Spectrum of Electrons with Inhomogeneous Energy Losses

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ABSTRACT

Diffusive cosmic-ray propagation of electrons with full spatial and energy dependence can be solved straightforwardly using a numerical scheme; this has the advantage over analytical methods of enabling us to compute the spectrum for quite general models based on empirical Galactic distributions of matter and radiation fields. The variation of the spectrum with distance from the Galactic plane, important for γ-ray studies, is derived. Preliminary results of an application to the 1 MeV - 100 GeV electron spectrum are presented.

1. INTRODUCTION

Observational data provide a great variety of information about Galactic cosmic-ray origin and propagation: gamma-rays, radio, composition etc. all give constraints of different kinds. Meanwhile the standard model of propagation has matured from the homogeneous model to a more realistic one including many processes: diffusion, convection, spatially varying energy losses and re-acceleration, occuring in a Galaxy with properties derived from observations of gas distributions and potential cosmic-ray sources (see e.g. Jones 1990, Berezinsky et al. 1990). Further, although attention is usually focussed on the local CR spectrum, predictions of the diffuse galactic γ-ray intensity require the spectrum at all points in the Galaxy. In face of this complexity the problem becomes increasingly intractable by analytical methods (Lerche and Schlickeiser 1982ab; Webber et al. 1992; Bloemen et al. 1993) ; therefore we became interested in investigating an alternative: direct numerical solution of the propagation equations. Monte Carlo simulations (e.g. van der Walt and Steenkamp 1991) would be an alternative but appear to be far too time consuming in this context, at least for 3-D models, due to the very large number of scatterings. We restrict attention here to computing the electron spectrum since this is of relevance to recent COMPTEL measurements of the low-energy gamma-ray diffuse emission (Strong et al. 1993, and this conference). We have developed an efficient method which can be used to obtain solutions for quite general spatial variations in the model parameters.

2. MODEL AND NUMERICAL METHOD

The cosmic-ray transport equation for primaries is written in the form:

$$\frac{\partial\psi(\vec{r},E,t)}{\partial t} = q(E) + \vec{\nabla}.[D\vec{\nabla}\psi \ - \vec{V}\ \psi] + \frac{\partial}{\partial E}[\psi\frac{dE}{dt}] \qquad (1)$$

The solution is generated on a grid in $(\vec{r},E)$; starting from the initial condition $\psi(\vec{r},E,0) = 0$ we integrate forwards in time until a steady-state is reached. Although we are interested only in the steady-state the time-evolution method has the advantage of being stable numerically and straightforward to implement. By adjustment of the time and space increments it was found possible to achieve accurate solutions without excessive computer time. In this preliminary investigation we treat only 1-D propagation in z; the cylindrically symmetric 3-D problem will be a simple extension. The energy loss term includes the (spatially dependent) effects of ionization, bremsstrahlung, synchrotron and inverse-Compton losses. We impose the boundary condition $\psi(-z_h) = \psi(z_h) = 0$ for a 'halo' thickness of $2z_h$. The diffusion coefficient is assumed to have the form $D = D_o(E/E_o)^{-\delta}$ for $E > E_o$ and $D = D_o$ for $E < E_o$. Values of $E_o = 5$ GeV, $D_o = 3\ 10^{28}$ cm^2 s^{-1}, δ=0.6 and z_h= 3kpc were adoted † (Webber, Lee and Gupta 1992; Bloemen et al. 1993). Bloemen et al. (1993) show that for a convection term of the form $V = 3V_oz$, V_o <15 km s^{-1} kpc^{-1} ; this term is set to zero in this first analysis.

Since we consider only the z-coordinate the gas densities are averaged over galactocentric radius. The HI density profile is taken from Dickey and Lockman (1990); their

† In 'halo' models with z_h = several kpc the age of CR is typically 10^8 years and the average density 0.03 cm^{-3} (e.g. Ginzburg et al. 1980; Berezinsky et al. 1990; Webber et al. 1992; Bloemen et al. 1993), in contrast to homogeneous models where the grammage and radioactive nuclei data together with the assumption of homogeneity leads to an average density of 0.3 cm^{-3} and a full thickness of only 0.5-1 kpc for an 'age' of 1-2 10^7 years (e.g. Webber 1990; Shapiro et al 1991).

distribution has $\int n_H dz = N_H$=6.2 10^{20} cm^{-2}, FWHM = 0.23 kpc ; for molecular hydrogen we use the profile of Bronfman et al. (1988) with N_H= 5 10^{20} cm^{-2}, FWHM =0.14 kpc; we take the extensive (T≈ 10^4K, $\rho \approx$ 0.025 cm^{-3}) warm ionized medium from Reynolds (1990,1991), Cordes et al. (1991) with N_H= 2 10^{20} cm^{-2}, FWHM = 3 kpc. In a model with a large CR halo the particles spend most of their time in the extended component.

The grammage for a 1-D diffusion model with thin matter disk only is given by (Berezinsky et al. 1990) $X_H = \rho z_g z_h v/D = N_H z_h v/2D$; with our parameters, X_H= 8 g cm^{-2} , consistent with CR nucleon composition measurements. This estimate neglects the broad ionized component which will increase the grammage by a small factor.

The electron injection spectrum $q(E)$ is assumed to be a power-law with index -2.4 (Webber 1983). As normalization for $\psi(E,0)$ we use the value at 1 GeV (2 10^{-5} cm^{-2}sr^{-1} s^{-1} MeV^{-1}) derived from an analysis of COS-B γ-ray data (Strong 1985). For inverse-Compton losses the radiation field is based on Youssefi and Strong (1991); for synchrotron losses the magnetic field is taken as 6 e$^{-z/5kpc}\mu$G.

3. ELECTRON SPECTRUM: EXAMPLE CALCULATION

To illustrate the method, we present some sample results. Fig 1 shows the time evolution of $\psi(E,z)$ for E=10 MeV. The flux converges asymptotically to the steady-state solution (reached after about 2 10^8 years). Fig 2 shows the steady-state spectrum $\psi(E,z)$ for various z, and Fig 3 the spectral index variations. Averaged over z the spectrum resembles the predictions of the homogeneous model, with an asymptotic change in index of ± 1 at low and high energies due to energy losses. The situation is different at high z where the electrons have an age of about 10^8 years and energy losses cause a steep cutoff at both ends of the spectrum. The variation of the spectrum with z indicates that the homogeneous model calculations used to predict bremsstrahlung emission (Strong et al. 1993 and this conference; Skibo and Ramaty 1993) are an over-simplification and that the spatial dependence should be included. To illustrate the effect of varying model parameters Fig 4 shows the electron spectrum at z=0 for various diffusion coefficients in the range 0.5-10 10^{28} cm^2 s^{-1}.

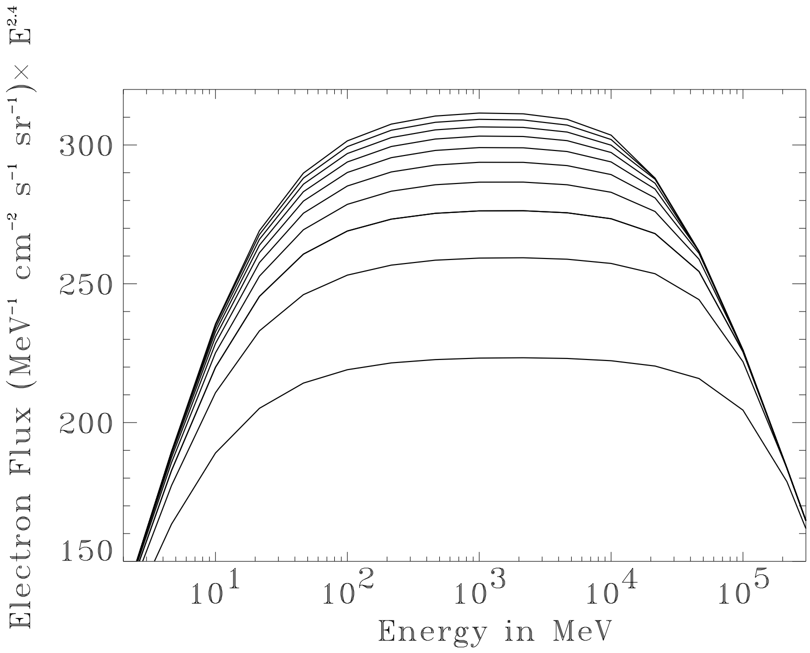

Fig 1: Electron spectrum at z=0 as function of time in steps of 10^5 years. Spectrum has been multipled by E$^\gamma$ where γ=2.4 is the injection spectral index, to show clearly the effect of energy losses.

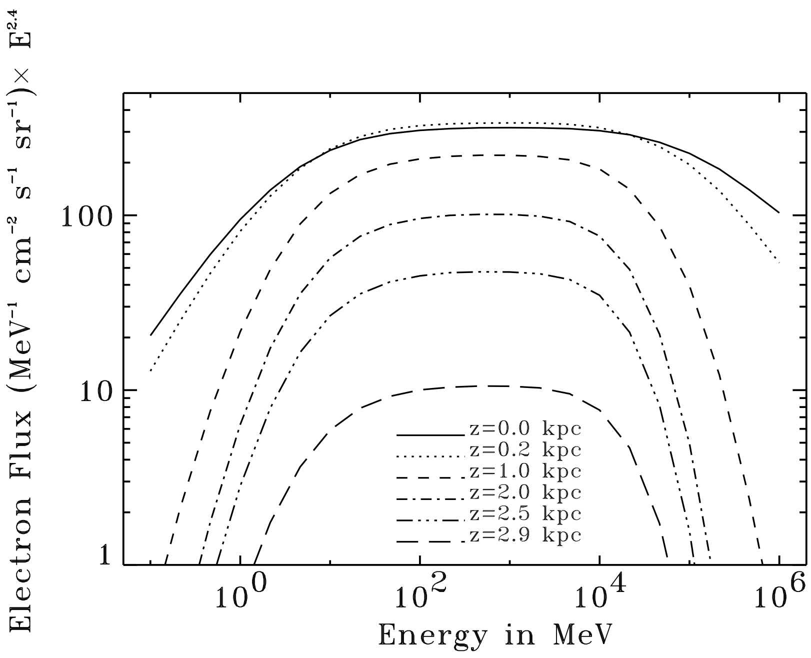

Fig 2: Steady-state electron spectrum as function of height z from Galactic plane.

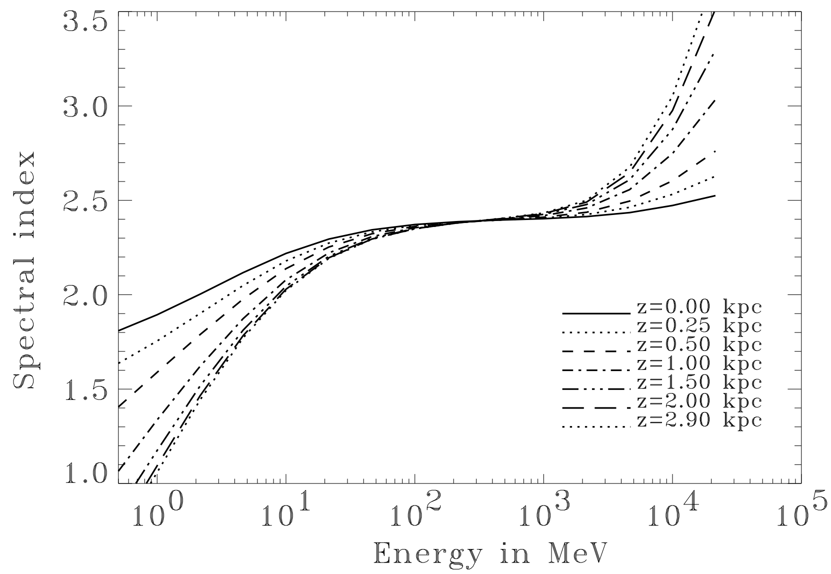

Fig 3: Spectral index as function of energy and height z from Galactic plane.

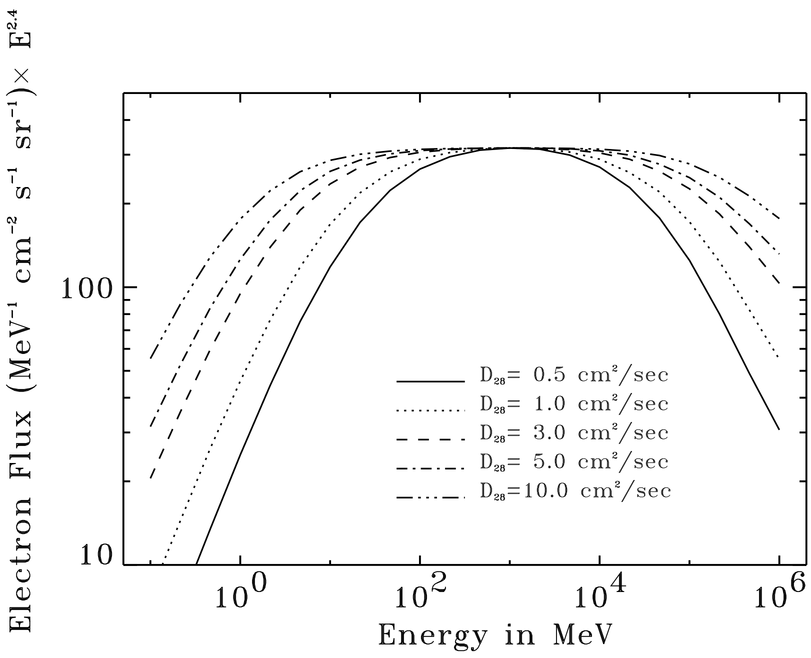

Fig 4: Steady-state electron spectrum at z=0 for various diffusion coefficients. Other parameters as given in text.

4. CONCLUSIONS

The numerical method appears to be a practical way to compute the observational consequences of 'realistic' propagation models. The extension of the method to 3-D propagation is straightforward. Other processes such as reacceleration and the effect of a cloudy ISM could be easily incorporated. For nuclei we will include fragmentation and radioactive nuclei; the positron component can then also be included. The goal is to allow flexible and consistent modelling of both electrons and nuclei which can be compared with γ-ray , radio, nuclear composition and other observational data.

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