The peculiar motions of early-type galaxies in two distant regions – V. The Mg–σ relation, age and metallicity

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ABSTRACT
We have examined the Mg–σ relation for early-type galaxies in the EFAR sample and its dependence on cluster properties. A comprehensive maximum likelihood treatment of the sample selection and measurement errors gives fits to the global Mg–σ relation of $Mg_b = 0.131 \log \sigma - 0.131$ and $Mg_2 = 0.257 \log \sigma - 0.305$. The slope of these relations is 25 per cent steeper than that obtained by most other authors owing to the reduced bias of our fitting method. The intrinsic scatter in the global Mg–σ relation is estimated to be 0.016 mag in $Mg_b$ and 0.023 mag in $Mg_2$. The Mg–σ relation for cD galaxies has a higher zero-point than for E and S0 galaxies, implying that cDs are older and/or more metal-rich than other early-type galaxies with the same velocity dispersion.

We investigate the variation in the zero-point of the Mg–σ relation between clusters. We find that it is consistent with the number of galaxies observed per cluster and the intrinsic scatter between galaxies in the global Mg–σ relation. We find no significant correlation between the Mg–σ zero-point and the cluster velocity dispersion, X-ray luminosity or X-ray temperature over a wide range in cluster mass. These results provide constraints for models of the formation of elliptical galaxies. However, the Mg–σ relation on its own does not place strong limits on systematic errors in Fundamental Plane (FP) distance estimates resulting from stellar population differences between clusters.

We compare the intrinsic scatter in the Mg–σ and Fundamental Plane relations with stellar population models in order to constrain the dispersion in ages, metallicities and M/L ratios for early-type galaxies at fixed velocity dispersion. We find that variations in age or metallicity alone cannot explain the measured intrinsic scatter in both Mg–σ and the FP. We derive the joint constraints on the dispersion in age and metallicity implied by the scatter in the Mg–σ and FP relations for a simple Gaussian model. We find upper limits on the dispersions in age and metallicity at fixed velocity dispersion of 32 per cent in $\delta t/t$ and 38 per cent in $\delta Z/Z$ if the variations in age and metallicity are uncorrelated; only strongly anticorrelated variations lead to significantly higher upper limits. The joint distribution of residuals from the Mg–σ and FP relations is only marginally consistent with a model having no correlation between age and metallicity, and is better matched by a model in which age and metallicity variations are moderately anticorrelated ($\delta t/t = 40$ per cent, $\delta Z/Z = 50$ per cent and $\rho \approx -0.5$), with younger galaxies being more metal-rich.

Key words: galaxies: clusters: general – galaxies: distances and redshifts – galaxies: elliptical and lenticular, cD – galaxies: evolution – galaxies: formation – galaxies: stellar content.

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1 INTRODUCTION

The primary aim of the EFAR project (Wegner et al. 1996, hereafter Paper I) is to use the tight correlations between the global properties of early-type galaxies embodied in the Fundamental Plane (FP) (Djorgovski & Davis 1987; Dressler et al. 1987) to measure relative distances to clusters of galaxies in order to investigate peculiar motions and the mass distribution on large scales. However, these global relations also constrain the dynamical properties and evolutionary histories of early-type galaxies. For example, Renzini & Ciotti (1993) show that the tilt of the FP implies a range in mass-to-light ratio $M/L$ among ellipticals of less than a factor of 3, while the low scatter about the FP implies a scatter in $M/L$ at any location in the plane of less than 0.1 dex. Similar reasoning has been used to constrain the star formation history of cluster ellipticals using the metallicity–mass relation (Bower, Lucey & Ellis 1992; Kodama & Arimoto 1997). Recently the FP, $Mg\alpha$ and colour–mass relations have been followed out to higher redshifts and used to show that the early-type galaxies seen at $z\sim 1$ differ from present-day early-type galaxies in a manner consistent with passive evolutionary effects (van Dokkum & Franx 1996; Ellis et al. 1997; Kelson et al. 1997; Kodama & Arimoto 1997; Ziegler & Bender 1997; Bender et al. 1998; Kodama et al. 1998; Stanford, Eisenhardt & Dickinson 1998; van Dokkum et al. 1998).

In this paper we consider the relation between the central velocity dispersion $\sigma$ and the strength of the magnesium lines at a rest wavelength of 5174 Å for the early-type galaxies in the EFAR sample. This $Mg\alpha$ relation connects the dynamical properties of galaxy cores with their stellar populations. The remarkably small scatter about this relation (Burstein et al. 1988; Guzmán et al. 1992; Bender et al. 1993; Jørgensen, Franx & Kjærgaard 1996; Bender et al. 1998) and its distance-independent nature make it a potentially useful constraint on models of the star formation history of early-type galaxies and a test for environmental variations in the FP (Burstein et al. 1988; Bender, Ziegler & Bruzual 1996).

There are, however, some problems with using the $Mg\alpha$ relation for probing galaxy formation. Two of these problems are apparent from the stellar population models (e.g. Worthey 1994; Vazdekis et al. 1996): (i) both age and metallicity contribute to the $Mg$ line strengths to a comparable degree, so that a spread in line-strengths could be a result of either a range of ages or a range of metallicities, or some combination; (ii) the $Mg$ line strengths are not particularly sensitive indicators – at fixed metallicity a difference in age of a factor of 10 only results in a change of $0.05$–$0.1$ mag, while at fixed age a difference of 1 dex in metallicity gives a change of $0.1$–$0.2$ mag. Thus $Mg$ line-strength measurements must be accurate in order to yield useful constraints on the ages and metallicities of stellar populations, and the $Mg\alpha$ relation on its own can only supply constraints on combinations of age and metallicity, and not one or the other separately.

Recently, Trager (1997) suggested that the tightness of the $Mg\alpha$ relation may be the result of a ‘conspiracy’, in that there appears to be an anticorrelation between the ages and metallicities of the stellar populations in early-type galaxies at fixed mass which acts to reduce the scatter in the $Mg$ line-strengths. Trager takes the accurate $H\beta$, $Mg$ and Fe line-strengths from González (1993) and applies the stellar population models of Worthey (1994) to derive ages and abundances from line indices with different dependences on age and metallicity. He finds that at fixed velocity dispersion the ages and abundances lie in a plane of almost constant $Mg$ line-strength, leading him to predict little scatter in the $Mg\alpha$ relation even for large differences in age or metallicity – a factor of 10 in age (from 1.5 to 15 Gyr) gives a spread in $Mg_2$ of only $0.01$–$0.02$ mag. This conclusion depends on the appropriateness of the single stellar population models and requires confirmation from further high-precision line-strength measurements. It can also be tested using the high-redshift samples now becoming available.

In a similar vein, a number of authors (Bower, Kodama & Terlevich 1998; Ferreras, Charlot & Silk 1998; Shioya & Bekki 1998) have recently re-examined whether the apparent passive evolution of the colour–magnitude relation out to $z\sim 1$ really implies a high redshift for the bulk of the star formation in elliptical galaxies. They conclude that, in fact, such evolution can be consistent with a rather broad range of ages and metallicities if the galaxies assembling more recently are on average more metal-rich than older galaxies of similar luminosity.

As well as studies focusing on the evolution of the galaxy population, there have been investigations of possible variations with local environment. Guzmán et al. (1992) have suggested that there are systematic variations in the $Mg\alpha$ relation which affect estimates of relative distances based on the FP. They report a significant offset in the zero-point of the $Mg\alpha$ relation between galaxies in the core of the Coma cluster and galaxies in the cluster halo. Jørgensen and coworkers (1996, 1997) examined a sample of 11 clusters and found a weak correlation between $Mg$ line-strength and local density within the cluster which is consistent with this result. Similar offsets are claimed between field and cluster ellipticals by de Carvalho & Djorgovski (1992) and Jørgensen (1997), although Burstein, Faber & Dressler (1990) found no evidence of environmental effects. Such systematic differences could result from different star formation histories in different density environments, producing variations in the mass-to-light ratio of the stellar population. FP distance measurements would then be subject to environment-dependent systematic errors leading to spurious peculiar motions. Where data for field and cluster ellipticals come from different sources, however, the possibility also exists that any zero-point differences are due to uncertainties in the relative calibrations rather than intrinsic environmental differences.

The $Mg\alpha$ relation has thus become an important diagnostic for determinations of both the star formation history and the peculiar motions of elliptical galaxies. Here we examine the $Mg\alpha$ relation in the EFAR sample, which includes more than 500 early-type galaxies drawn from 84 clusters spanning a wide range of environments. In Section 2 we summarize the relevant properties of the sample and the techniques used to determine the $Mg\beta$, $Mg_2$ line-strength indices, the central velocity dispersions $\sigma$, and the errors in these quantities. We present the $Mg\alpha$ relation in Section 3 and investigate how it varies from cluster to cluster within our sample, and with cluster velocity dispersion, X-ray luminosity and X-ray temperature. In Section 4 we compare our results with the predictions of stellar population models in order to derive constraints on the ages, metallicities and mass-to-light ratios of early-type galaxies in clusters. In particular, we consider the constraints on the dispersion in the ages and metallicities from the intrinsic scatter in the $Mg\alpha$ relation on its own, and in combination with the intrinsic scatter in the FP. Our conclusions are given in Section 5.

2 THE DATA

Here we give a short description of our sample and data set, with emphasis on the velocity dispersions and line indices used in this paper. The interested reader can find more detail on the sample selection in Paper I (Wegner et al. 1996); on the measurement, calibration and error estimation procedures for the spectroscopic
parameters in Paper II (Wegener et al. 1999); and on the structural and morphological properties of the galaxies in Paper III (Saglia et al. 1997).

2.1 The sample

The EFAR sample of galaxies comprises 736 mostly early-type galaxies in 84 clusters. These clusters span a range of richnesses and lie in two regions towards Hercules–Corona Borealis and Perseus–Pisces–Cetus at distances of between 6000 and 15 000 km s\(^{-1}\). In addition to this programme sample we have also observed 52 well-known galaxies in Coma, Virgo and the field in order to provide a calibrating link to previous studies.

The EFAR galaxies are listed in table 2 of Paper I, and comprise an approximately diameter-limited sample of galaxies larger than about 20 arcsec with the visual appearance of ellipticals. Photometric imaging (Paper III) shows that 8 per cent are cDs, 12 per cent are pure Es and 49 per cent are bulge-dominated E/S0s; thus 69 per cent of the sample are early-type galaxies, with the remaining 31 per cent being spirals or barred galaxies. We have obtained spectroscopy for 666 programme galaxies, measuring redshifts, velocity dispersions and line strengths (indices) (Paper II). We have used the redshifts that we obtained together with literature redshifts for other galaxies in the clusters in order to assign programme galaxies to physical clusters. We have used the combined redshift data for these physical clusters to estimate cluster mean redshifts and velocity dispersions. The early-type galaxies in our sample span a wide range in luminosity, size and mass: they have absolute magnitudes from \(M_{R} = -24\) to \(-18\) (\(M_{R} = -21.6; H_{0} = 50\) km s\(^{-1}\) Mpc\(^{-1}\)), effective radii from 1 to 70 kpc (\(R_{e} = 9.1\) kpc) and central velocity dispersions from less than 100 to over 400 km s\(^{-1}\) (\(\sigma = 220\) km s\(^{-1}\)). The sample is thus dominated by early-type galaxies with luminosities, sizes and masses typical of giant ellipticals.

2.2 The measurements

We summarize here the procedures used in measuring the redshifts, velocity dispersions and Mg line strengths; full details are given in Paper II.

Redshifts and velocity dispersions were measured from each observed galaxy spectrum using the IRAF task FXCOR. Line strength indices on the Lick system were determined using the prescription given by González (1993). The Mg\(_{b}\) and Mg\(_{2}\) indices were both measured: Mg\(_{b}\) because it is the index most commonly measured in previous work, and Mg\(_{b}\) because it could be measured for more objects (as it requires a narrower spectral range) and is better determined (being less susceptible to variations in the non-linear continuum shape). We find it more convenient to express the ‘atomic’ Mg\(_{b}\) index in magnitudes like the ‘molecular’ Mg\(_{2}\) index rather than as an equivalent width in Å, since this puts these two indices on similar footing. The conversion is

\[
M_{gb} = -2.5 \log_{10} \left( 1 - \frac{M_{gb}}{\Delta \lambda} \right),
\]

where \(\Delta \lambda\) is the index bandpass (32.5 Å for Mg\(_{b}\)).

Error estimates for each quantity were derived from detailed Monte Carlo simulations, calibrated by comparisons of the estimated errors with the results obtained from repeat measurements (over 40 per cent of our sample had at least two spectra taken). Two sorts of corrections were applied to the dispersions and line strengths: (i) an aperture correction, based on that of Jørgensen, Franx & Kjærgaard (1995), to account for different effective apertures sampling different parts of the galaxy profile, and (ii) a run correction to remove systematic errors between different observing setups. After applying these corrections, individual measurements for each galaxy were combined using a weighting scheme based on the estimated errors and the overall quality of the spectrum.

The median estimated errors in the final combined values are \(\Delta \sigma / \sigma = 9.1\) per cent (i.e. \(\Delta \log \sigma = 0.040\) dex), \(\Delta M_{gb} = 0.013\) mag and \(\Delta M_{g2} = 0.015\) mag. The distribution of estimated errors for each quantity is shown in the upper panel of Fig. 1. The lower panel of the figure shows how the error estimates were calibrated against the repeat observations: the distribution of the ratio of rms error to estimated error for objects with repeat measurements is compared with the predicted distribution assuming that the estimated errors are the true errors. The initial error estimates from the simulations have been rescaled to give the best match (under a Kolmogorov–Smirnov test) to the rms errors from the repeat measurements. A rescaling by factors of 0.85 and 1.15, respectively, gives good agreement for the errors in \(\sigma\) and Mg\(_{b}\); adding 0.005 mag likewise gives good agreement for the errors in Mg\(_{2}\).

A comparison with the literature (Paper II, fig. 13) shows that our dispersions are consistent with previous measurements by Davies et al. (1987), Guzmán (1993), Jørgensen (1997), Lucey et al. (1997) and Whitmore et al. (1985). For the subset of galaxies in common, we compared our line strengths with the definitive Lick system measurements of Trager et al. (1998) in order to derive the small zero-point corrections required to calibrate our measurements to the Lick system (Paper II, figs 14 and 15); the overlap of our Mg\(_{2}\) measurements with those of Lucey et al. (1997) also shows consistency (fig. 16, Paper II).

3 THE Mg–\(\sigma\) RELATION

3.1 The global relation

In this section we investigate the global Mg–\(\sigma\) relation found
amongst the entire sample of EFAR galaxies with early-type morphological classifications (cD, E or E/S0; see definitions in Paper III) for which we obtained linestrength measurements. The Mg\textsubscript{b}–\sigma relation is shown in Fig. 2(a) and the Mg\textsubscript{2}–\sigma relation in Fig. 2(b).

In order to fit linear relations with intrinsic scatter in the presence of significant measurement errors in both variables, arbitrary censoring of the data set and a broad sample selection function, we have developed a comprehensive maximum likelihood (ML) fitting procedure (Saglia et al., in preparation). Excluding galaxies with dispersions less than 100 km s\textsuperscript{-1} or selection probabilities less than 10 per cent, and also outliers with low likelihoods, the ML fits to the Mg\textsubscript{b}–\sigma relation (490 galaxies) and the Mg\textsubscript{2}–\sigma relation (423 galaxies) are

\[
\text{Mg}\textsubscript{b} \approx 0.131 \pm 0.017 \log \sigma - 0.131 \pm 0.041, \quad (2)
\]

\[
\text{Mg}\textsubscript{2} \approx 0.257 \pm 0.027 \log \sigma - 0.305 \pm 0.064. \quad (3)
\]

These fits are shown in Fig. 2 as solid lines. The ratio of the slopes of these relations is consistent with the Mg\textsubscript{2}–Mg\textsubscript{b} relation that we obtained in Paper II: Mg\textsubscript{2} \approx 1.94Mg\textsubscript{b} – 0.05. Monte Carlo simulations of the data set and fitting process, the results of which are displayed in Fig 3, show that there is no bias in the ML estimates of the slopes and zero-points, and provide reliable estimates of the uncertainties in the fit.

The ML fits can be compared with simple regressions of Mg\textsubscript{b} and Mg\textsubscript{2} on \log \sigma. These regressions are shown in the figure as dashed lines, and are

\[
\text{Mg}\textsubscript{b} \approx (0.104 \pm 0.011) \log \sigma - 0.067 \pm 0.026, \quad (4)
\]

\[
\text{Mg}\textsubscript{2} \approx (0.199 \pm 0.016) \log \sigma - 0.168 \pm 0.038. \quad (5)
\]

As expected, the simple regressions yield slopes which are biased low as a result of the presence in the data of errors in the abscissa as well as the ordinate, and also the intrinsic scatter in the relation. Slightly less-biased results are obtained by least-squares regression minimizing the orthogonal residuals (cf. Jørgensen et al. 1996):

\[
\text{Mg}\textsubscript{b} \approx (0.109 \pm 0.012) \log \sigma - 0.078 \pm 0.027, \quad (6)
\]

\[
\text{Mg}\textsubscript{2} \approx (0.215 \pm 0.017) \log \sigma - 0.205 \pm 0.041. \quad (7)
\]
Hereafter we adopt the ML fits to the Mg– relation being fitted had significant errors in the velocity dispersions (as of the Mg– functions. We conclude that previous determinations of the slope properly account for the measurement errors or the selection uncertainties are underestimated because these regressions do not described in Isobe et al. (1990) and Feigelson & Babu (1992). The regression fits are shown in the insets to Figs 2(a) and (b). In order to minimize for both measurement errors and run correction errors). These least-squares fits and their uncertainties are obtained using the ML fit may be exaggerated by outliers or by deviations of the underlying distribution of galaxies in the Mg– plane from a bivariate Gaussian. We therefore drop the assumption of an intrinsic bivariate Gaussian distribution in the Mg– plane and use Monte Carlo simulations based on the observed distribution of dispersions and line-strengths and their estimated errors (accounting for both measurement errors and run correction errors). These simulations assume only that there is a global linear Mg– relation about which there is Gaussian intrinsic scatter. We vary this intrinsic scatter and compute the robust estimate of the observed scatter about the fit (the half-width of the central 68 per cent of the residuals) for the simulated distributions. The results of these simulations are presented in Fig. 4, which shows the normalized likelihood distributions for the intrinsic scatter in Mg– and Mg2 given the observed scatter. We find that to account for the observed scatter in the relations we require an intrinsic scatter of 0.016±0.001 mag for Mg– and 0.023±0.002 mag for Mg2–. The ratio of the intrinsic scatter in Mg2 to the intrinsic scatter in Mg– is slightly lower than expected from the observed Mg2–Mg– relation, Mg2≈1.94Mg––0.05 (see Paper II).

Table 1. Comparison of Mg– relation fits.

<table>
<thead>
<tr>
<th></th>
<th>Slope</th>
<th>Intercept</th>
<th>δMgobs</th>
<th>δMgint</th>
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</thead>
<tbody>
<tr>
<td>Ziegler &amp; Bender (1997)</td>
<td>0.106</td>
<td>−0.079</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>EFAR (this work)</td>
<td>0.131</td>
<td>−0.131</td>
<td>0.022</td>
<td>0.016</td>
</tr>
<tr>
<td>±0.020</td>
<td>±0.048</td>
<td>±0.002</td>
<td>±0.001</td>
<td></td>
</tr>
<tr>
<td>Mg2– relation...</td>
<td>Burstein et al. (1988)</td>
<td>0.175</td>
<td>−0.110</td>
<td>0.016</td>
</tr>
<tr>
<td>Guzmán et al. (1992)</td>
<td>0.260</td>
<td>−0.316</td>
<td>0.016</td>
<td>0.013</td>
</tr>
<tr>
<td>±0.027</td>
<td>±0.003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bender et al. (1993)</td>
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<td>−0.166</td>
<td>0.025</td>
<td>0.018</td>
</tr>
<tr>
<td>Jørgensen et al. (1996)</td>
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<td>−0.155</td>
<td>0.025</td>
<td>0.020</td>
</tr>
<tr>
<td>±0.016</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EFAR (this work)</td>
<td>0.257</td>
<td>−0.305</td>
<td>0.031</td>
<td>0.023</td>
</tr>
<tr>
<td>±0.028</td>
<td>±0.067</td>
<td>±0.003</td>
<td>±0.002</td>
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</tr>
</tbody>
</table>

3.2 Cluster-to-cluster variations

We do not have enough galaxies per cluster to fit both the slope and
the zero-point of the Mg–σ relations on a cluster-by-cluster basis, even in our best-sampled clusters. We therefore limit ourselves to investigating the variation in the Mg–σ zero-point. To this end we measured the median offset in Mg\(^{b}_0\) and Mg\(^2\) from the global fits given above for the clusters with three or more linestrength measurements (75 clusters for Mg\(^{b}_0\) and 72 for Mg\(^2\)). Note that we only used galaxies that are cluster members based on their redshifts (see Paper II). The results are not changed significantly if we use all clusters, or only clusters with five or more measurements.

The top panels of Fig. 5 show these zero-point offsets as a function of cluster ID number (CID), while the middle panels show the distributions of the offset values. The robustly estimated scatter in the zero-point offsets is 0.012 mag in Mg\(^{b}_0\) and 0.019 mag in Mg\(^2\), showing that the relations are remarkably uniform among the aggregates of galaxies in the EFAR sample. The bottom panels in the figure plot the same offsets as a function of redshift, showing that there is no dependence of the relations on relative distance within the sample.

This scatter in the zero-point offsets could purely be a consequence of a galaxy-to-galaxy scatter in a global Mg–σ relation, or it could also require a variation in the zero-point of the relation from cluster to cluster. These possibilities were examined by extending the simulations described in the previous section, adding a further source of scatter to the Mg–σ relation in the form of an intrinsic variation between clusters in the zero-point of the relation. For simplicity we assume that this variation also has a Gaussian distribution.

We find that if we make the extreme assumption that there is cluster-to-cluster scatter but no intrinsic scatter between galaxies within a cluster, then zero-point variations between clusters with an rms of 0.009 mag in Mg\(^{b}_0\) and 0.015 mag in Mg\(^2\) are required to recover the observed cluster-to-cluster scatter. However this model underpredicts the observed scatter about the global relation, giving 0.017±0.001 mag for Mg\(^{b}_0\) and 0.025±0.002 mag for Mg\(^2\), compared with the actual values of 0.022±0.002 mag and 0.031±0.003 mag. On the other hand, if we assume that there is no zero-point variation between clusters, then the intrinsic scatter between galaxies required to recover the observed scatter in the global relation (0.016 mag in Mg\(^{b}_0\) and 0.023 mag in Mg\(^2\); see previous section) predicts a scatter in the cluster zero-points of 0.012±0.001 mag in Mg\(^{b}_0\) and 0.016±0.002 mag in Mg\(^2\), which is consistent with the observed values of 0.012±0.002 and 0.019±0.004 mag within the joint errors.

We conclude that there is no evidence for significant intrinsic zero-point variations between clusters, because sampling a galaxy population drawn from a single global relation with intrinsic scatter consistent with the observations can account for the zero-point differences between our clusters.

3.3 Variation with cluster properties

As there is very little change in the zero-point of the Mg–σ relation from cluster to cluster, it follows that there can be at most only a weak dependence of the zero-point on the properties of the clusters.

Here we investigate the effect of cluster properties on the stellar populations as reflected in the Mg–σ zero-points, considering
cluster velocity dispersions, X-ray luminosities and X-ray temperatures (all indicators of cluster mass). The cluster dispersions come from table 7 of Paper II, using redshifts both from EFAR and from the ZCAT catalogue (Huchra et al. 1992; version of 1997 May 29). X-ray luminosities and temperatures are available for 26 of our 84 clusters in the homogeneous and flux-limited catalogue of X-ray properties of Abell clusters by Ebeling et al. (1996) based on ROSAT All-Sky Survey data. The X-ray luminosities are determined to a typical precision of about 20 per cent. In order to have comparable precision in the cluster velocity dispersions, we only use clusters with dispersions computed from at least 20 galaxy redshifts; this also leaves 26 clusters, 17 of which are in common with the X-ray subsample.

Fig. 6 shows the offsets in the Mg–j relations as functions of log j, log LX and kT. Applying the Spearman rank correlation statistic, we find that there is no significant correlation between the Mg–j offsets and any of these quantities, and thus no evidence for a trend in the zero-point of the Mg–j relation with cluster mass.

Weighted regressions give best-fitting relations and their uncertainties:

\[ \Delta M_{b0} = (-0.002 \pm 0.009) \log \sigma_{\text{cluster}} + (0.016 \pm 0.050), \]
\[ \Delta M_{2} = (-0.001 \pm 0.011) \log \sigma_{\text{cluster}} + (0.002 \pm 0.060). \]

If we take a complementary approach, splitting the clusters into two subsamples about the median value of LX and fitting the Mg–σ relations to the galaxies of the high-LX and low-LX clusters separately, we again find no significant differences in the slopes or the zero-points of the fits, which are compatible with the global fits obtained above.

4 DISCUSSION

There are at least four main questions that can be addressed using the above results.

(i) What are the theoretical implications of the lack of correlation between the mass of a cluster and the zero-point of the Mg–σ relation for cluster galaxies?

(ii) What effect do the stellar population differences implied by the observed variations in the Mg–σ relation have on FP estimates of distances and peculiar velocities?

(iii) What constraint does the intrinsic scatter about the Mg–σ relation place on the spread in age, metallicity and mass-to-light ratio amongst early-type galaxies in clusters?

(iv) What further constraints on these quantities result from combining the scatter about the Mg–σ relation with the intrinsic scatter about the FP?

4.1 Mg–σ zero-point and cluster mass

The small scatter in the zero-point of the Mg–σ relation from cluster to cluster, and in particular the lack of correlation between the Mg–σ zero-point and the cluster mass, seems to imply that the mass overdensity (on Mpc scales) in which an early-type galaxy is found has little connection with its stellar population and star formation history.

The variation of the Mg–σ relation with cluster properties has previously been studied in a sample of 11 nearby clusters by Jørgensen et al. (1996) and Jørgensen (1997). Following Guzmán et al. (1992), these authors look for a trend in Mg2–σ offsets with

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the ‘local density’ within clusters. The estimator of local density used is 
\( \rho_{\text{cluster}} = \log \sigma_{\text{cluster}} - \log R \), where \( R \) is the projected distance of the galaxy from the cluster centre. Since \( R \) is only a lower limit on the true distance of the galaxy from the cluster centre, this is a rather poor estimator of the true local density. Jørgensen et al. found that the residuals in \( M_g \) show a weak trend with local density, \( \Delta M_g \approx 0.009 \rho_{\text{cluster}} \). Since the residuals do not correlate with radius within the cluster [see fig. 3 of Jørgensen (1997)], but do show a significant correlation with cluster velocity dispersion, \( \Delta M_g \approx 0.02 \log \sigma_{\text{cluster}} \) (least-squares fit to the data in fig. 5 of Jørgensen (1997)), we would argue that a more straightforward interpretation of their results is a correlation of \( M_g - \sigma \) zero-point with total cluster mass rather than local density.

A correlation of this amplitude is formally consistent at the 2σ level with the distribution of \( M_g \) offsets versus \( \log \sigma_{\text{cluster}} \) for the EFAR data [see equation (9)]; transforming Jørgensen’s result via the \( M_g - M_{gb'} \) relation gives a correlation which is consistent at the 1.4σ level with equation (8). We conclude that any correlation between the \( M_g - \sigma \) relation zero-point and the cluster mass is sufficiently weak (of order \( \Delta M_g \approx 0.02 \log \sigma_{\text{cluster}} \) or less) that it is not reliably established by the existing data, which are consistent with no correlation at all.

Semi-analytic models for the formation of elliptical galaxies, which previously neglected metallicity effects (see Baugh, Cole & Frenk 1996; Kauffmann 1996), are only now beginning to incorporate chemical enrichment and successfully reproduce the general form of the observed colour–magnitude and \( M_g - \sigma \) relations (Kauffmann & Charlot 1998). In consequence, there are as yet no reliable predictions for the variation of the \( M_g - \sigma \) relation zero-point with cluster mass. The limits given above, together with limits on the difference in \( M_g - \sigma \) zero-points for field and cluster ellipticals (Burstein et al. 1990, de Carvalho & Djorgovski 1992; Jørgensen 1997), should provide valuable additional constraints and encourage further development of chemical enrichment models within a hierarchical framework for galaxy and cluster formation.

### 4.2 Systematic effects on FP distances

We now consider the effects on FP distance estimates of systematic differences in the stellar populations of early-type galaxies from cluster to cluster. In Section 3.2 we found that the observed cluster-to-cluster variations in the \( M_g - \sigma \) zero-point were consistent with sampling a single global \( M_g - \sigma \) relation with intrinsic scatter between galaxies, and did not require intrinsic scatter between clusters. Here we turn the question around and ask how much intrinsic cluster-to-cluster scatter is allowed by the observations.

From simulations using the model described in Section 3.2, incorporating intrinsic scatter both between galaxies and between clusters, we find that the maximum cluster-to-cluster scatter allowed within the 1σ uncertainties in the scatter in the global \( M_g - \sigma \) relation and the cluster zero-points is approximately 0.005 mag in \( M_{gb'} \) and 0.010 mag in \( M_g \). For our best-fitting ML \( M_g - \sigma \) relations and a FP given by \( R = \alpha \sigma^p \) with \( \alpha = 1.27 \), this level of cluster-to-cluster scatter would lead to rms errors in FP distance estimates of up to 10 per cent. These systematic errors, resulting from differences in the mean stellar populations between clusters, would apply even to clusters in which the FP distance errors arising from stellar population differences between galaxies had been made negligible by observing many galaxies in the cluster.

\(^{1}\)The sign of the trend in equation (6) of Jørgensen (1997) is incorrect; the coefficient should be +0.009 (Jørgensen, private communication).

We emphasize that our results here do not require any cluster-to-cluster scatter, but are consistent with cluster-to-cluster scatter corresponding to systematic distance errors between clusters with an rms of up to 10 per cent. We therefore cannot determine from the \( M_g - \sigma \) relation alone whether systematic differences in the mean stellar populations between clusters contribute significantly (or at all) to the errors in FP estimates of distances and peculiar velocities. A more effective way of testing for such systematic differences is by directly comparing the zero-point offset of each cluster from the global \( M_g - \sigma \) relation with the ratio of its FP and Hubble distance estimates; this approach will be investigated in a future paper.

### 4.3 Stellar population models

To answer the questions concerning the typical age, metallicity and mass-to-light ratio of early-type galaxies which were raised at the start of this discussion, we need to employ stellar population models. We use the predictions from the single stellar population models of Worthey (1994) and Vazdekis et al. (1996), noting the many caveats given by these authors regarding their models. To simplify our analysis, we fit \( M_{gb'} \), \( M_g \) and \( ML_{LR} \) as linear functions of logarithmic age (\( \log t \), with \( t \) in Gyr) and metallicity (\( \log Z/Z_\odot \)), for galaxies with ages greater than 4 Gyr and metallicities in the range −0.5 to +0.5. For the model of Worthey (1994; Salpeter IMF) we obtain

\[
M_{gb'} \approx 0.058 \log t + 0.086 \log Z/Z_\odot + 0.077, \quad (10)
\]

\[
M_g \approx 0.107 \log t + 0.182 \log Z/Z_\odot + 0.147, \quad (11)
\]

\[
\log ML_{LR} \approx 0.825 \log t + 0.184 \log Z/Z_\odot - 0.169. \quad (12)
\]

Fig. 7 compares this fit with Worthey’s model in the case of the predicted dependence of \( M_{gb'} \) and \( ML_{LR} \) on age and metallicity. The figure shows that for ages of 5 Gyr or greater the fit and the model are in satisfactory agreement for all metallicities.

For the model of Vazdekis et al. (1996; bimodal initial mass function, \( \mu = 1.35 \)) we have

\[
M_{gb'} \approx 0.051 \log t + 0.083 \log Z/Z_\odot + 0.084, \quad (13)
\]

\[
M_g \approx 0.115 \log t + 0.187 \log Z/Z_\odot + 0.137, \quad (14)
\]

\[
\log ML_{LR} \approx 0.673 \log t + 0.251 \log Z/Z_\odot - 0.216, \quad (15)
\]

in agreement with the fit obtained by Jørgensen (1997). There is good agreement between the predictions of the two models for the dependence of \( M_g \) and \( M_{gb'} \) on age and metallicity, and moderately good agreement for the dependence of \( ML_{LR} \).

Note that the same change in the Mg indices is produced by changes in age, \( \Delta \log t \), and metallicity, \( \Delta \log Z/Z_\odot \), if \( \Delta \log t/\Delta \log Z/Z_\odot \approx 3/2 \). This is the ‘3/2 rule’ of Worthey (1994), which applies to many of the Lick line indices, leaving them degenerate with respect to variations in age and metallicity. However age and metallicity produce the same change in \( ML_{LR} \) only if \( \Delta \log t/\Delta \log Z/Z_\odot \approx 1/3 \) or 1/4, so that measurements of mass-to-light ratios can in principle be combined with Mg line strengths to break the age/metallicity degeneracy.

### 4.4 Dispersion in age, metallicity and \( M/L \)

In the following analysis we infer the dispersion in the ages and metallicities of early-type galaxies by comparing the scatter in the \( M_g - \sigma \) relation with the predictions of the single stellar population models described in the previous section. This analysis uses the stellar population models to predict differential changes in the
Figure 7. The relation between Mg$b'$ and log $M/I_R$ as a function of age and metallicity in the model of Worthey (1994). The solid lines are contours of constant age (1.5, 2, 3, 5, 8, 12 and 17 Gyr), with increasing line thickness indicating increasing age. The dashed lines are contours of constant metallicity (−0.5, −0.25, 0, 0.25, 0.5), with increasing line thickness indicating increasing metallicity. The dotted grid is the linear fit to the model.

 qtities of interest, and not absolute values. It is also important to remember that by the dispersion in age or metallicity we mean the dispersion in these quantities at fixed log $\sigma$, or, equivalently, the dispersion after the overall trend with log $\sigma$ is accounted for. Thus the dispersion in age or metallicity that we infer is the dispersion at fixed galaxy mass, not the distribution of ages and metallicities as a function of galaxy mass (which is related to the slope of the Mg−$\sigma$ relation and the distribution of galaxies along it).

Single stellar populations models specified by (amongst other parameters) a unique age and a unique metallicity can only provide an approximation to real galaxies, the stellar contents of which must necessarily span a range (although perhaps a narrow one) of ages and metallicities. Since the global Mg indices can be quite sensitive to the detailed metallicity distribution (Greggio 1997), some of the scatter that we observe may be due to galaxy-to-galaxy differences in the shape of the metallicity distribution rather than a dispersion in the mean metallicity or age.

A further complication is presented by the overabundance of Mg with respect to Fe (compared with the solar ratio) in the cores of early-type galaxies (e.g. Peletier 1989; Gorgas, Efstathiou & Aragón-Salamanca 1990; Worthey, Faber & González 1992). As a comparison of Figs 2 and 7 shows, the models discussed in the previous section are unable to account for the highest observed Mg line strengths. Tantalò, Chiosi & Bressan (1998) have produced single stellar population models including the effects of [Mg/Fe] variations and find that

$$\Delta M_{g2} = 0.099 \Delta [Mg/Fe] + 0.089 \Delta \log t + 0.166 \Delta \log Z/I_Z$$  

Comparing this equation with those above, we see that the differential dependence on age and metallicity is similar to that predicted by Worthey (1994) and Vazdekis et al. (1996). However, any intrinsic scatter in the [Mg/Fe]−$\sigma$ relation will contribute additionally to the intrinsic scatter in the $M_{g2}−\sigma$ relation and reduce the dispersion in age and metallicity required to account for the observations.

For these reasons, and also because of other potential sources of intrinsic scatter such as dark matter, rotation, anisotropy, projection effects and broken homology, the estimates of the dispersion in age and metallicity derived here must be considered as upper limits.

With these caveats in mind, we proceed to use the model fits given in the previous section to infer the dispersion in age or metallicity based on the observed intrinsic scatter of 0.016 mag in Mg$b'−\sigma$ and 0.023 mag in Mg$g−\sigma$. For ease of interpretation we quote the dispersions in age and metallicity as the fractional dispersions $\delta \log t / \log e$ and $\delta Z/I_Z / \log e$. In applying the models in what follows, we adopt the mean of the coefficients for the two models and give the dispersions in age and metallicity corresponding to the intrinsic scatter about the Mg$b'−\sigma$ relation. Using the intrinsic scatter obtained from the Mg$g−\sigma$ relation would give results that are ~30 per cent smaller, because the observed ratio of the intrinsic scatters is $\delta M_{g2}/\delta M_{b'} = 1.4$, rather than about 2 as would be expected either from the observed Mg$g−Mg_{b'}$ relation or from the models. We use the scatter in Mg$b'$ rather than Mg$g$, because our goal is to establish upper limits on the dispersions in age and metallicity. The estimated errors in the intrinsic scatter lead to uncertainties in the dispersions of 5–10 per cent.

If age variations in single stellar populations are the only source of scatter then the dispersion in age is $\delta \log t = 67$ per cent, whereas if metallicity variations are the sole source then the dispersion in metallicity is $\delta Z/I_Z = 43$ per cent. Similarly, the observed difference in the Mg−$\sigma$ relation zero-point for the cD galaxies implies that these objects are either older or more metal-rich than normal E or E/S0 galaxies. If the zero-point differences are interpreted as age differences, cDs are on average 40 per cent older than typical E or E/S0 galaxies (i.e. as old as the oldest early-type galaxies); if the zero-point differences are interpreted as metallicity differences, cDs have metallicities on average 25 per cent higher than typical E or E/S0 galaxies (i.e. as high as the most metal-rich early-type galaxies).

We can also use the model fits to estimate the approximate change in $M/I_R$ corresponding to a change in the Mg line indices. If these changes are caused by age variations alone, then we find that $\Delta \log M/I_R = 7 \Delta M_{g2}$ and $\Delta \log M/I_R = 14 \Delta M_{b'}$; if, however, they are due only to variations in metallicity we have $\Delta \log M/I_R = 1.2 \Delta M_{g2}$ and $\Delta \log M/I_R = 2.6 \Delta M_{b'}$. Thus the change in $\log M/I_R$ is about five times larger if the observed change in the Mg indices is due to age differences rather than metallicity differences. The intrinsic scatter in the Mg$b'−\sigma$ relation implies a dispersion in mass-to-light ratio of 50 per cent if due to age variations, but only 10 per cent if due to metallicity variations.

This predicted scatter in $M/I_R$ is in fact a scatter in luminosity or surface brightness (as that is all the models deal with). We can therefore readily establish the effect of this scatter on distances estimated using the FP if the scatter in $M/I_R$ is uncorrelated with the sizes and dispersions of the galaxies, as indeed is the case for the EFAR sample (at least for galaxies with $\sigma > 100$ km s$^{-1}$). For a FP given by $R = \alpha \pi^2$, with $R$ the effective radius and $I$ the mean surface brightness within this radius, if the scatter in $M/I_R$ is simply a scatter in $I$ we have $\Delta \log R = \beta \Delta \log M/I_R$. Most determinations of the FP, including our own, yield $\beta = -0.8$ (e.g. Dressler et al. 1987, Jorgensen et al. 1996, Saglia et al. 1998).

Combining this relation with the dependence of $M/I_R$ on the Mg
line indices obtained above, we find that the scatter in the $Mg-\sigma$ relation corresponds to an intrinsic scatter in relative distances estimated from the FP of 40 per cent if due to age variations, or 8 per cent if due to metallicity variations. As the intrinsic scatter in the FP is found to be in the range 10–20 per cent (Djorgovski & Davis 1987, Jørgensen et al. 1993; Jørgensen et al. 1996), one cannot explain both the scatter in the $Mg-\sigma$ relation and the scatter in the FP as the result of age variations alone or metallicity variations alone (unless the single stellar population models are incorrect or there are significant galaxy-to-galaxy differences in the metallicity distributions). Suitable combinations of age variations and metallicity variations can, however, account for the measured intrinsic scatter in both the $Mg-\sigma$ and FP relations.

4.5 Combined $Mg-\sigma$ and FP Constraints

As a simple model, we assume that the scatter in the FP and the $Mg-\sigma$ relations (at fixed $\log \sigma$) is entirely due to variations in age and metallicity (at fixed galaxy mass). These variations are further assumed to have Gaussian distributions in $\log t$ and $\log Z/Z_0$, with dispersions $\delta \log t = \delta \log \epsilon$ and $\delta \log Z = \delta Z/Z_0 \log \epsilon$ and correlation coefficient $\rho$ ($-1 \leq \rho \leq 1$). While a Gaussian distribution of metallicities at fixed galaxy mass is a reasonable initial hypothesis for describing variations in the chemical enrichment process, the single-peaked shape of the assumed lognormal distribution for the mean ages may not realistically represent the star formation history (even for galaxies of the same mass). The dispersion in age inferred under this model should therefore be considered only as a general indication of the time-span over which early-type galaxies of fixed mass formed the bulk of their stellar population.

Writing the scatter in $Mg$ line strengths and FP residuals as $\delta_{Mg}$ and $\delta_{FP}$ and the dispersion in $\log t$ and $\log Z/Z_0$ as $\sigma_t$ and $\sigma_Z$, this simple model relates the scatter in the observed quantities to the dispersion in age and metallicity by

$$\delta_{Mg}^2 = a_0^2 + 2pa_1a_2\sigma_t\sigma_Z + a_2^2\sigma_Z^2,$$

$$\delta_{FP}^2 = b_t^2\sigma_t^2 + 2pb_t\sigma_t\sigma_Z + b_Z^2\sigma_Z^2.$$  

Here $a_t$ and $a_Z$ are the coefficients of $\log t$ and $\log Z/Z_0$ for $Mg$, and $b_t$ and $b_Z$ the coefficients for $\log R = -0.8 \log M/L$, derived from the mean of the linear fits to the two stellar population models given in Section 4.3.

Fig. 8 shows the constraints on the variations in age and metallicity (assumed, for now, to be uncorrelated) which are imposed by the measured intrinsic scatter in the $Mg-\sigma$ relations and the intrinsic dispersion in $\log M/L$ inferred from the intrinsic scatter in the FP. The intrinsic scatter that we find about the $Mg_1-\sigma$ and $Mg_2-\sigma$ relations is then consistent with dispersions in age and metallicity on an elliptical locus defined by equation (17) (with $\rho = 0$) in the $\delta t l t-\delta Z/Z_0$ plane. The different loci for $Mg_1$ and $Mg_2$ (the solid lines in Fig. 8) result from the difference between the observed ratio of the scatter in $Mg_2$ to that in $Mg_1$ and the predicted ratio from the model, and give some indication of uncertainties both in the intrinsic scatter about the $Mg-\sigma$ relations and in the model predictions. A second constraint is similarly obtained from the intrinsic scatter in distance (i.e. in $\log R$) for the FP using equation (18) (again with $\rho = 0$). The dashed lines in Fig. 8 correspond to intrinsic scatter about the FP of 10, 15 and 20 per cent.

The important point to note about this figure is that, as mentioned in Section 4.3, the dependences of the $Mg$ line strengths and mass-to-light ratio on age and metallicity are quite different, so that (if variations in age and metallicity are uncorrelated) the two sets of constraints are nearly orthogonal. Thus the region of the $\delta t l t-\delta Z/Z_0$ plane that is consistent with the scatter in both the $Mg-\sigma$ relation and the FP is quite limited. If we use the intrinsic scatter in the $Mg_1-\sigma$ relation and assume a 20 per cent intrinsic scatter in log $R$ about the FP [at the upper end of the quoted range -- see e.g. Djorgovski & Davis (1987) or Jørgensen et al. (1996)], we obtain approximate upper limits on the dispersions in age and metallicity of $\delta t l t = 32$ per cent and $\delta Z/Z_0 = 38$ per cent. If, however, we use the intrinsic scatter in the $Mg_2-\sigma$ relation and adopt an intrinsic FP scatter of 10 per cent (as obtained for Coma by Jørgensen et al. 1993), then we obtain approximate lower limits of $\delta t l t = 15$ per cent and $\delta Z/Z_0 = 27$ per cent.

Similar arguments allow us to evaluate the relative contributions of the dispersions in age and metallicity to the errors in distance estimates derived from the FP. For the fiducial case ($\delta FP = 20$ per cent, $\delta Mg_1 = 0.016$ mag and $\rho = 0$), where $\delta t l t = 32$ per cent and $\delta Z/Z_0 = 38$ per cent, the mean stellar population model implies that the dispersion in age gives an intrinsic FP scatter of 19 per cent while the dispersion in metallicity gives 7 per cent. In fact for most of the plausible range of dispersions in age and metallicity shown in Fig. 8, it is the dispersion in age that dominates the intrinsic scatter about the FP. Only for the lowest plausible age dispersion and the highest plausible metallicity dispersion ($\delta t l t = 11$ per cent and $\delta Z/Z_0 = 43$ per cent, corresponding to $\delta FP = 10$ per cent and $\delta Mg_1 = 0.016$ mag) does the contribution to the FP scatter from the dispersion in metallicity achieve equality with the contribution from the dispersion in age.

The constraints on the dispersions change if there is a significant correlation (or anticorrelation) between the variations in age and metallicity. Fig. 9 shows how the constraints corresponding to the
the observed distribution, although there is a weak but significant anticorrelation between the residuals (owing to the dominance of the age variations in the FP residuals) which is not apparent in the EFAR data. Over 100 such simulations, a two-dimensional Kolmogorov–Smirnov test (Press et al. 1992) gives a median probability of 0.3 per cent that this distribution and the observed distribution are the same.

Figs 10(c) and (d) show simulated distributions for the cases where the intrinsic scatter in Mg\(_b\)–\(\sigma\) is due to age alone or metallicity alone. Neither case is consistent with the observed distribution, supporting the claim that neither age nor metallicity can be solely responsible for the scatter in both the Mg\(_b\)–\(\sigma\) relation and the FP. Figs 10(e)–(h) show simulated distributions for four cases where the variations in age and metallicity are correlated (with \(\rho = +1, +0.5, -0.5\) and \(-1\), respectively). The perfectly correlated and perfectly anticorrelated cases are not consistent with the observed distribution. However Fig. 10(g) shows that a distribution with no significant correlation between the Mg\(_b\)–\(\sigma\) and FP relation residuals is produced when \(\rho = -0.5\). A two-dimensional Kolmogorov–Smirnov test gives a median probability over 100 such simulations of 1.7 per cent that this distribution and the observed distribution are the same. This relatively low probability may reflect a problem with the model, although it may simply be due to sampling uncertainty (the probabilities under this test vary between simulations with an rms of a factor of 6) or non-Gaussian outliers in the EFAR residuals. The point to be emphasized is that a model with a moderate degree of anticorrelation between age and metallicity appears to give significantly better agreement with the observed distribution than a model in which age and metallicity are not correlated.

## 5 Conclusions

We have examined the Mg–\(\sigma\) relation for early-type galaxies in the EFAR sample. We fit global Mg\(_b\)–\(\sigma\) and Mg\(_z\)–\(\sigma\) relations [equations (2) and (3)] that have slopes about 25 per cent steeper than those obtained by most previous authors. This difference results not from the data itself but from an improved fitting procedure: we apply a comprehensive maximum likelihood approach which correctly accounts for the biases introduced by both the sample selection function and the significant errors in both Mg and \(\sigma\). The observed scatter about the Mg–\(\sigma\) relations is 0.022 mag in Mg\(_b\) and 0.031 mag in Mg\(_z\); the intrinsic scatter in the global relations, estimated from Monte Carlo simulations, is 0.016 mag in Mg\(_b\) and 0.023 mag in Mg\(_z\).

With too few galaxies per cluster to reliably determine the full relation for each cluster separately, we fix the slopes of the relations at their global values in order to investigate the variation in the zero-point from cluster to cluster. We find that the zero-point has an observed scatter between clusters of 0.012 mag in Mg\(_b\) and 0.019 mag in Mg\(_z\), and that this observed scatter is consistent with the small number of galaxies sampled in each cluster being drawn from a single global relation with intrinsic scatter between galaxies as given above – i.e. the observations do not require any scatter in the Mg–\(\sigma\) zero-point between clusters. The allowed range for the intrinsic scatter between clusters corresponds to cluster-to-cluster systematic errors in FP distances and peculiar velocities with an rms anywhere in the range 0–10 per cent. We therefore cannot determine from the Mg–\(\sigma\) relation alone whether systematic differences in the mean stellar populations between clusters contribute significantly (or at all) to the errors in distances and peculiar velocities obtained using the FP.
We have also examined the variation in the Mg–σ relation with cluster properties. Our cluster sample ranges from poor clusters to clusters as rich as Coma, having velocity dispersions from 300 to 1000 km s\(^{-1}\) and X-ray luminosities spanning 0.3–8×10\(^{44}\) erg s\(^{-1}\). We do not detect a significant correlation of Mg–σ zero-point with cluster velocity dispersion, X-ray luminosity or X-ray temperature, nor is there any significant difference in the Mg–σ relations obtained by fitting the galaxies in the high-L\(_X\) clusters and low-L\(_X\) clusters separately. The predominant factor in the production of Mg in these early-type galaxies (and presumably other α-elements and perhaps their metallicity and star formation history in general) is thus galaxy mass and not cluster mass. These observations place constraints on semi-analytic models for the formation of elliptical galaxies, which are now beginning to incorporate chemical enrichment and should soon be able to make reliable predictions for the variation of the Mg–σ relation with cluster mass.

We apply the single stellar population models of Worthey (1994) and Vazdekis et al. (1996) to place upper limits on the global dispersion in the ages, metallicities and [M/H] ratios of early-type galaxies of given mass using the intrinsic scatter in the global Mg–σ relation. We infer an upper limit on the dispersion in [M/H] of 50 per cent if the scatter in Mg–σ is due to age differences alone, or 10 per cent if it is due to metallicity differences alone. These correspond to upper limits on the dispersion in relative galaxy distances estimated from the FP of 40 per cent (age alone) or 8 per cent (metallicity alone). Since the intrinsic scatter in the FP is found to be 10–20 per cent, one cannot (within the context of the single stellar population models) explain both the scatter in the Mg–σ relation and the scatter in the FP as the result of age variations alone or metallicity variations alone.

We therefore determine the joint range of dispersions in age and metallicity that are consistent with the measured intrinsic scatter in both the Mg–σ and FP relations. For a simple model in which the galaxies have independent Gaussian distributions in log t and log Z/Z\(_{⊙}\), we find upper limits of δt/t = 32 per cent and δZ/Z = 38 per cent at fixed galaxy mass. If the variations in age and metallicity are not independent, but have correlation coefficient ρ, we find that so long as ρ is in the range −0.5 to 1 these limits on the dispersions in age and metallicity change by only ±6 and ±12 per cent, respectively. Only if the age and metallicity variations are strongly anticorrelated (ρ = −1) do we obtain significantly higher upper limits, with δt/t as large as 57 per cent and δZ/Z as large as 80 per cent. The distribution of the residuals from the Mg–σ and FP relations is only marginally consistent with a model having no correlation between age and metallicity, and is better matched by a model in which age and metallicity variations are moderately anticorrelated (δt/t = 40 per cent, δZ/Z = 50 per cent and ρ = −0.5), with younger galaxies being more metal-rich. Stronger bounds on the dispersion in age and metallicity amongst

![Figure 10. The joint distribution of residuals from the Mgσ relation and a FP scatter of 20 per cent assuming the variations in age and metallicity are uncorrelated; (c) a simulation with δt/t = 67 per cent and δZ/Z = 0 (from the Mgσ intrinsic scatter assuming a dispersion in age only); (d) a simulation with δt/t = 0 and δZ/Z = 43 per cent (from the Mgσ intrinsic scatter assuming a dispersion in metallicity only). The bottom panel shows simulations consistent with the intrinsic scatter in the Mgσ relation and a FP scatter of 20 per cent, and with various assumed correlations between age and metallicity; (e) a simulation with δt/t = 26 per cent, δZ/Z = 27 per cent and ρ = 1; (f) a simulation with δt/t = 28 per cent, δZ/Z = 31 per cent and ρ = 0.5; (g) a simulation with δt/t = 38 per cent, δZ/Z = 50 per cent and ρ = −0.5; (h) a simulation with δt/t = 57 per cent, δZ/Z = 80 per cent and |ρ| = −1. The dotted lines are the expected correlations for a dispersion in age alone (Δ log Z = 0) or metallicity alone (Δ log t = 0). The dashed line is the correlation expected if the distribution is dominated by the errors in log σ.](image-url)
early-type galaxies of given mass will require more precise measurements of the deviations from the Mg–σ relation and the FP and also improved models for the dependence of the line indices and mass-to-light ratio on age and metallicity. Further powerful constraints can also be obtained by measuring the intrinsic scatter in the Mg–σ and FP relations at higher redshifts, since the linestrengths and mass-to-light ratio have different dependences on age.

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