# Orbital fitting with a potential integrator

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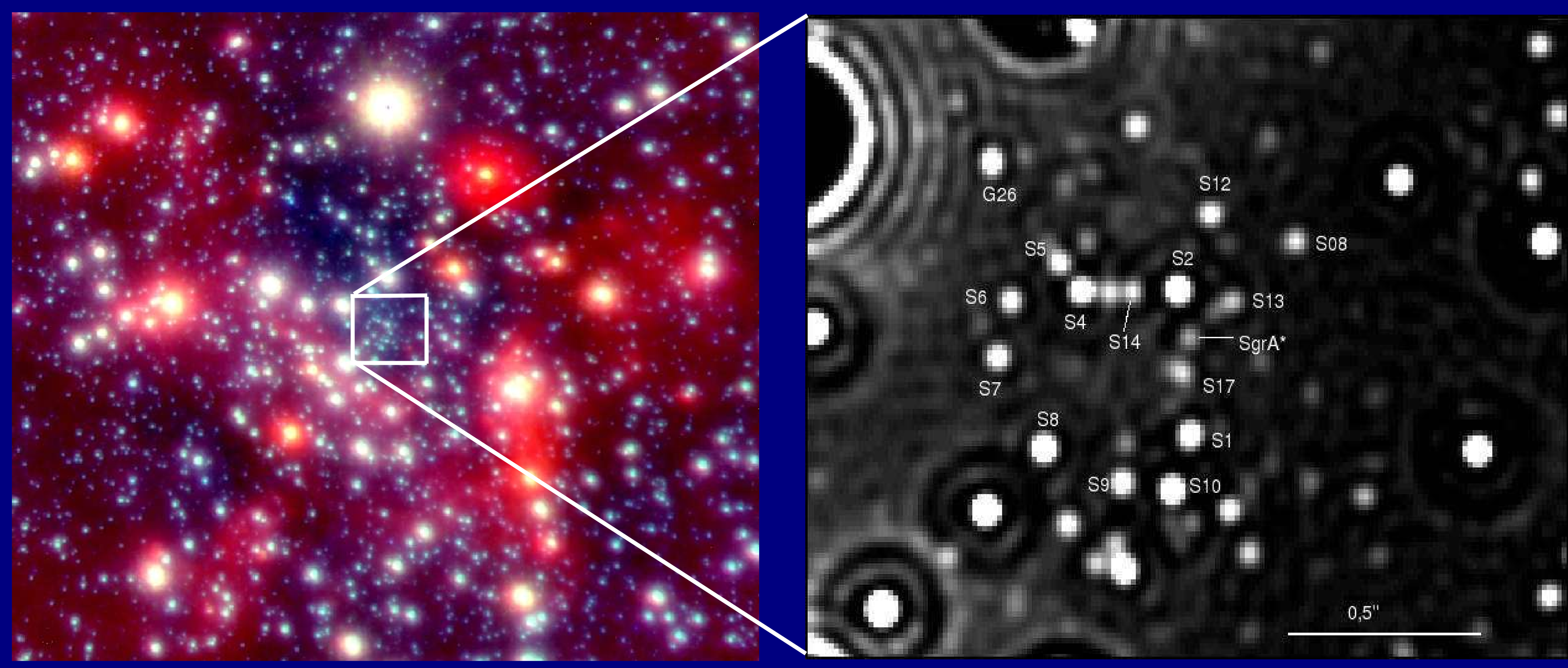
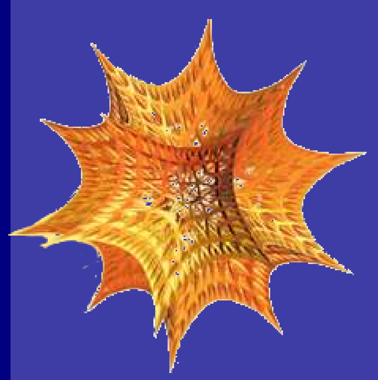
## Abstract

After nearly 15 years of high-precision astrometry of the innermost stars in our galaxy and a few years of Doppler-based radial velocity measurements the accuracy of the available data has reached a level at which one might hope to dectect deviations from the Keplerian orbits on which the stars apparently move due to the existence of a supermassive black hole at the dynamical center of the milky way. Such deviations are due to relativistic effects and due to an extended mass component. Both cases are scientifically highly interesting.

In order to analyze these post-Newtonian effects we have implemented a general orbital fitting routine that allows to fit orbits in an arbitrary potential and that can take into account relativistic effects. This poster presents the newly developped tool.

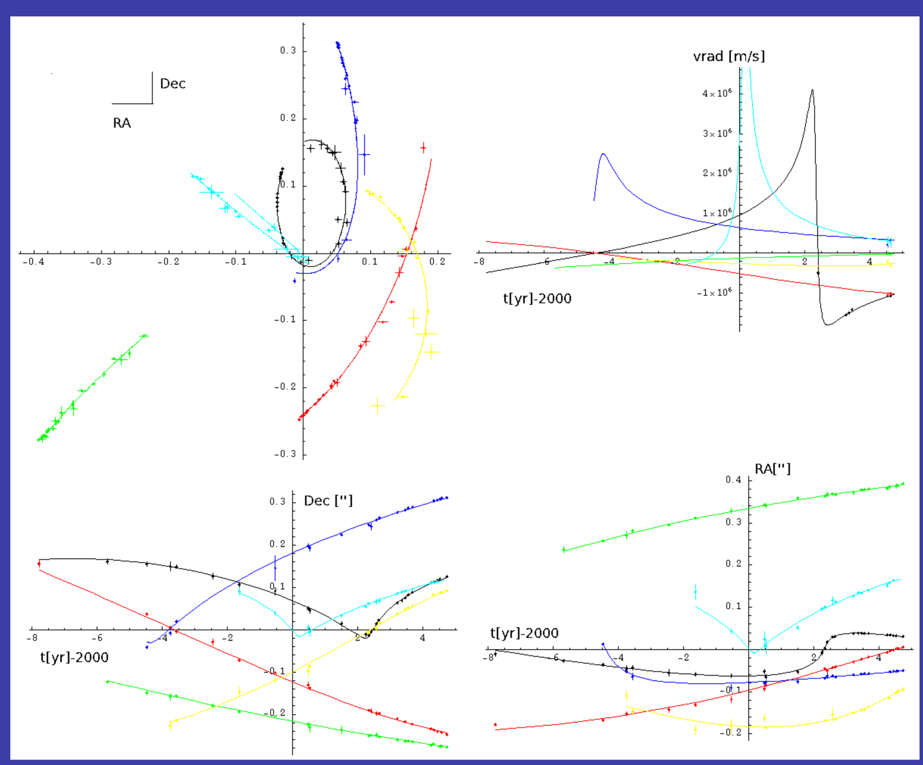



## The problem is computationally demanding



For a 1/r potential it is well-known that the equations of motion lead to the famous Kepler ellipses. Assuming such a potential, orbits can be fit by adjusting the orbital elements, as the motion is known analytically. However, a more general approach is needed if a general potential determines the dynamics. Then the trajectory has to be determined numerically. The problem can be described by the initial conditions of each test particle plus the parameters describing the potential. For each set of parameters a $\chi^2$ with respect to the measured data can be calculated. One seeks for the parameter values which minimize the $\chi^2$. This is a computationally demanding problem as at each step of the high-dimensional minimization the equations of motion are solved numerically. We have chosen the high-level tool *Mathematica* for the implementation.

## Fitting the Keplerian case yields back the known results



A first test of the new tool was to check that the results agree with a fit that implicetely uses the ellipse-shaped orbits. Here a multiple fit of six S-stars is shown, using the data from Eisenhauer et al. (2005) [1]. Both the orbital elements and the parameters describing the gravitational potential are completely consistent with the findings from the corresponding fit which uses the analytical solution of the equations of motion.

| | Parameters | Data points | $\chi^2$ | reduced $\chi^2$ | no relativity |
|---|---|---|---|---|---|
| Fit | 40 | 263 | 292.2 | 1.31 | |

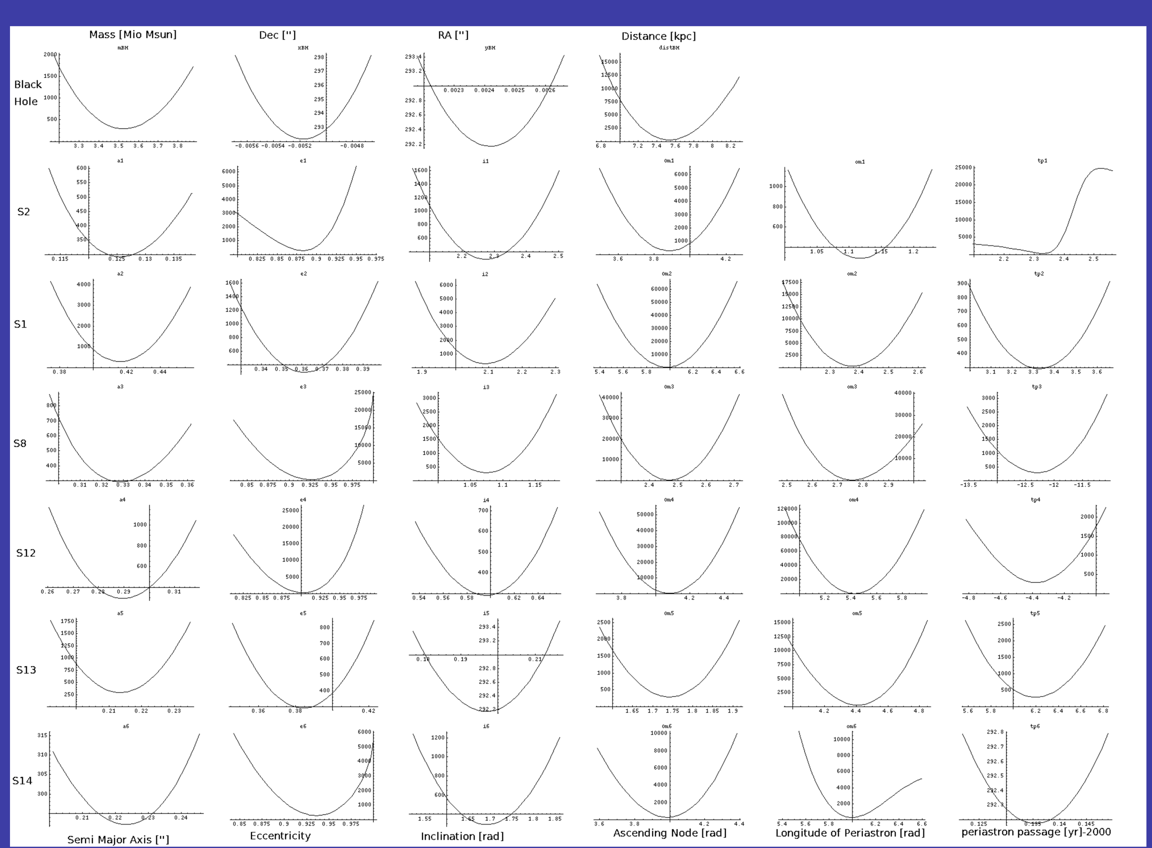
| | Mass [Mio Msun] | Pos Dec [mas] | Pos RA [mas] | Distance [kpc] |
|---|---|---|---|---|
| Potential | 3.53 ± 0.27 | -5.2 ± 1.2 | -2.4 ± 1.2 | 7.54 ± 0.25 |

| | s. maj. ax. ["] | eccentricity ["] | inclination [°] | $\Omega$ [°] | $\omega$ [°] | Peri. Passage |
|---|---|---|---|---|---|---|
| S2 | 0.126 ± 0.002 | 0.883 ± 0.007 | 130.3 ± 1.2 | 222.6 ± 1.4 | 64.0 ± 1.3 | 2002.33 ± 0.01 |
| S1 | 0.42 ± 0.03 | 0.361 ± 0.037 | 119.7 ± 0.9 | 342.3 ± 0.8 | 136.1 ± 4.1 | 2003.33 ± 0.49 |
| S8 | 0.33 ± 0.02 | 0.926 ± 0.018 | 61.6 ± 4.9 | 141.5 ± 2.0 | 158.0 ± 1.8 | 1987.70 ± 0.79 |
| S12 | 0.290 ± 0.012 | 0.903 ± 0.005 | 34.2 ± 1.4 | 233.7 ± 4.4 | 310.8 ± 3.4 | 1995.62 ± 0.02 |
| S13 | 0.21 ± 0.02 | 0.384 ± 0.016 | 11.3 ± 9.8 | 100 ± 51 | 253 ± 40 | 2006.20 ± 0.41 |
| S14 | 0.22 ± 0.02 | 0.936 ± 0.008 | 96.9 ± 2.1 | 228.4 ± 1.7 | 343.4 ± 2.1 | 2000.14 ± 0.05 |

[1] Eisenhauer, F. et al., astro-ph/0502129, accepted by ApJ

## Further numerical tests show that the method works as expected

- Test of the numerical integrator
  - Energy is conserved better than $10^{-5}$
  - Angular momentum conserved better than $10^{-5}$
  - Orientation of orbit stable to double precision
- Test of the minimization routine
  - Fitted parameter values mark a minimum in parameter space



Slices through the $\chi^2$-function. In each plot $\chi^2$ is calculated for all parameters fixed to the minimizing values, except for one which is varied by a few percent. This illustrates that the minimization has found a minimum in the 40-dimensional parameter space.

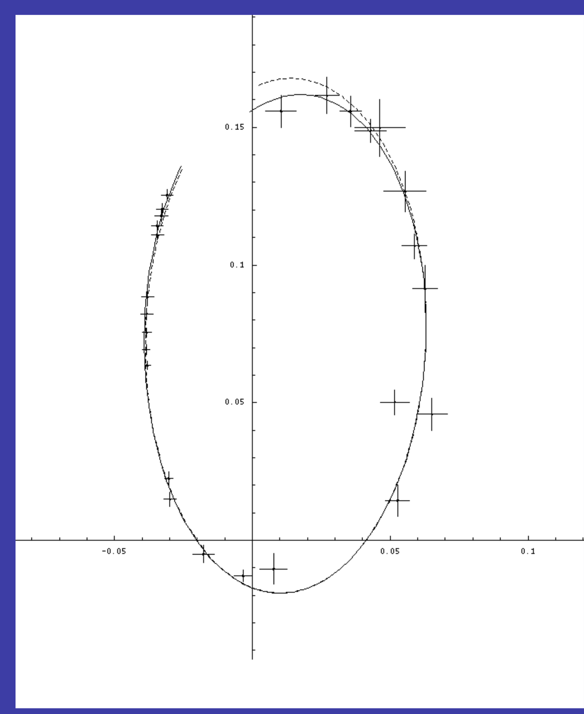
## Features of the new tool

- 7 parameters for the point mass built-in: Mass, distance, projected position, 3D-velocity
- Additional potential given as a mass distribution with arbitrary number of parameters
  - solves Poisson equation
- Arbitrary number of stars, each determined by 6 orbital elements
- Built-in, user-selectable relativistic effects
  - Schwarzschild-term
  - Light retardation (Roemer effect)
  - Relativistic Doppler formula
  - Gravitational redshift
- Full covariance matrix information
- Richness in numerical methods adopted *Mathematica*
  - Many integrators available: Runge-Kutta, symplectic, ...
  - Many minimization routines: (Quasi-)Newton, simulated annealing, ...

## The downside

- Fitting is relatively slow
  - 20..30 minutes for 40 parameters on a normal PC

## Probing for an extended mass component is possible



For the purpose of demonstrating what the tool can do we have chosen the most simple extended mass component, which is given by a constant density: $\rho(r) = \rho_0$.

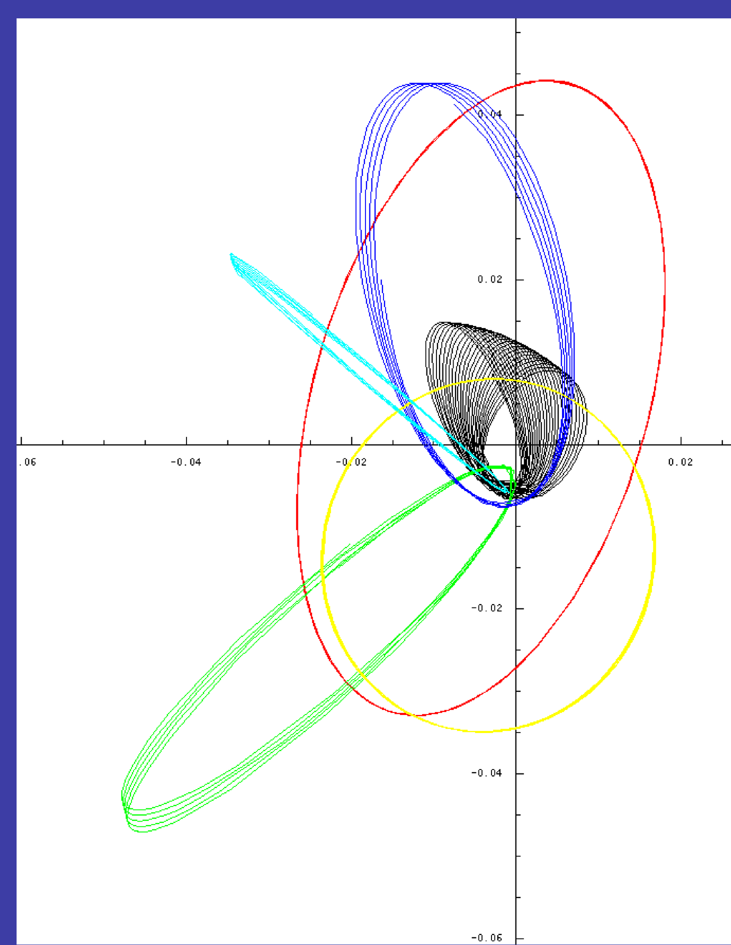
We get an $1\sigma$-upper limit of: $\rho_0 < 33\ M_\odot / \mathrm{mpc}^3$

This corresponds to 6% of the fitted central point mass inside the central arcsecond. For a more realistic analysis see Mouawad et al. (2005) [1].

The plot to the right shows the orbit of S2 if the extended mass component were 3 times the derived upper limit - assuming the Keplerian parameters were exact at J2000.0 (solid curve). The dashed line is the original S2-orbit. This illustrates that the orbital fitting will soon allow to constrain possible extended mass distributions at the few percent level.

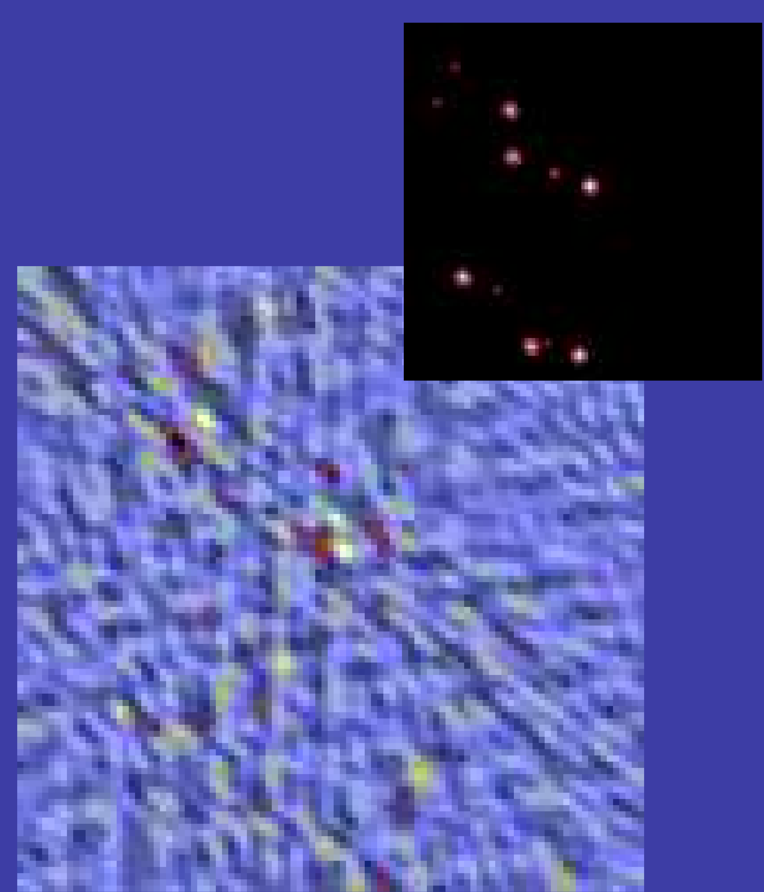
[1] Mouawad, N. et al., AN, Vol. 326, Issue 2, p. 83-95

## The VLTI will measure the Schwarzschild periastron shift within few years



For S2 the expected periastron shift is 0.18° per orbital revolution. The current error on the orientation of the orbit in its plane is $\Delta\omega = 1.2°$. Thus, in a few years the effect will become measurable with existing instruments.

If the semi major axes of the S-stars were 10 times smaller than they are, the orbits would have undergone significant periastron shifts in the past 15 years, as the plot on the right demonstrates. These orbits at the mas-scale are well within the accessible domain of the VLTI.



We could show in simulations that imaging such a scaled-down version of the S-stars 3 times per year with 6 baselines (4 telescopes) for a few years is sufficient to repeat the S-stars experiment in this relativistic environment.

The plot to the left shows an synthezised image from the simulations[1], from which a deconvolution technique can retrieve the astrometric positions of the stars with an accuracy of 1mas.

[1] Paumard et al. (2005), proceedings of the ESO workshop held in Garching, Germany, 4-8 April 2005

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