Structure Formation, Backreaction and Weak Gravitational Fields

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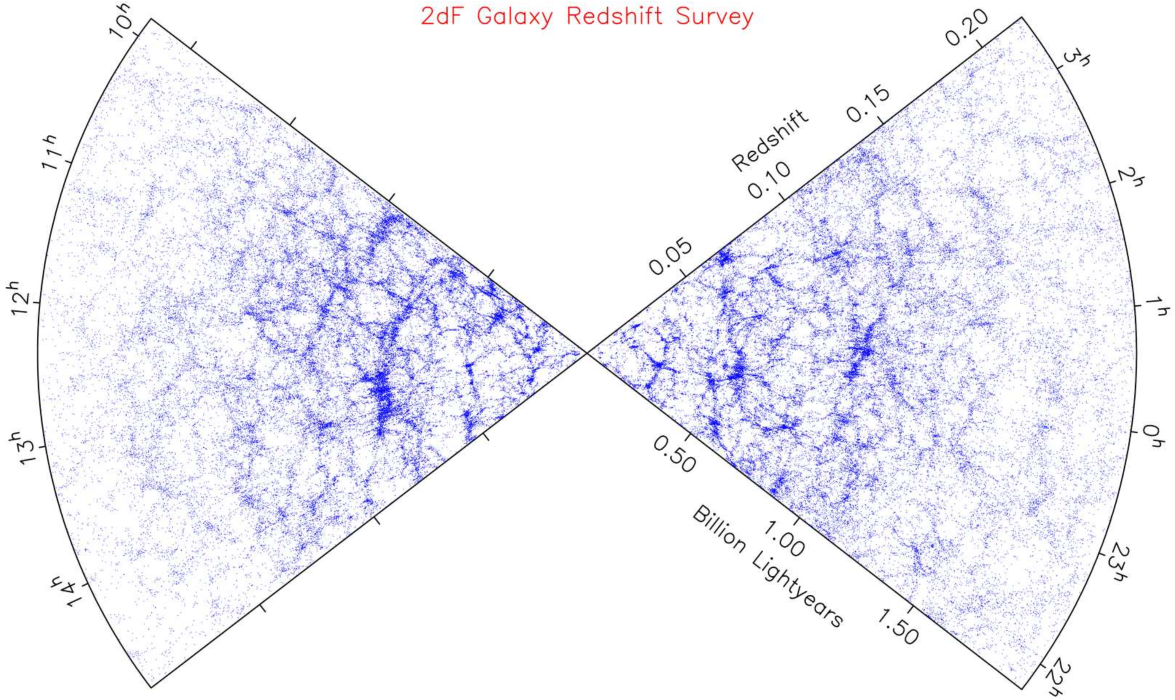
AP and T. P. Singh, arXiv:0806.3497, 2008, **PRL** in press;

AP, **PRD**78, 063522, 2008, arXiv:0806.2755;

AP and T. P. Singh, **JCAP** 0803:023, 2008, arXiv:0801.1546.

Introduction

The Universe is inhomogeneous on small scales...



... but homogeneous when **averaged** on large scales.

Introduction

Can inhomogeneities affect the cosmological expansion?

Two main approaches in understanding these effects :

- Effect on light propagation (“special observer” assumption).
- Effect on average expansion via correlations in fluctuations or backreaction, arising from the fact that in general $E[\langle g\rangle] \neq \langle E[g]\rangle$, where $E[g]$ is Einstein tensor for metric g.
  (G. F. R. Ellis, *Gen. Rel. and Grav.*, 1984.)

Focus here is on the latter.

Argument: Although technically possible, in the real world backreaction *does not* significantly influence the average cosmic expansion.

Structure of backreaction

Regardless of the averaging scheme used, broad structure of the backreaction is

$$\mathcal{C} \sim \langle\, \widetilde{\Gamma}^2 \,\rangle - \langle\, \widetilde{\Gamma} \,\rangle^2$$

Here Γ is the Christoffel connection and tilde represents any processing required by the averaging operation.

Such terms appear on the RHS of the corrected cosmological equations.

Results from linear theory

Assumption: Linear perturbation theory (PT) in the metric *and* matter fluctuations, is a good approximation at early times.

$$ds^2 = -(1+2\varphi)d\tau^2 + a(\tau)^2(1-2\varphi)d\vec{x}^2\,.$$

Justified by amplitude of CMB anisotropies, coupled with Copernican belief.

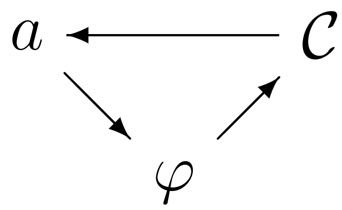
The dominant contribution to the backreaction $\mathcal{C}$ (e.g. in a sCDM universe) is

$$\mathcal{C} \sim \frac{1}{a^2}\langle\, \nabla\varphi \cdot \nabla\varphi \,\rangle \sim \frac{1}{a^2}(10^{-4})H_0^2\,.$$

Broadly speaking, contribution of the backreaction at early times is expected to be small.

Results from linear theory [AP, arXiv:0806.2755]

What we really want is a self consistent solution of the loop



i.e. – Background affects perturbations, perturbations give backreaction and backreaction affects background.

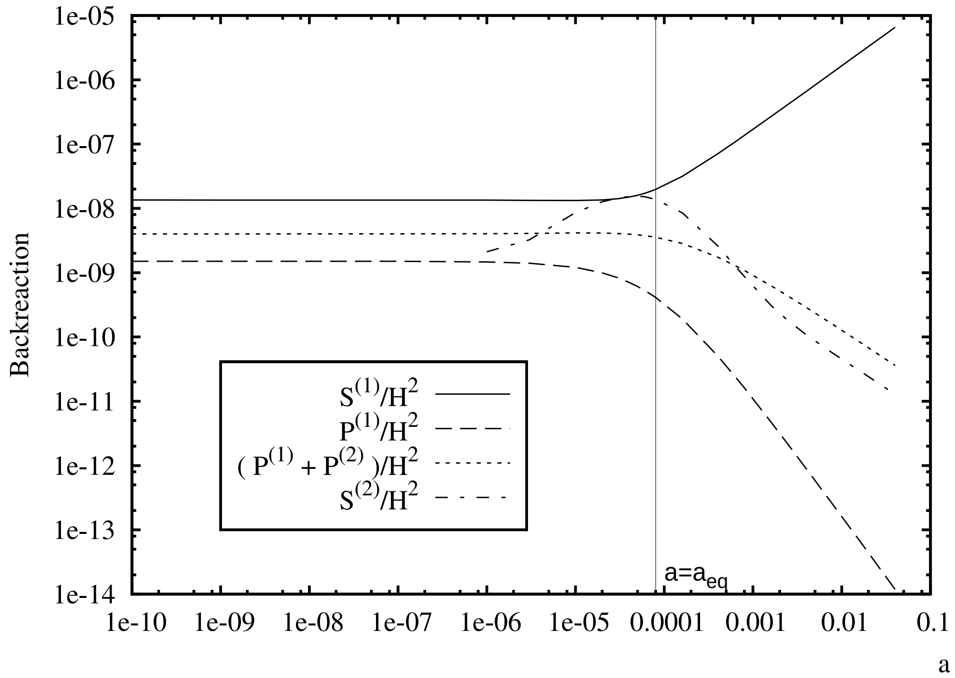
Smallness of $\mathcal{C}$ at early times gives the hope that an iterative approach might work

$$a^{(0)} \longrightarrow \varphi^{(0)} \longrightarrow \mathcal{C}^{(0)} \longrightarrow a^{(1)} \longrightarrow \varphi^{(1)} \longrightarrow \dots$$

Here $a^{(0)}$ and $\varphi^{(0)}$ are computed assuming $\mathcal{C} = 0$.

Results from linear theory [AP, arXiv:0806.2755]

An exact calculation in the Zalaletdinov averaging framework [Zalaletdinov, GRG 24,1015,1992; GRG 25,673,1993] gives



Here $S^{(1)} + P^{(1)}$ appears in Friedmann eqn., and $P^{(1)} + P^{(2)} + S^{(2)}$ appears in the acceleration eqn. Only magnitudes are plotted.

The nonlinear regime

So linear PT gives a tiny backreaction. Is PT for the metric valid at late times also? Suppose that it is. Then the relevant equation is

$$a^{-2}\nabla^2\varphi = 4\pi G\bar{\rho}\delta \ \ ; \ \ \delta \equiv \rho(t,\vec{x})/\bar{\rho}(t) - 1\,.$$

As before,

$$\mathcal{C} \sim a^{-2}\langle\, \nabla\varphi\cdot\nabla\varphi\,\rangle\,.$$

For an over/under-density of physical size R, treating $a^{-1}\nabla \sim R^{-1}$ and $G\bar{\rho} \sim H^2$, we have

$$|\varphi| \sim (HR)^2\,|\delta|\ .$$

For voids, $\delta \sim -1$, and then $\mathcal{C} \sim H^2(HR)^2 \ll H^2$.

The nonlinear regime

For overdensity, more care is needed. In a typical spherical collapse situation,

$$R \sim (1-\cos u)r \;;\; H^{-1} \sim (G\bar{\rho})^{-1/2} \sim t \sim H_0^{-1}(u-\sin u)$$

$$G\rho \sim \frac{(H_0 r)^2}{R^2 R'} \sim \frac{H_0^2}{(1-\cos u)^3} \;;\; \delta \sim (\rho/\bar{\rho}) \sim \frac{(u-\sin u)^2}{(1-\cos u)^3}$$

$$|\varphi| \sim \frac{(H_0 r)^2}{(1-\cos u)} \;;\; \mathcal{C} \sim H^2 \left[(H_0 r)^2 \frac{(u-\sin u)^2}{(1-\cos u)^4}\right] .$$

At late times we apparently have $\mathcal{C} \sim H^2$ implying large corrections and $\varphi \sim 1$ implying a breakdown of perturbation theory.

The nonlinear regime [AP and Singh, arXiv:0801.1546]

Crucial question is :
Is this situation actually realised, or are we taking these simple models too far?

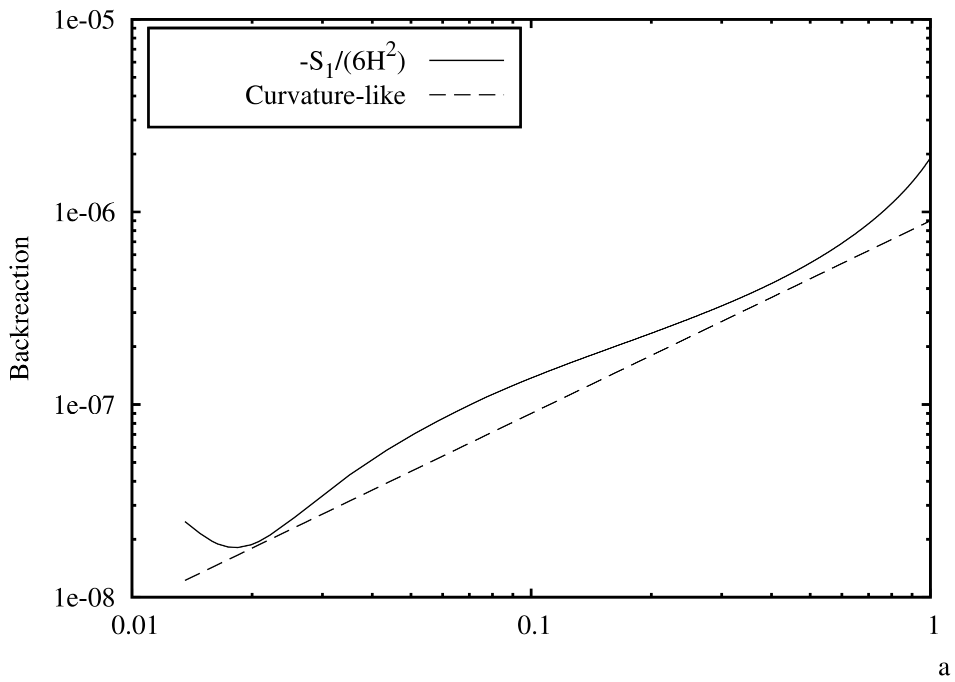
Claim : Perturbation theory in the metric does not break down at late times, since observed peculiar velocities remain small. The spherical collapse model is not a good approximation when model peculiar velocities grow large.

We study an exact relativistic spherical collapse model, and show that the perturbed FLRW form for the metric can be recovered by an explicit coordinate transformation *even in the nonlinear regime*, provided peculiar velocities remain small.

The nonlinear regime [AP and Singh, arXiv:0801.1546, arXiv:0806.3497]

Our model contains a central overdense region of Eulerian size $\sim 5h^{-1}$Mpc surrounded by an underdensity of Eulerian size $\sim 25h^{-1}$Mpc. Density contrasts reach $\delta \sim 15$ in the overdensity and $\delta \sim -0.8$ in the underdensity. Peculiar velocities remain less than $\sim 10^{-2}$.

The dominant backreaction component is shown below



Conclusions

- Backreaction due to averaging of inhomogeneities, is real and nontrivial, but appears to be negligible given realistic initial conditions and evolution models.
  - Backreaction is small provided PT in the metric is valid.
  - This appears to be the case provided peculiar velocities remain small.
  - Observed peculiar velocities are, in fact, nonrelativistic.
- Conclusions here are based on linear PT and exactly solved toy models of structure formation.
- More accurate calculations may be possible using N-body simulations.