent width ratio of the $^{12}\text{CO}/^{13}\text{CO}$ lines. Using the Eggleton formula (1983) to determine the Roche lobe size and hereafter the density of the lobe-filling star, we derive $R_d = 19 R_\odot$ and $\rho = 2 \times 10^{-4} \text{ g cm}^{-3}$, using an orbital period of $P = 33.5$ days and a mass ratio of $q = 0.058$. This compares well with a single-giant KIII1-5 classification ($R = 11 - 28 R_\odot$ and $\rho = 13 - 4 \times 10^{-4} \text{ g cm}^{-3}$). Thus, the luminosity class – giant – indicates that the star’s envelope can be contained within the Roche lobe, and thus evolutionary expansion of the donor can sustain Roche lobe mass transfer rate. Wind-fed accretion alone cannot power the luminosities we observe from this system.

The long orbital period suggests that the evolution of GRS 1915+105 is further advanced than V404 Cyg, a similar system with a stripped-subgiant KOIV donor at a period of 6.5 days around the black hole ($M_d = 0.7^{0.3}_{0.2} M_\odot$; Shahbaz et al. 1994). The more evolved giant star in GRS 1915+105 is also indicated by the smaller mass of the donor star compared to V404 Cyg due to increased mass loss sustained by nuclear evolution (King et al. 1996). Indeed, the mass of the donor star, a “stripped-giant”, is lower by a factor of three compared to a single giant star. King (1993) has shown that the mass of the donor star can have a maximum mass of $1.3 M_\odot$ while its actual mass depends on the mass of the helium core. Adapting the relation $R_d = 12.55 R_\odot (M_c/0.25)^{5.1}$ from King (1993), we derive $M_c = 0.27 M_\odot$ for $R_d = 19 R_\odot$. The helium core has a

mass $M_c = 0.27 M_\odot$, one third of the remaining mass of the star at $M_d = 0.81 M_\odot$. More than $1 M_\odot$ mass of the donor star has already been transferred to the black hole. The orbital period of GRS 1915+105 is increasing and will reach at least 45 days (Sect. 3 in King 1993), a point when the donor star will reach its helium-core mass and the mass-transfer rate will cease. Actually, the stellar size for the helium-core mass of $M_c = 0.27 M_\odot$ is $R_d = 18.6 R_\odot$ (relation (1) in King 1993), suggesting that the stellar size is close to its possible minimum, thus not far from the time the mass transfer rate will cease.

GRS 1915+105 is in perpetual outburst since its discovery in 1992 whereas V404 Cyg has been in quiescence after its X-ray outburst. This is consistent with a higher mass transfer rate expected towards the end of the nuclear evolution of the donor star. Indeed, a higher mass-transfer rate is expected in GRS 1915+105 by 4–8 times compared to V404 Cyg, by using the relation, $-\dot{M}_2 \approx 5.4 \times 10^{-9} (M_c/0.25)^{7.11} M_d M_\odot \text{yr}^{-1}$ (King 1993), compared to an Eddington accretion-rate of $\dot{M}_{\text{Edd}} \approx 1.5 \times 10^{-8} \text{ erg s}^{-1}$ (for a $14 M_\odot$ black-hole mass). An increased orbital period, for example from the 6.5 days of V404 Cyg to the 33.5 days of GRS 1915+105, due to evolution would result in longer outbursts and a decreased duty cycle as well as higher super-Eddington accretion rates (King et al. 2002). The observed X-ray luminosity during outburst is dependent on inclination and anisotropic radiation patterns, therefore it is not easy to show that the accretion rate is higher in GRS 1915+105. The observed X-ray luminosity is super-Eddington at the outburst maximum ($L_{\text{Edd}} \approx 1.8 \times 10^{39} \text{ erg s}^{-1}$ for a $14 M_\odot$ black hole mass) in both V404 Cyg at $L_x(\text{max}) \approx 3.75 \times 10^{39} (D/5 \text{ kpc})^2 \text{ erg s}^{-1}$ (Tanaka 1989) and in GRS 1915+105 at $L_x(\text{max}) \sim 5 \times 10^{39} (D/12 \text{ kpc})^2 \text{ erg s}^{-1}$ (Rau et al. 2003). Interestingly, King (2002) recently proposed that GRS 1915+105, a low-mass X-ray binary showing super-Eddington accretion, may well be the best representative in our galaxy of ultraluminous X-ray sources in elliptical galaxies.

It is clear that, as far as the physical mechanisms involved in the radiation emitted, this rich system will be explored more efficiently once the binary parameters are known more accurately. A better sampling of the orbital period will decrease the uncertainty in the stars' masses which mostly arises from the radial velocity uncertainty, as well as the independent determination of the binary inclination from the IR ellipsoidal modulations for comparison to the inclination derived from the radio jets will constrain further evolutionary considerations of this peculiar system.

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