Reconstructing the Expansion History of the Universe with SNe Ia: a Model-Independent Approach

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OPINAS seminar

June 17, 2014

Outline

Introduction

Method

Tests

Real Data

Other Applications

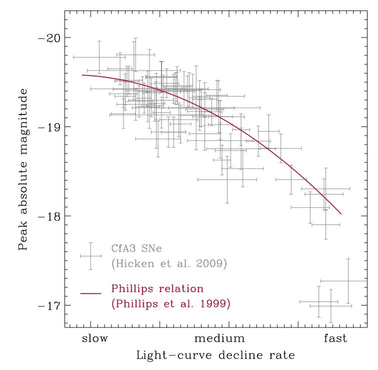
Summary

Type Ia Supernovae

- SNe Ia are Standardizable candles. Can be calibrated through the width-luminosity relation *(Phillips 1993)*
- Brighter events decline slower, fainter decay faster
- $\mu(z) = m - M$ is the distance modulus

$$D_L(a) = 10^{\mu/5+1}$$

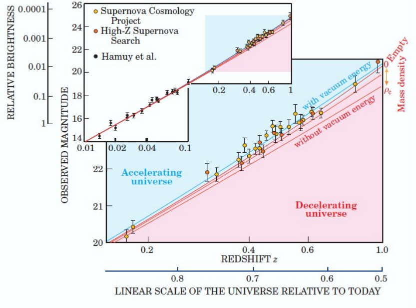
- Good distance indicators out to $z \sim 1.5$



(S. Taubenberger)

Cosmology with SN Ia

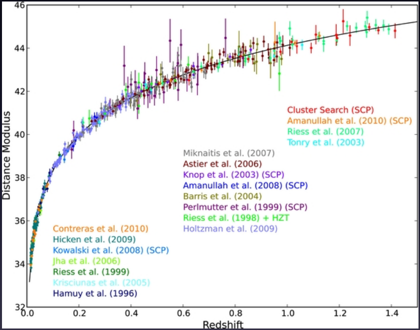
- SNe beyond $z \sim 0.5$ appear fainter than expected for an empty universe.
- First evidence of accelerated expansion
- Later confirmed by CMB and BAO measurements
- Nobel Prize in Physics 2011
  S. Perlmutter, B. Schmidt, A. Riess

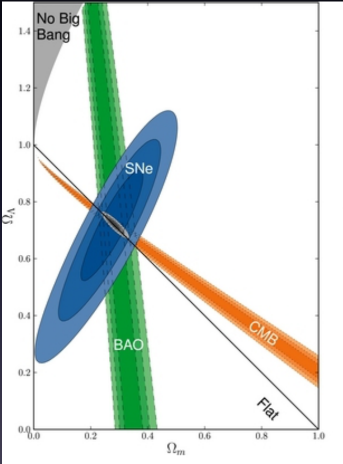


(Perlmutter et al. 1999)

Current SN constraints

- Largest Hubble diagram





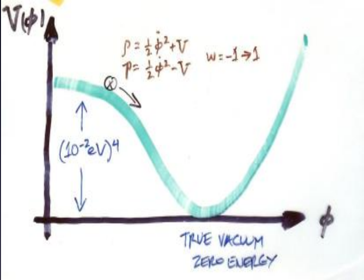
- Consistent with a ΛCDM with $\Omega_{\rm m} = 0.27$, $w = -1$ *(Suzuki et al. 2012)*

Beyond ΛCDM

Possible explanations for acceleration:

- cosmological constant: $w = -1$
- dynamically evolving scalar-field: $w(a) = w_0 + w_a(1 - a)$
- Modified Gravity: terms on the left side of Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$



Degeneracy of scenarios difficult to break with current data!

Why "model-independent"?

- SNe Ia directly test geometry. No need to define a Friedman Equation a priori
- Literature: *Starobinsky 1998; Huterer & Turner 1999, 2001; Shafieloo 2006, 2007, Ishida et al. 2011...*
- Direct reconstruction of $D_\mathrm{L}(z)$ and $H(z)$
- Geometrical framework to compare different cosmological theories, e.g. $f(R)$ or DGP models
- Additionally: check for systematics, test redshift intervals, inhomogeneity...

The Method *(Mignone & Bartelmann 2008)*

From Robertson-Walker metric $[a = (1+z)^{-1}$ and $e(a) = 1/E(a)]$

$$D_\mathrm{L}(a) = \frac{c}{H_0}\frac{1}{a}\int_a^1 \frac{e(x)}{x^2}dx \xrightarrow[d/da]{} D'_\mathrm{L}(a) = -\frac{1}{a^2}\int_a^1 \frac{e(x)}{x^2}dx - \frac{e(a)}{a^3}$$

Volterra Integral Equation

$$e(a) = f(a) + \lambda \int_1^x K(x,t)e(t)dt$$

$$e(a) = -a^3 D'_\mathrm{L} + a\int_1^a x^{-2}e(t)dt$$

Neumann Series: unique solution!

$$e(a) = e_0 + \sum_{n=1}^{\infty}\lambda^n e_n(a)$$

$$e_0 = -a^3 D'_\mathrm{L}(a)$$

$$e_n = \int_1^a x^{-2}e_{n-1}(t)dt$$

The Method

Problem: equation involves $D'_{\rm L}$!

- First: smooth the data

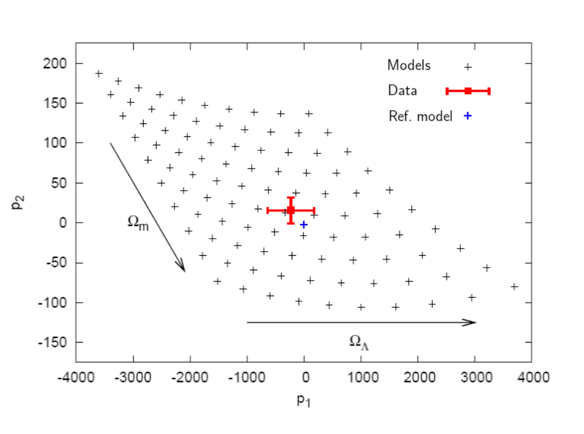
$$D_{\rm L}(a) = \sum_{j=0}^{J} c_j p_j(a) \quad \rightarrow \quad D'_{\rm L}(a) = \sum_{j=0}^{J} c_j \, p'_j(a)$$

- Approximate this derivative to the derivative of the real data
- Principal Component Analysis to derive an *optimal* basis
- The expansion coefficients are found minimizing

$$\chi^2 = \sum_{i}^{\rm SNe} \frac{[D_{\rm L}|_{{\rm obs},i} - D_{\rm L}(a_i, c_j, H_0)]^2}{\sigma_i^2}$$

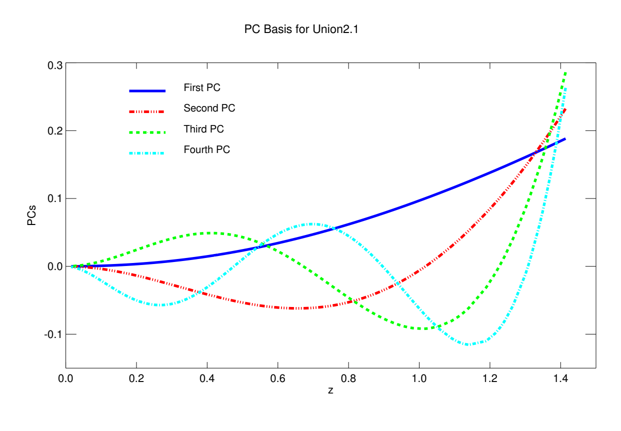
The PC Basis

- Derived theoretically from an ensemble of *behaviours* for D_L, using redshift distribution of the data: Training set



The PC Basis

- PCs for the Union2.1 catalog



Uncertainties

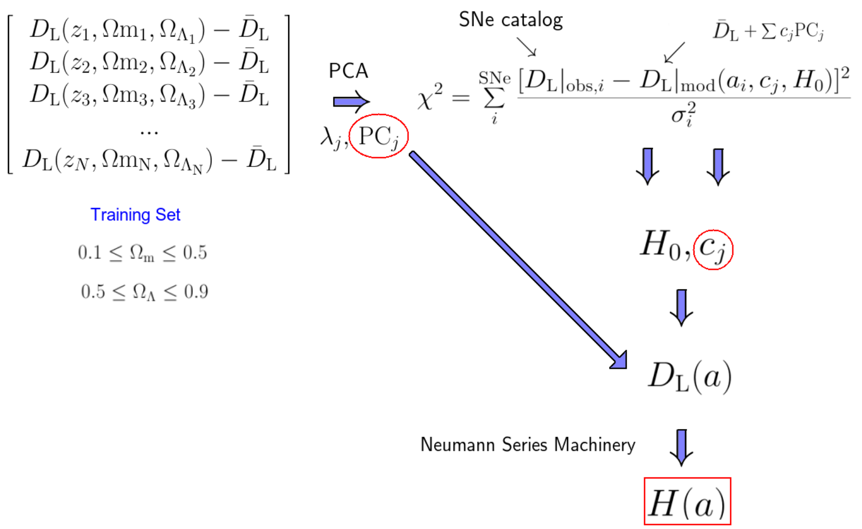
- Fisher matrix of the χ^2 function

$$F_{ij} \equiv \left\langle \frac{\partial^2 \chi^2}{\partial c_i \partial c_j} \right\rangle \quad \rightarrow \quad F_{ij} = \sum_{k=0}^{\mathrm{SNe}} \frac{p_i(a_k) p_j(a_k)}{\sigma_k^2}$$

- Errors satisfy $(\Delta c_i)^2 = (F^{-1})_{ii}$ and propagate into estimate of $\Delta D_{\mathrm{L}}(a)$ and $\Delta e(a)$
- Error in determination of PCs: $\Delta\lambda = \frac{1}{\sqrt{\lambda}}$
- Total error budget

$$\boxed{\Delta_{\mathrm{T}}^2 = \Delta_{\mathrm{coeff}}^2 + \Delta_{H_0}^2 + \Delta_{\lambda}^2}$$

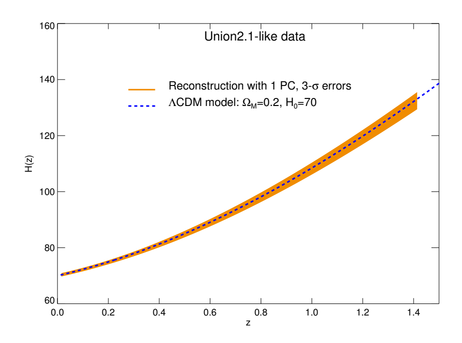
Method Summary



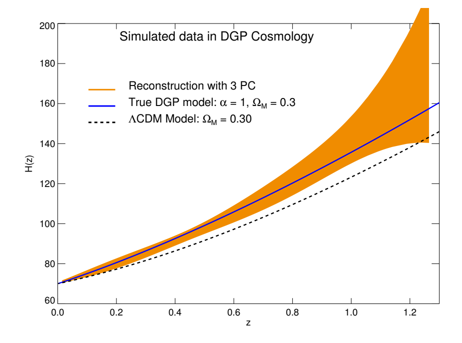
Tests with Simulations

- IF higher number of PCs need to be calculated, underlying cosmology is NOT ΛCDM

ΛCDM simulated data

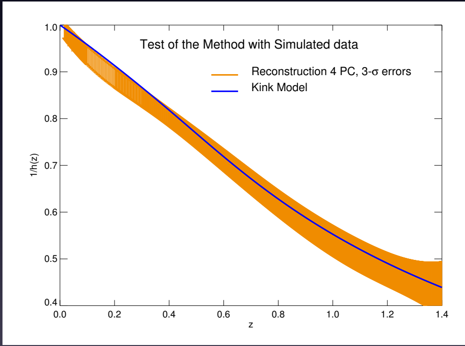


DGP simulated data

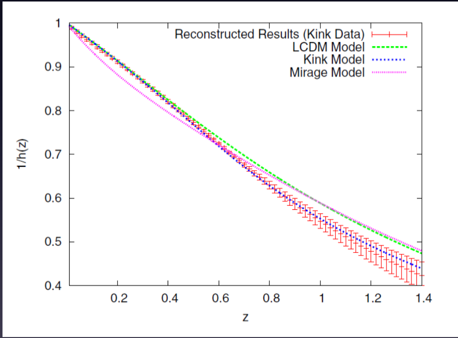


The Kink Model: a test case

- Extreme model with rapidly varying equation of state
- Reconstruction consistent with other techniques



(Benitez-Herrera et al. in prep.)

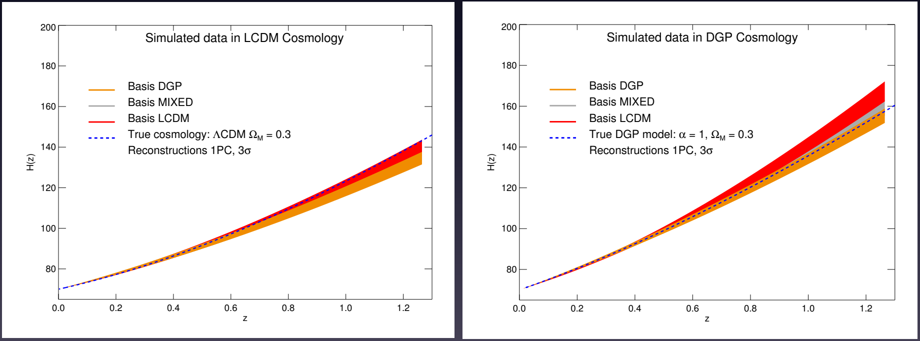


(Shafieloo & Kim 2011)

Extending the Training Set

- Training Set composed by ΛCDM behaviours only
- Make this initial matrix as general as possible
- Three different basis: ΛCDM, DGP and mixed

ΛCDM simulated data | DGP simulated data

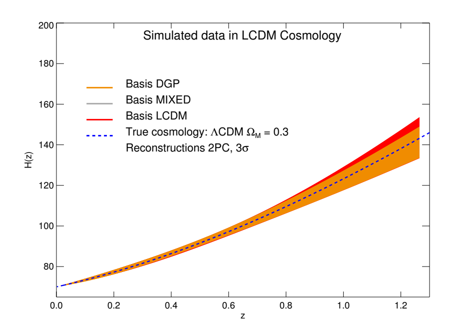


(Benitez-Herrera et al. in prep)

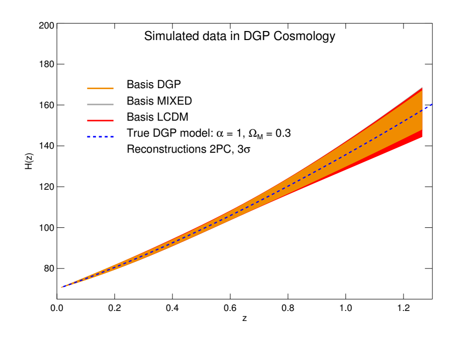
Extending the Training Set

- With 2PCs, the dependance with basis is negligible

ΛCDM simulated data



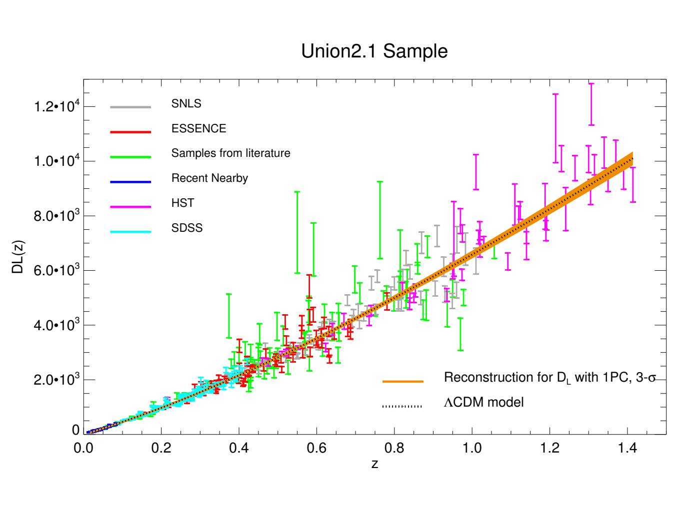
DGP simulated data



(Benitez-Herrera et al. in prep)

Union2.1 Sample *(Suzuki et al. 2012)*

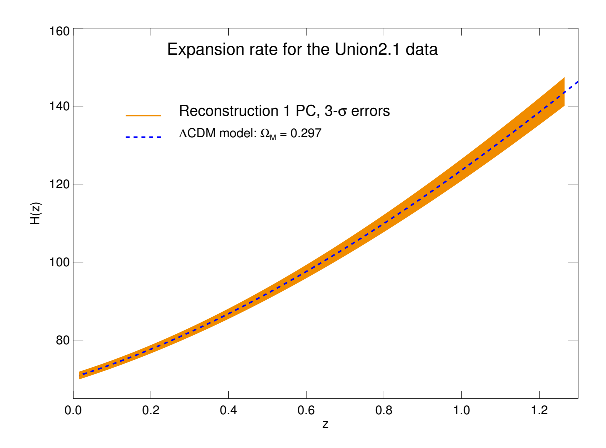
$D_L(z)$



z *(Benitez-Herrera et al. 2013)*

The Reconstructed Expansion Rate

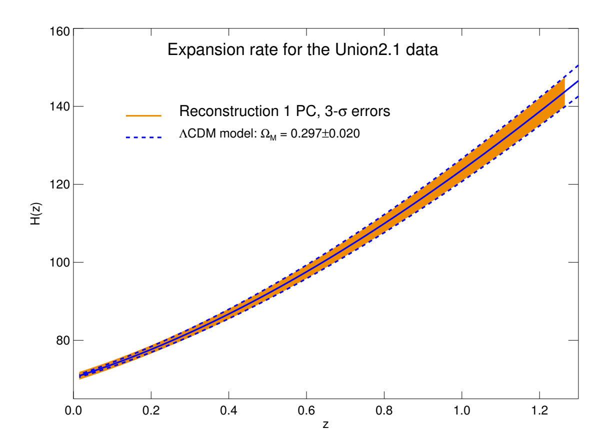
$H(z)$



z

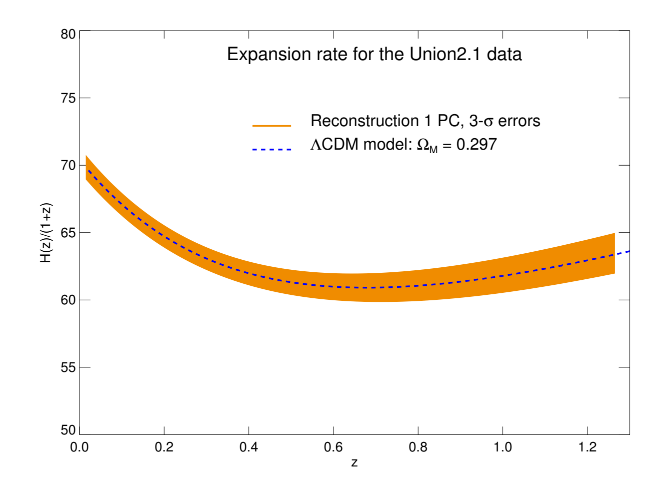
The Reconstructed Expansion Rate

$H(z)$



z

The Reconstructed Expansion Rate



(Benitez-Herrera et al. in prep)

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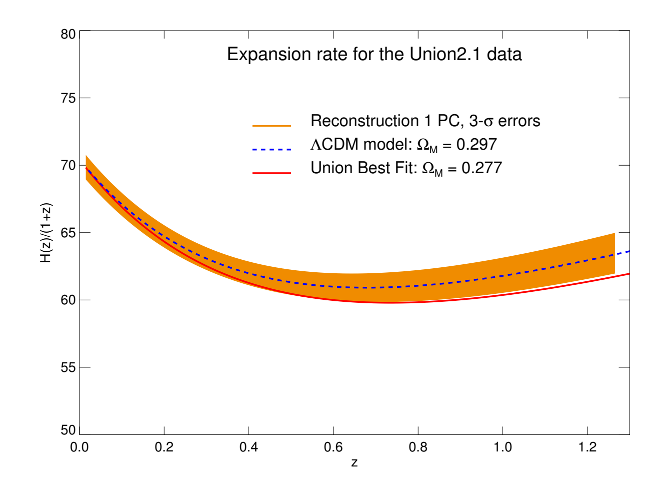
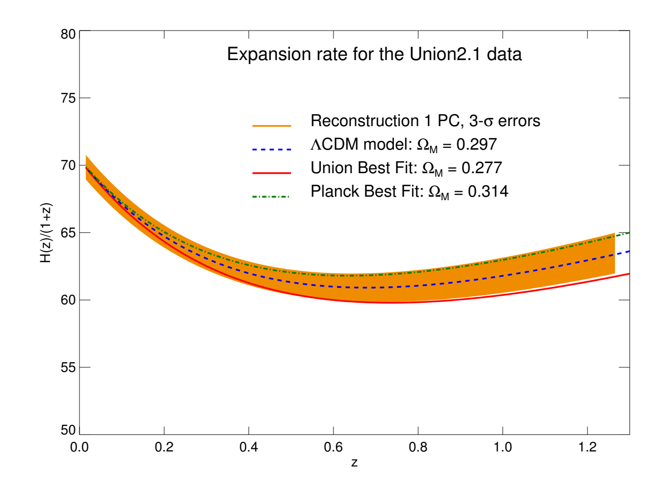


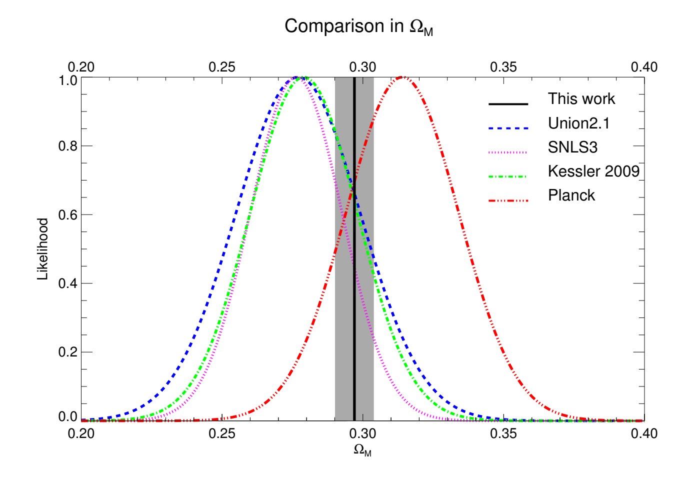
(Benitez-Herrera et al. in prep)

The Reconstructed Expansion Rate



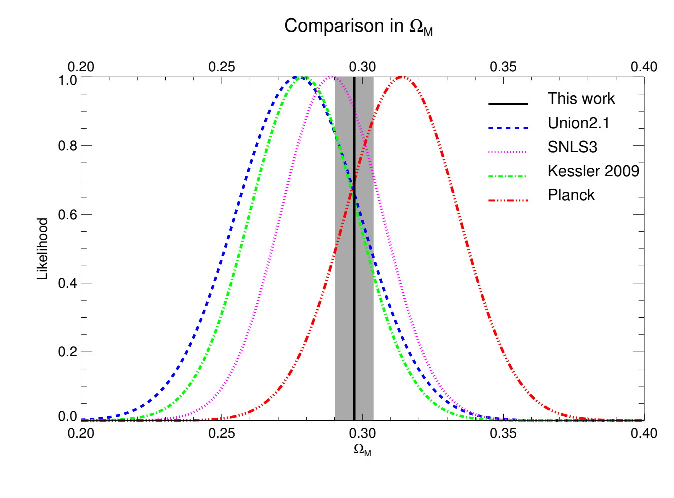
(Benitez-Herrera et al. in prep)

Cosmological Constraints



(Benitez-Herrera et al. 2013)

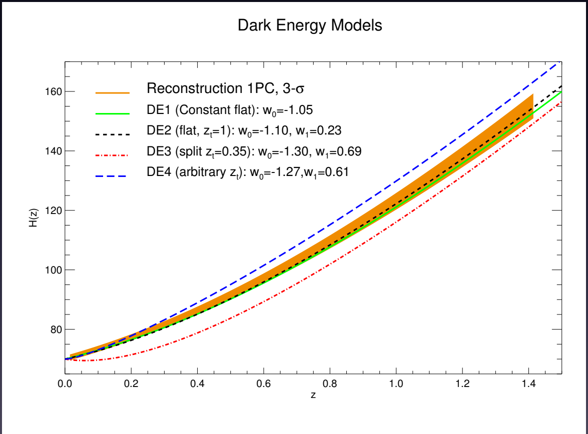
Cosmological Constraints



(Benitez-Herrera et al. 2013)

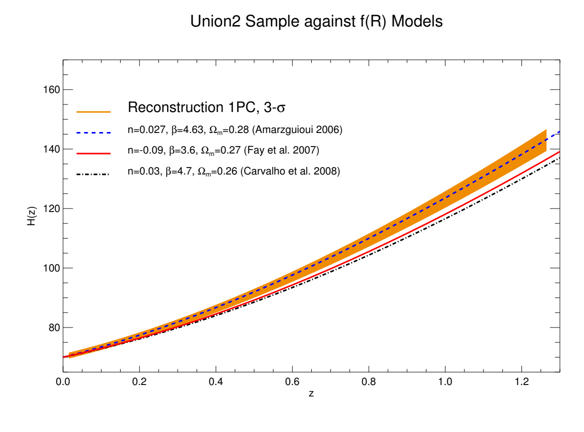
Other Cosmologies

1. Dark Energy models: extension of Chevallier+Polarski parameterization with varying z_t *(Rapetti et al. 2005)*



Other Cosmologies

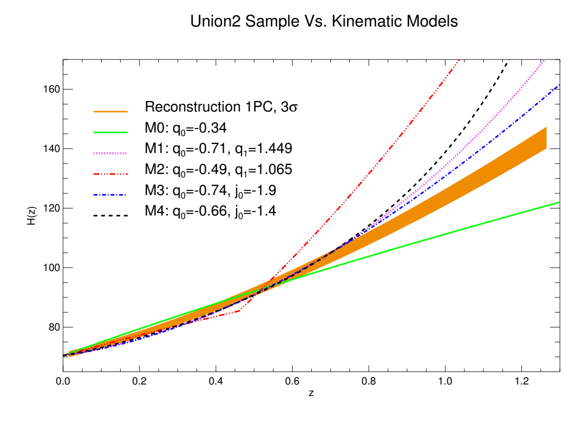
2. f(R) models *(Carvalho et al. 2008)*: $f(R) = R - \beta/R^n$



Other Cosmologies

3. Kinematic models *(Guimaraes et al. 2009)*

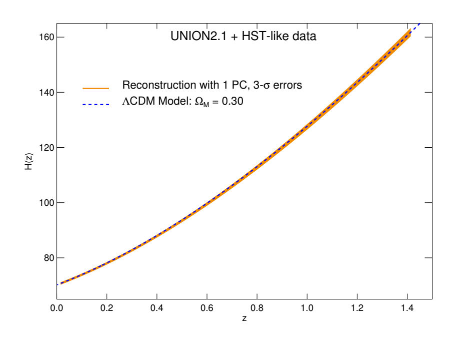
Several parameterizations of the deceleration parameter $q(z)$



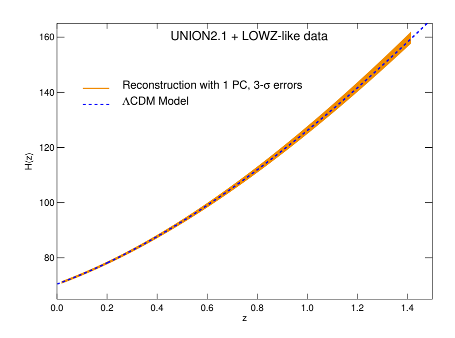
Testing Redshift Ranges

- Design a survey that is most sensitive to the expansion history in some specific redshift interval
- Simulations with SuperNova ANAlysis software

Union2.1 + High redshift

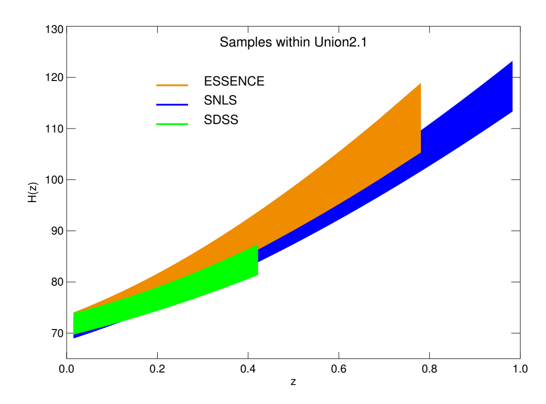


Union2.1 + Low redshift



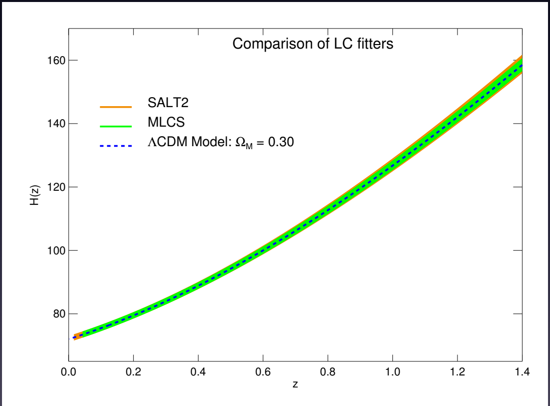
Systematic Errors

- Aquiles heel of SN cosmology
- Do different subsamples provide the same expansion history?

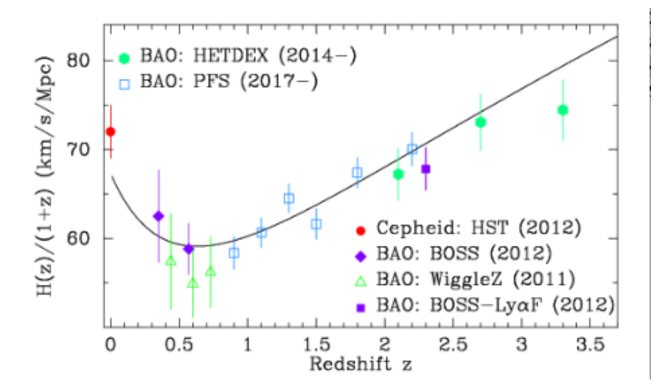


Systematic Errors

- Aquiles heel of SN cosmology
- Impact of light curve fitters in cosmological results!



Other Cosmological Probes



(Kim et al. 2013)

- Include BAOs: $D_L = (1+z)^2 D_A$

Summary

1) Model-independent approach to recover $H(z)$

2) Vased on inversion of the distance-redshift relationship + PCA to build an *optimal basis* for D_{L}

3) Framework to compare and constrain cosmological scenarios

4) Value for Ω_{m} agrees with Planck results

Looking ahead:

- Analysis of SNLS3 + SDSS-II catalog *(Betoule et al. 2014)*
- BAO data and forecasts from future surveys, e.g. eBOSS
- Inhomogeneity: forecasts from LSST for ihnomogeneous models

THANK YOU

FOR YOUR ATTENTION!

APPENDIX

Equations

Hilbert-Einstein action

$$S = \frac{-1}{16\pi G}\int d^4xR\sqrt{-g}$$

Friedmann Equation:

$$H^2(a) = H_0^2\left[\frac{\Omega_{\mathrm{r}0}}{a^4} + \frac{\Omega_{\mathrm{m}0}}{a^3} - \frac{\Omega_{k0}}{a^2} + \Omega_{de0}F(a)\right]$$

$$F(a) = \exp\left[-3\int_a^1 [1 + w(a')]\frac{da'}{a'}\right]$$

Equations II

Dark Energy

$$w(a) = \frac{w_{\rm et} z + w_0 z_{\rm t}}{z + z_{\rm t}} = \frac{w_{\rm et}(1-a)a_{\rm t} + w_0(1-a_{\rm t})a}{a(1-2a_{\rm t}) + a_{\rm t}}$$

DGP

$$H^2 - \frac{H^\alpha}{r_c^{2-\alpha}} = H_0^2 \left[\frac{\Omega_{\rm m}}{a^3}\right]; \quad r_c = (1-\Omega_{\rm m})^{\frac{1}{\alpha-2}} H_0^{-1}$$

$f(R)$

$$H^2 = H_0^2 \left[\frac{3\Omega_{\rm m0}(1+z)^3 + f/H_0^2}{6f'\xi^2}\right]$$

where ξ^2 depends on first and second derivatives of $f(R)$

Light Curve fitter models

SALT2

$$F(SN, p, \lambda) = x_0 \times [M_0(p, \lambda) + x_1 \times M_1(p, \lambda)] \times \exp[c \times CL(\lambda)]$$

$$\mu_B = m_B + \alpha x_1 - \beta c - M_B$$

MLCS

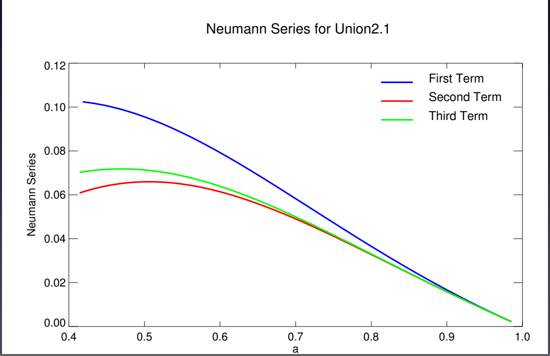
$$m_{\mathrm{model}}^{e,f} = M^{e,f'} + p^{e,f'} \Delta + q^{e,f'} \Delta^2 + X_{\mathrm{host}}{}^{e,f'} + K_{ff'}^{e} + \mu + X_{\mathrm{MW}}^{e,f'}$$

where $f \equiv$ observer frame filters; $f' \equiv$ rest-frame filters.
$M^{e,f'} \equiv$ absolute mag. for a SN with $\Delta = 0$

Neumann Series

$$e(a) = -a^3 D'_{\mathrm{L}} + a \int_1^{a_{\mathrm{min}}} x^{-2} e_{n-1}(x) dx$$

$$e(a) = \sum_{j=1}^{J} c_j e^{(j)}(a) = \sum_{j=1}^{J} c_j \left[-a^3 PC_j + \sum_{n=1}^{3} a^n \int_1^{a_{\mathrm{min}}} x^{-2} e_{n-1}(x) dx \right]$$



Principal Component Analysis

Mathematically X_i^k k supernovae, i components (wavelength bins)

One computes the covariance matrix:

$$\Sigma_{ij} = \frac{\sum_k (X_i^k - \overline{X}_i)(X_j^k - \overline{X}_j)}{k}$$

The matrix is symmetric. Then, you diagonalize it and sort the eigenvalues.

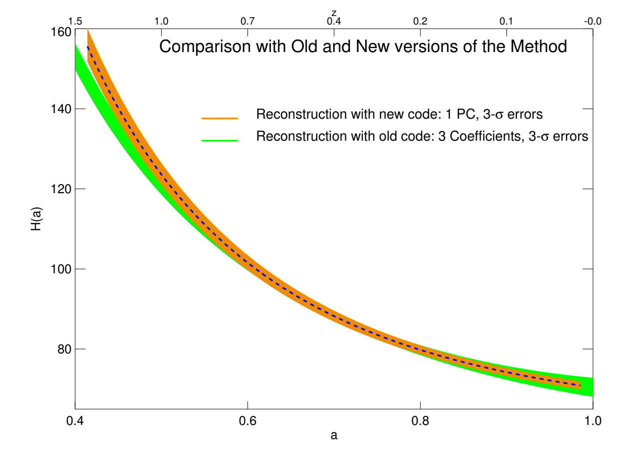
The first corresponding eigenvectors are the first PCs, they preserve most of the variance in the data, hence they well reproduce the initial data in a lower-dimensional space.

PC Approach *(Maturi & Mignone 2009)*

1. Construct **Training set** t_i that samples all possible behaviours of the observable
2. Define a **Reference vector** $\bar{t}$ which will be the origin of the new parameter space
3. Construct **Scatter Matrix** (analogous to a covariance matrix): $S = \Delta\Delta^T$ with $\Delta = (t_1 - \bar{t}, t_2 - \bar{t}, ..., t_M - \bar{t})$ containing the differences between each training vector t_i and $\bar{t}$
4. Principal Components, w_i, calculated by solving usual eigenvalue problem: $w_i = \lambda_i S w_i$
5. Optimal basis for that training set $\rightarrow$ concentrates information about deviations from the reference model in a few features (by definition)

The Reconstructed $H(a)$

$H(a)$



a

- For D_{L}:

$$[\Delta D_{\mathrm{L}}(a)]^2 = \sum_{j=0}^{J} (\Delta c_j)^2 \left[p_j(a)\right]^2$$

- For $e(a)$:

$$[\Delta e(a)]^2 = \sum_{j=0}^{J} (\Delta c_j)^2 [e_n^j(a)]^2$$

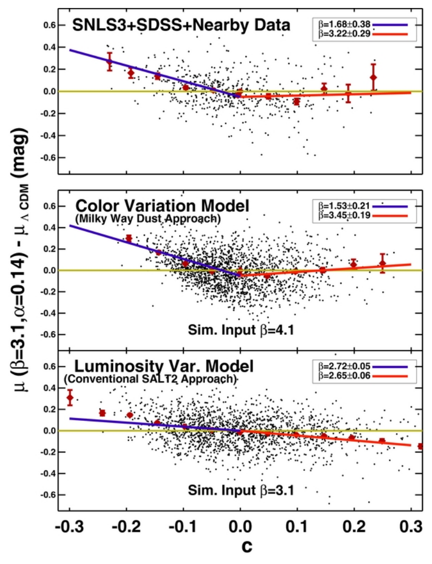
- Cumulative percentage of total variance

$$t_M = \left(\frac{\sum_{i=1}^{L} \lambda_i}{\sum_{j=1}^{N_{\rm PC}} \lambda_j} \right) \times 100$$

- Example: Union2.1 data
  1st PC accounts for 99.90% of t_M

Systematic Errors

- SNANA-simulated samples with different systematics
- Test if detectable with PC algorithm



Systematic Errors II

Source	Error on w
Vega	0.033
All instrument calibration	0.030
Color correction	0.020
Mass correction	0.016
Contamination	0.016
Intergalactic extinction	0.013
Galactic extinction	0.010
Light curve shape	0.006
Quadrature sum	**0.061**

KINEMATIC MODELS

$M0 : q(z) = q_0$

$M1 : q(z) = q_0 + q_1 z$

$M2 : q(z) = q_0 forz < z_t; q(z) = q_1 forz > z_t$

$M3 : j(z) = j_0$