Kinetic Theory Revision Guide

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This document is merely a laundry list of what you need to know. It is not a set of revision notes but, combined with the official syllabus and tutorial work, it may be useful for checking things off as you compose your own revision notes, which you should of course do.

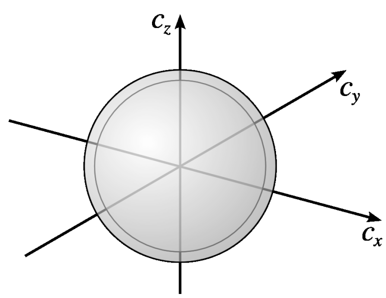
1 Maxwell-Boltzmann distribution

B&B Ch. 5, Questions 1.1 - 2.2

- The fraction of particles with x-velocity between c_x and $c_x + dc_x$ (from Boltzmann factor) is

$$g\left(c_x\right) dc_x \propto exp\left(-\frac{\frac{1}{2}mc_x^2}{k_B T}\right)$$

- Velocity → speed:



A shell centred on the origin, of radius c, is one of constant speed c. Shell volume $\propto c^2 dc$. Assuming isotropic speed distribution, fraction of particles with *speed* between c and $c + dc$ is $\propto c^2 dc$ as well as $exp\left(-\frac{\frac{1}{2}mc^2}{k_B T}\right) \therefore$

$$f\left(c\right) dc \propto c^2 exp\left(-\frac{\frac{1}{2}mc^2}{k_B T}\right).$$

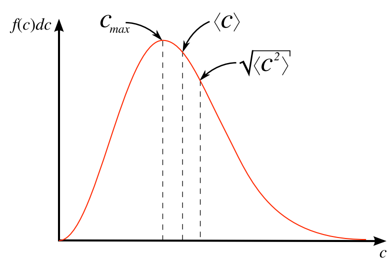
- Maxwell disribution: Normalise $f\left(c\right) dc \rightarrow f\left(c\right) dc = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_B T}\right)^{\frac{3}{2}} exp\left(\frac{-mc^2}{2k_B T}\right)$

- Averages (also known as 'moments') of $g(c_x)\,dc_x$ and $f(c)\,dc$.
  Calculate using:

$$\begin{aligned} \langle f(x)\rangle &= \int f(x)\,p(x)\,dx && \text{(if } p = \text{prob } x \rightarrow x+dx \text{ is already normalised)} \\ \text{or} \quad \langle f(x)\rangle &= \frac{\int f(x)p(x)dx}{\int p(x)dx} && \text{(if } p \text{ is not normalised – the denominator corrects this)} \end{aligned}$$

  Know how to calculate:

$$\begin{array}{cc} \langle c_x\rangle = 0 & \left\langle c_x^2\right\rangle = \frac{k_B T}{m} \\ \langle |c_x|\rangle = \sqrt{\frac{2k_B T}{\pi m}} & \langle c\rangle = \sqrt{\frac{8k_B T}{\pi m}} \\ \left\langle c^2\right\rangle = \frac{3kT}{m} & \left(\langle \text{K.E.}\rangle = \frac{1}{2}m\left\langle c^2\right\rangle = \frac{3}{2}k_B T\right) \\ c_{max} = \sqrt{\frac{2k_B T}{m}} & \left(\text{set } \frac{df}{dc} = 0\right) \end{array}$$



2 Pressure, effustion & the mean free path

B&B Ch. 6 - 8 Questions 2.3 - 2.10

- Assumptions of kinetic theory:
  - elastic collisions
  - no interactions, molecules move in streat lines between collisions
  - enough molecules that statistics are reasonable
  - molecules are small with respect to the separation between them
  - container large with respect to mean free path
  - thermal equilibrium (allows use of the Boltzmann factor)
  - Newtonian mechanics
- Know how to argue

$$d\Phi = \frac{1}{2}nf(c)\,dc\,\sin\theta\,d\theta.cdt\,\cos\theta$$

- Use $d\Phi$ to prove

$$p = \frac{1}{3}nm\left\langle c^2\right\rangle$$

- Use $p = \frac{1}{3} nm \left\langle c^2 \right\rangle$ to prove

$$p = nk_BT \qquad\qquad pV = Nk_BT$$

- Use $d\Phi$ to prove

$$\Phi = \frac{1}{4} n \left\langle c \right\rangle$$

- Use $\Phi = \frac{1}{4} n \left\langle c \right\rangle$ to prove

$$\Phi = \frac{p}{\sqrt{2\pi m k_B T}}$$

- Note that effusion rate is a function of mass, $\therefore$ a small hole can be used to separate different isotopes, e.g. U^{238} and U^{235}.

- Be able to argue that if

$$f\left(c\right) dc \propto c^2 exp\left(-\frac{mc^2}{2k_BT}\right)$$

  inside the container, then

$$f\left(c\right) dc \propto c^3 exp\left(-\frac{mc^2}{2k_BT}\right)$$

  outside small hole, and be able to show (through moments of $f\left(c\right)$) that this means faster particles are more likely to leave container (assume hole $\ll \lambda$).

- Know example 7.6 in B&B:

$$\begin{array}{lcc} D \gg \lambda & \Rightarrow & p_1 = p_2 \\ D \ll \lambda & \Rightarrow & \frac{p_1}{\sqrt{T_1}} = \frac{p_2}{\sqrt{T_2}} \end{array}$$

- Mean free path: be able to show

$$\tau = \frac{1}{n\sigma v} \qquad\qquad \lambda = \frac{1}{\sqrt{2} n\sigma}$$

  Have a feel for

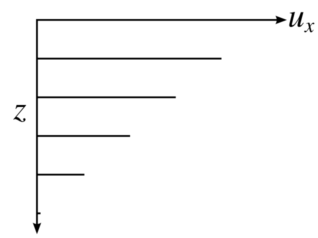
$$\left.\begin{array}{c} \sigma = \pi D^2 \\ D \sim 10^{-10} \text{ m} \\ p \sim 1 \text{ atm} \\ T \sim 300\text{K} \end{array}\right\} \Rightarrow n \sim 10^{25} \text{ (ideal gas)} \Rightarrow \lambda \sim 10^{-7}\text{m}$$

- Know how to calculate the area of a circle from its *radius* and *diameter*!

3 Transport Properties

B&B Ch. 9 - 10, Questions 3.1 - 3.7, 4.2,7.1

- Know the definition of viscosity



$$\frac{F}{A} = \eta \frac{\partial \left(\langle u_x \rangle \right)}{\partial z}$$

($\frac{F}{A}$ is the net momentum transfer across the plane per unit area per unit time)

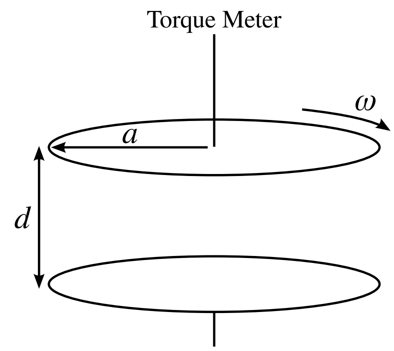
- Use this to show that

$$\eta = \frac{1}{3} nm\lambda \langle c \rangle$$

(If your derivation gives you a factor of $\frac{1}{4}$, see B&B's!) Know the limitations of this derivation: $\langle c \rangle$ is assumed to be a linear function of height, λ assumes hard shells, derivation explicitly assumes that all particles travel exactly λ between collisions.

- Note behaviour of η
  - Independent of n $\therefore$ independent of p
  - $\propto \sqrt{T}$
- Inaccurate at:
  - low p ($\lambda \sim$container size)
  - high p ($\lambda \ngtr\!\!\!\!\gg$separation of molecules)

- Know method for measuring η (see question 3,4, and HT collection question B1)



  - Proof:

$$\Gamma = -\frac{\eta \pi \omega a^4}{2d}$$

- Know:

$$I \frac{\partial \omega}{\partial t} = \Gamma \qquad\qquad I_{disc} = \tfrac{1}{2} m a^2$$

  (should be able to prove I_{disc}, and other I's for, e.g. spheres. Remind yourself of the definition of I!)

- Know definition of thermal conductivity

$$Q = -\kappa \frac{\partial T}{\partial z}$$

  where Q is the heat flux per unit area per unit time

- Prove

$$\kappa = \frac{1}{3} n \lambda \langle c \rangle C_{mol}$$

  (C_{mol} is molecular heat capacity; $N_A C_{mol} = C_V$ per mole)

- Know how to solve time-independent heat diffusion equation

$$Q = -\kappa \frac{\partial T}{\partial z}$$

  see (for example) HT collection B2. Know how to include a constant heat source.

- Know how to solve the time-separable heat diffusion equation with sinusoidally-varying heat source, for example, question 3.7

- Know Newton's Law of Cooling.