On the Clustering of GRBs on the Sky

Shiv K. Sethi[1], S. G. Bhargavi[2], and Jochen Greiner[3]

[1]*Mehta Research Institute, Chhatnag Road, Jhusi*
Allahabad 221 506, India
[2]*Indian Institute for Astrophysics, Koramangla*
Bangalore 560034, India
[3]*Astrophysikalisches Institut Potsdam*
D-14482 Potsdam , An der Sternwarte 16, Germany

Abstract. The two-point correlation of the 4th (current) BATSE catalog (2494 objects) is calculated. It is shown to be consistent with zero at nearly all angular scales of interest. Assuming that GRBs trace the large scale structure in the universe we calculate the angular correlation function for the standard CDM (sCDM) model. It is shown to be $\leq 10^{-4}$ at $\theta \simeq 5^\circ$ if the BATSE catalog is assumed to be a volume-limited sample up to $z \simeq 1$. Combined with the error analysis on the BATSE catalog this suggests that nearly 10^5 GRBs will be needed to make a positive detection of the two-point angular correlation function at this angular scale.

INTRODUCTION

Recent optical identification of Gamma-ray bursts (GRBs) has established the cosmological origin of GRBs and redshifts have been measured in 9 cases. (For a comprehensive list of references on this subject see [1]). However, the physical origin of these bursts, their environment, and their relationship with other astrophysical objects still remains an unsolved puzzle. If these bursts are associated with the underlying large scale structure in the universe, then they should show clustering in their positions on the sky as expected of cosmological objects.

One way to search for the clustering is to determine the two-point angular auto-correlation function of the burst positions [2–4]. We compute this quantity for the 4th (current) BATSE catalog (2494 objects) in the next section. In §3 we calculate the two-point correlation function from existing, viable, theoretical models of structure formation. §4 summarizes the main results.

TWO-POINT CORRELATION FUNCTION

Given a two-dimensional distribution of N point objects in a solid angle Ω, the two-point angular correlation function is defined using the relation [5]:

$$n_{\rm DD} = nd\Omega N(1 + w(\theta)) \tag{1}$$

Here $n_{\rm DD}$ is the total number of pairs between angular separation θ and $\theta + d\theta$; $n = N/\Omega$; and $d\Omega$ is an infinitesimal solid angle centered around θ. $w(\theta)$ is the two-point correlation function. It measures the excess of pairs over a random Poisson distribution at a given separation θ. Eq. (1) is not very convenient for estimating the two-point correlation function and several alternative estimators of the two-point angular correlation function have been suggested. We experimented with several estimators [6–9]. The advantage of using either [8] or [9] is that the error on the two-point function is nearly Poissonian; the leading term in the error for the other two estimators is $\propto 1/N$, which can dominate over the Poisson term for large bin size [9]. In this paper we report results using the estimator given by Landy and Szalay [9]:

$$\tilde{w}(\theta) = \frac{n_{\rm DD} - 2n_{\rm DR} + n_{\rm RR}}{n_{\rm RR}} \tag{2}$$

Here $n_{\rm DD}$ is the number of pairs (for a given θ) in the GRB catalog, $n_{\rm RR}$ is the number of pairs in a mock, random, isotropic sample, and $n_{\rm DR}$ is the catalog-random pair count. The variance of $\tilde{w}(\theta)$ is given by:

$$\delta\tilde{w}(\theta)^2 \simeq \frac{1}{n_{\rm DD}}. \tag{3}$$

In Figure 1 (Left Panel) we show the angular correlation function with 1σ error bars for current BATSE (2494 objects) catalog. We also plot the 1σ errors (Eq. (3)). The main conclusions of our analysis are:

1. The two-point angular correlation function is consistent with zero on nearly all angular scales of interest.

2. From Figure 1 (Left Panel) it is seen that at several angular scales a 1σ detection of the correlation function seems to be possible. To make a definitive statement about a detection we need to take into account several uncertainties in our analysis. One of the dominant source of uncertainty is the heterogeneity of the sample with respect to the error in angular positions of the GRBs (the localization uncertainty varies from $\simeq 1^\circ$–10°). This means that errors at $\theta \leq 10^\circ$ are much larger than seen in Figure 1 (Left Panel). Another major source of uncertainty comes from anisotropic exposure function of the BATSE instrument, which results in a non-zero correlation function even for a completely isotropic intrinsic distribution[1]. Though it is possible that some of the signal at large angular scales is not an artifact, more careful analysis would be required to confirm it.

---

[1] for more details see `http://www.batse.msfc.nasa.gov/batse/grb/catalog/`

THEORETICAL PREDICTIONS

The two-point angular correlation function can be related to the two-point three-dimensional correlation function $\xi(r)$ using Limber's equation (for details see [5]). If we assume that the GRBs constitute a volume-limited sample up to a distance $r_{\rm max}$ and that the comoving number density of objects is constant, the Limber's equation reduces to:

$$w(\theta) = \frac{\int_0^{r_{max}} \int_0^{r_{max}} r_1^2 r_2^2 dr_1 dr_2 \xi(r_{12}, z_1, z_2)}{\left[\int_0^{r_{max}} r^2 dr\right]^2} \tag{4}$$

Here

$$r_{12}^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta. \tag{5}$$

r is the coordinate distance in an isotropic, homogeneous universe. The two-point correlation function is related to the power spectrum $P(k)$ of the density fluctuations as:

$$\xi(r,t) = b^2 \frac{1}{2\pi^2} \int_0^\infty k^2 dk P(k,t) \frac{\sin(kr)}{kr}. \tag{6}$$

b, the bias factor, denotes the clustering of visible matter relative to the dark matter. While its absolute value is still uncertain, the relative bias between nearby rich clusters of galaxies and optically-identified galaxies is $\simeq 5$. And hence if GRBs originate in clusters rather than ordinary galaxies their correlation can be 25 times larger. In this paper, we use the linear perturbation theory predictions for $P(k,t)$. We have checked that for the angular scales of interest ($\theta \geq 5°$) it is a reasonable assumption. We use the BBKS fit [10] for the linear power spectrum of the standard CDM (sCDM) model and some of its variants. We normalize the power spectrum requiring $\sigma_8 = 0.7$. The time dependence of linear power spectrum is $P(k,t) \propto (1+z)^{-2}$, which is also the time dependence of the two-point correlation function. It should be noted that in general the two-point correlation function depends on both the separation between two points and their redshifts, as indicated in Eq. (4). However, the two-point correlation function is negligible for points separated by a large enough redshift difference. Therefore, for most purposes $\xi(r,t) \propto (1+z)^{-2}$, where z refers the redshift of any of the two points.

In Figure 1 (Right Panel) we show the theoretically predicted angular two-point correlation function for sCDM model. The bias b is taken to be one. If observed GRBs constitute a complete sample up to $z \simeq 1$ and they are assumed to be associated with highly biased structures like rich clusters, the value of correlation function is $\leq 10^{-4}$ at $5°$. This is the smallest angular scale at which information is possible in the BATSE catalog. At larger angles the correlation function typically scales as θ^{-1}.

CONCLUSIONS AND SUMMARY

The two-point correlation function of the 4th (current) BATSE catalog (2494 objects) is consistent with zero at nearly all angular scales that can be probed in the BATSE catalog. This result is consistent with theory if the GRBs are assumed to trace the dark matter distribution with some bias and are a complete sample up to $z \simeq 1$.

When can a detection of the two-point correlation function become possible? The error in the two-point correlation function scales as $n_{\rm DD}^{-1/2}$ (Eq. 3) and $n_{\rm DD} \propto N^2$, N being the number of objects in the catalog. Therefore the error in estimating the two-point correlation function scales as $1/N$. Theory suggests that the value of correlation function at $\theta = 5°$ is $\leq 10^{-4}$ if the GRB sample is assumed to be complete up to $z \simeq 1$. We check that $n_{\rm DD}$ at $\theta \simeq 5°$ is $\simeq 10^{-2}$ times the total number of pairs ($\simeq N^2/2$) in the GRB sample. This would suggest that a detection might become possible at this angular scale when the number of objects in the sample exceeds 10^5.

Future surveys like HETE-II and SWIFT will localize the GRBs to a few

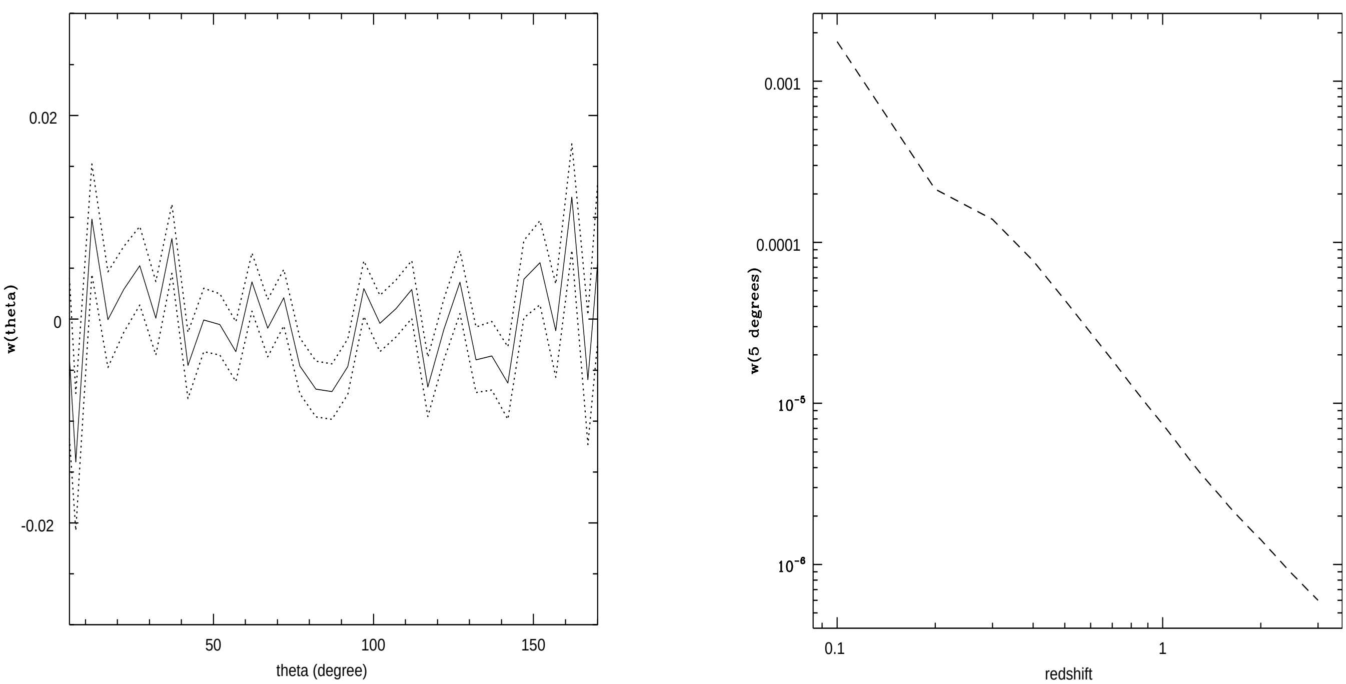

FIGURE 1. *Left Panel*: The two-point angular correlation function for the BATSE catalog (2494 objects) and the 1σ error bars are shown. The *solid* line corresponds to the two-point correlation function. The *dotted* lines show the 1σ errors given by Eq. (3). *Right Panel* : Theoretical prediction for the two-point angular correlation function is shown for the sCDM model as a function of depth (redshift) of the sample. The quantity plotted is the absolute value of the two-point correlation function at $\theta = 5°$.

arc-minutes. This means smaller angular scales could be probed. And as the theoretically-predicted two-point correlation function scales as $\sim \theta^{-1}$, the probability of detection will increase. SWIFT will detect nearly 1000 objects over a period of 3 years with an angular resolution $\leq 1''$. However, though the two-point correlation function is large at these angular scales, the average separation between 1000 objects on the complete sky is $\simeq 6^\circ$. Therefore as long as $w(\theta) \leq 1$, the probability of finding an object within a few arcseconds of the other is negligible. It is possible that $w(\theta) \gg 1$ at sub-arcsecond scales. However, detailed analysis, taking into account the non-linear correction to the power spectrum of density perturbation, is needed to make precise theoretical predictions for the future surveys.

Acknowledgements: JG is supported by the German Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie (BMBF/DLR) under contract No. 50 QQ 9602 3. RJG acknowledges a travel grant from DFG (KON 1973/1999 and GR 1350/7-1) to attend this conference. SKS thanks S. Bharadwaj for pointing out an error. We thank D. H. Hartman for valuable comments on the anisotropic exposure function of BATSE.

REFERENCES

1. Greiner, J. , `http://www.aip.de/People/JGreiner/grbgen.html` (1999).
2. Blumenthal, G. R., Hartmann, D. H., & Linder, E. V. , *AIP*, vol 307, p.117, New York (1994).
3. Hartmann, D. H. & Blumenthal, G. R. , *ApJ*, **342**, 521 (1989).
4. Lamb, D. Q. & Quashnock, J. M. , *ApJ*, **415**, L1 (1993).
5. Peebles, P. J. E. , *Large Scale Structure of the Universe*, Princeton University Press, Princeton (1980).
6. Peebles, P. J. E. & Hauser, M. G., *ApJS*, **28**, 19(1974).
7. Davis, M. & Peebles, P. J. E., *ApJ*, **267**, 465(1983)
8. Hamilton, A. J. S., *ApJ*, **417**, 19(1993)
9. Landy, S. D. & Szalay, A. S., *ApJ*, **412**, 64 (1993).
10. Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. , *ApJ*, **304**, 15 (1986).