Structure formation in presence of dark interactions

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Outline

Introduction:
  Introducing dark energy...
  Models of dark energy

Interacting dark energy:
  Motivations for dark interactions
  Basic equations

N-body simulations of interacting dark energy:
  Methods and implementation of dark interactions in GADGET
  Numerical tests and main features of the interaction

Results and open questions:
  Baryon-Dark Matter bias
  Halo density profiles
  Evolution of halo concentrations
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See talks by B. Schmidt and C. Wetterich
In the context of the presently established cosmological picture, we assume the dark energy being given by a quintessence scalar field:

$$\rho_{de} = \frac{1}{2} \dot{\phi}^2 + V(\phi)$$

[C. Wetterich, 1988]
[P. J. E. Peebles and B. Ratra, 1988]

and we introduce a coupling between the dark energy and the dark matter in the form:

$$\ddot{\phi} + 3H \dot{\phi} + \frac{dV(\phi)}{d\phi} = \kappa \beta(\phi) \rho_m$$

$$\dot{\rho}_m + 3H \rho_m = -\kappa \beta(\phi) \rho_m \dot{\phi}$$

$$\dot{\rho}_b + 3H \rho_b = 0$$

[C. Wetterich, 1995]
[L. Amendola, 2004]
Interacting Dark Energy - main features

from the modified background equations it is possible to derive:

✔ variable dark matter mass:

\[ m(\phi) = m_0 \exp\left[ \kappa \int_{\phi_0}^{\phi} \beta(\phi') d\phi' \right] = m_0 \Delta m_c(\phi) \]

✔ modified background evolution:

\[ H(a) \rightarrow H(a, \beta(\phi)) \]

For what concerns linear density fluctuations, the evolution equation is modified as follows:

\[
\ddot{\delta}_c + \left( 2H - 2\beta \frac{\dot{\phi}}{M} \right) \dot{\delta}_c - \frac{3}{2} H^2 \left[ (1 + 2\beta^2) \Omega_c \delta_c + \Omega_b \delta_b \right] = 0
\]

extra-friction term  modified gravitational interaction  varying DM particle mass

We implement these new features in GADGET [V. Springel, 2005]
Implementation of dark interactions in GADGET

1) The Hubble function and the mass variation are computed with a linear perturbation code (CMBEasy) and updated in the N-body code at each timestep;

2) An additional acceleration due to the cosmological extra friction is imprinted to all particles at each timestep;

3) The gravitational interaction is computed separately for CDM particles and for baryons, both in the tree algorithm and in the PM algorithm according to the interaction scheme:

\[ \tilde{G}_{ij} = G_N (1 + 2\beta_i \beta_j) \]

[\textit{L. Amendola, 2004}]
The parameters of our models

We consider a series of quintessence models with inverse power potential:

\[ V(\phi) = \frac{\Lambda^{4+\alpha}}{\phi^\alpha} \]

with constant coupling to CDM and no coupling to baryons, and with the cosmological parameters set according to the WMAP5 results:

<table>
<thead>
<tr>
<th>Model's parameters</th>
<th>Cosmological parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Slope $\alpha$</td>
</tr>
<tr>
<td>----------</td>
<td>----------------</td>
</tr>
<tr>
<td>$\Lambda_{CDM}$</td>
<td>0</td>
</tr>
<tr>
<td>RP1</td>
<td>0.143</td>
</tr>
<tr>
<td>RP2</td>
<td>0.143</td>
</tr>
<tr>
<td>RP3</td>
<td>0.143</td>
</tr>
<tr>
<td>RP4</td>
<td>0.143</td>
</tr>
<tr>
<td>RP5</td>
<td>0.143</td>
</tr>
<tr>
<td>RP6</td>
<td>2.0</td>
</tr>
</tbody>
</table>

The same models as in Macciò et al (2004)
Numerical tests – the Growth Factor

With a set of low resolution simulations we test the linear growth of density fluctuations by computing the evolution of the matter power spectrum amplitude at different redshifts.

\[ L_{\text{box}} = 320h^{-1} \text{Mpc} \]
\[ N = 2 \times 128^3 \]
\[ m_b \sim 1.9 \cdot 10^{11}h^{-1}M_{\odot} \]
\[ m_c(z = 0) \sim 9.2 \cdot 10^{11}h^{-1}M_{\odot} \]

the accuracy is at the percent level for all the values of the coupling.

\[ m_b \sim 1.9 \cdot 10^{11}h^{-1}M_{\odot} \]

\[ m_c(z = 0) \sim 9.2 \cdot 10^{11}h^{-1}M_{\odot} \]
The Simulations

For four of the models discussed before (ΛCDM, RP1, RP2, RP5) we run high resolution hydrodynamical simulations including all the modifications described above, normalizing density fluctuations with the same $\sigma_8$ today.

<table>
<thead>
<tr>
<th>$L_{\text{box}} = 80 , h^{-1}\text{Mpc}$</th>
<th>$m_c(z = 0) \sim 2 \cdot 10^8 , h^{-1}\text{M}_\odot$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 2 \times 512^3$</td>
<td>$m_b \sim 5 \cdot 10^7 , h^{-1}\text{M}_\odot$</td>
</tr>
<tr>
<td>$\epsilon_g = 3.5 , h^{-1}\text{kpc}$</td>
<td>$z_i = 60$</td>
</tr>
</tbody>
</table>

In addition, we run other two simulations with the same numerical settings but switching off the hydrodynamic forces acting on baryons (ΛCDM NO SPH, RP5 NO SPH).

Finally, we ran a last simulation with the largest coupling value but with the same initial conditions as the ΛCDM one (RP5 NO GF).

ALL the simulations have the same random phases.

All the simulations ran on 64 processors on the OPA cluster @RZG
Results: Baryon-CDM bias [M.Baldi et al., in prep.]

Integrated bias (as defined in Macciò et al. [2004]):

\[
B(< R) \equiv \frac{\rho_b(< R) - \bar{\rho}_b}{\bar{\rho}_b} \cdot \frac{\bar{\rho}_c}{\rho_c(< R) - \bar{\rho}_c}
\]

- At large radii the linear bias is recovered
- The bias is enhanced in the inner region both by hydrodynamic effects and by the extra scalar interaction
- The scalar field effect is clearly visible in case of purely collisionless simulations
Results: Halo density profiles

We compare density profiles of baryons and CDM for those halos in our group catalogue that can be identified as being the same object in the four different simulations.

- The inner density of both baryons and CDM decreases with increasing coupling;
- The same trend appears in the vast majority of the halos in our sample (more than 150 halos);
- This result is in contrast with Macciò et al [2004]
this work

\[ M_{200}(\Lambda CDM) = 2.82510^{-14} \, h^{-1} M_{\odot} \]

Macciò et al., 2004

\[ R \text{ (Mpc } h^{-1}) \]
Results: Halo concentrations [M. Baldi et al., in prep.]

We fit our density profiles with an NFW shape:
\[
\left( \frac{\rho(r)}{\rho_{\text{crit}}} \right)_{NFW} = \frac{\delta^*}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}
\]

and we compute halo concentrations for the 200 most massive halos in our group catalogue

\[
C = \frac{r_{\text{vir}}}{r_s}
\]

The decrease of concentration with coupling DOES NOT depend on the initial fluctuation amplitude!!

Deserves further investigation!!
Concluding...

We tested coupled dark energy cosmologies with constant coupling function to CDM particles for a range of possible coupling values by means of cosmological N-body simulations of structure formation, improving statistics with respect to previous works.

We find that the coupling imprints a universal linear bias between the amplitude of baryon and CDM density fluctuations, and that this bias is enhanced inside nonlinear structures.

We also find, contrary to previous works, that halo density profiles get shallower in the inner part of massive halos with increasing value of the coupling.

As a consequence, halo concentrations at z=0 are significantly reduced in coupled dark energy models with respect to ΛCDM, proportionally to the value of the coupling.
Our modified version of GADGET can handle:

- Specific expansion history for any parametrization of dark energy;

- Variation in time of the gravitational interaction of baryons and CDM separately;

- Variation in time of particles’ mass separately for baryons and CDM;

New features will keep being added...

So, if you have any interesting dark energy model with one, some, or all of these features, and you are interested in testing its effect on cosmic structure formation...

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this work

Integrated Bias for Group nr. 1

\[ z = 0.00000 \]

\[ M_{200}(\Lambda CDM) = 1.57836 \times 10^{14} h^{-1} M_{\odot} \]

Macciò et al., 2004

\[ \beta = 0.25 \quad \alpha = 0.143 \]

\[ \beta = 0.15 \quad \alpha = 0.143 \]
introducing dark energy...

what is the fundamental nature of DE and DM?
what makes the DE density today so small (but not zero)?
why DE and DM density are comparable exactly now?

from the WMAP team
Models of dark energy

dark energy must have:

\[ w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} < - \frac{1}{3} \]

\[ \rho_{DE,0} = \rho_{DE,0}^{obs} \sim 10^{-10} \text{erg/cm}^3 \]

Cosmological constant: \[ w_\Lambda = -1 \]

cosmological term (classical) \[ \rho_\Lambda \sim 10^{110} \text{erg/cm}^3 \]

Vacuum energy (quantum) \[ \rho_{vac} \sim 10^{110} \text{erg/cm}^3 \]

120 orders of magnitude off!!
Models of dark energy

Scalar fields \( w_\phi \neq -1 \)

Generalized Lagrangian: \( p(\phi, \chi) \)

k-essence: \( p(\phi, \chi) = K(\phi)\tilde{p}(\chi) \)

C. Armendariz-Picon, V. Mukhanov, P. J. Steinhardt (2000)

kinetic phantom: \( p(\phi, \chi) = -\chi + V(\phi) \)

R. R. Caldwell (2002)

quintessence: \( p(\phi, \chi) = \chi + V(\phi) \)

C. Wetterich (1988)

Quintessence equation of motion:
\[
\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0
\]
Quintessence – scaling solutions

A scaling solution is a scalar field trajectory on which the equation of state is constant: \( w_\phi = \text{const.} \).

Scaling solutions are attractors!!

2 potentials realize a scaling:

**Exponential**

\[
V(\phi) = M^4 e^{-\mu \phi / M}
\]

\[
w_\phi = w_B \Rightarrow \frac{\rho_\phi}{\rho_B} = \text{const.}
\]

😊 Solves the fine tuning
😊 Natural from ST
😊 Why ever?

**Ratra-Peebles**

\[
V(\phi) = M^{\alpha+4} \phi^{-\alpha}
\]

\[
w_\phi = \frac{\alpha w_B - 2}{\alpha + 2} \Rightarrow \frac{\rho_\phi}{\rho_B} \propto \alpha^2/(\alpha+2)
\]

😊 Solves the fine tuning
😊 Late DE domination
😊 Why now?
**Interacting Dark Energy - Motivations**

Why speculating about a possible interaction between Dark Energy and Dark Matter?

✔ Why not?

No symmetry principle requires DE and DM being uncoupled (coupling to baryons tightly constrained by EP tests, coupling to DM constrained by CMB to $\beta < 0.15$)

✔ Fundamental problems: why now?

Dynamical DE eases the fine tuning problem but doesn’t address the coincidence problem. **Coupling might help!**

✔ Discrepancies between theory and observations for LCDM

Satellite problem and “cusp-core” problem could disappear (or get worse!) in presence of coupling.
Interacting Dark Energy - basic equations

Einstein field equations: \[ G_{\mu\nu} = \kappa^2 T_{\mu\nu} \]

General Covariance requires: \[ \nabla_{\mu} G^{\mu}_{\nu} = 0 \Rightarrow \nabla_{\mu} T^{\mu}_{\nu} = 0 \]

BUT...

\[ T_{\mu\nu} = \sum_{i} T_{\mu\nu}^{(i)} \Rightarrow \nabla_{\mu} T^{\mu}_{\nu}^{(i)} = -\nabla_{\mu} T^{\mu}_{\nu}^{(j)} \]

is allowed

so we can have a coupling between the DE scalar field and the DM fluid in the form:

\[ \nabla_{\mu} T^{\mu}_{\nu}^{(\phi)} = \sqrt{\frac{2}{3}} \kappa \beta(\phi) T^{\alpha}_{\alpha}^{(m)} \nabla_{\nu} \phi \]

\[ \nabla_{\mu} T^{\mu}_{\nu}^{(m)} = -\sqrt{\frac{2}{3}} \kappa \beta(\phi) T^{\alpha}_{\alpha}^{(m)} \nabla_{\nu} \phi \]

modified conservation equations
modified dynamics of the fluids

...with a little algebra we get the (coupled!) dynamic equations for the scalar field and the matter fluid:

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{dV(\phi)}{d\phi} = \sqrt{\frac{2}{3}} \kappa \beta(\phi) \rho_m \]

\[ \dot{\rho}_m + 3H \rho_m = -\sqrt{\frac{2}{3}} \kappa \beta(\phi) \rho_m \dot{\phi} \]

from these equations it is possible to derive:

- **variable dark matter mass:**

  \[ m(\phi) = m_0 \exp \left[ \sqrt{\frac{2}{3}} \kappa \int_{\phi}^{\phi_0} \beta(\phi') d\phi' \right] = m_0 F_M(\phi) \]

- **modified background evolution through a phase-space analysis**
variation of particles’ mass

\[ F_M(\phi) = \exp\left[ \sqrt{\frac{2}{3}} \kappa \int_{\phi_0}^{\phi} \beta(\phi') d\phi' \right] \]
Modified background evolution

The full set of dynamic equations + the Friedmann equation can be analyzed in phase space to find the critical points of the system.

This shows the existence of two distinct regimes:

weak coupling regime ( \(|\beta| < 3/2\) )

✔ late-time accelerated DE attractor

✔ \(\phi\)-MDE epoch (Early DE, \(\Omega_{\phi}^{\text{early}} = 4\beta^2/9\))
Modified background evolution

strong coupling regime ($|\beta| > 3/2$)

☑ late-time scaling
   (solves the “why now?” problem)

☑ coupling too strong!!

☑ no MDE = no structure formation!!

strong coupling: $\beta = 4.02$ (top)
$\beta = 2.37$ (bottom)
weak coupling regime: the $\phi$-MDE epoch
weak coupling regime: the effect on equivalence
weak coupling regime: the evolution of the Hubble function

![Graph showing the Hubble function for different CQ models. The graph plots the ratio of the Hubble function to its present value against redshift (z). The models include LambdaCDM, RP1, RP2, RP3, RP4, RP5, and RP6, each represented by a different color line.](image-url)
Perturbations in CDE - main features

if we now perturb to the first order in all the quantities the coupled dynamic equations:

\[ \ddot{\delta}_c + (2H - 2H \beta x) \dot{\delta}_c - \frac{3}{2} H^2 \left[ (1 + \frac{4}{3} \beta^2) \Omega_c \delta_c + \Omega_b \delta_b \right] = 0 \]

after some algebra and introducing we get the perturbations eq. for the matter fluid (now including also uncoupled baryons):

- **extra-friction term (anti-friction)**
- **modified gravitational interaction**
- **varying DM particle mass**
Perturbations in CDE - the growth factor

Density perturbations will grow in a different way in a coupled dark energy cosmology:

\[ GF \equiv \frac{1}{a} \frac{\delta^+(a)}{\delta_0} = f(a, \Omega_i, \beta_i) \]

Initial conditions for N-body simulations will have to be modified.
Interaction scheme in a CDE cosmology

N-body codes of structure formation must be modified in order to account for this new physics in the dark sector, implementing:

- **modified background evolution**
- **variable DM particle mass**
- **different gravitational strength** for baryons and DM
- **extra-friction**

\[
\tilde{G}_{ij} = G_N (1 + \frac{4}{3} \beta_i \beta_j)
\]

\[
\tilde{G}_{bb} = \tilde{G}_{cc} = m_0 F_M
\]

\[
m = m_0 F_M
\]
implementation of CDE in GADGET: 4 main steps

I - the relevant quantities (Hubble function, coupling, mass correction, DE kinetic term, growth factor) as a function of $z$ are computed with a linear perturbation code (CMBEasy, M. Doran & G. Robbers) and given as an input to GADGET.

II - the initial conditions are rescaled according to the different growth factors and cosmological evolution (normalizing to the same $\sigma_8$ today). Also velocities have to be rescaled accordingly.

III - the properties of tree nodes and the gravitational potential on the grid (for the PM part) are evaluated separately for the DM and baryon distributions.

IV - The gravitational acceleration is computed for each particle according to the new "species-dependent gravity"
the Mass Function of dark matter halos
The coupling bias: Power Spectrum

The baryon bias due to the coupling affects all scales
growing matter – a coupled neutrino scenario


if we have 2 distinct DM families, one of them can also be strongly coupled
late time scaling attractor → solution of the “why now?” problem [Huey & Wandelt, PRD 74 (2006)]

...however:

- the coupling and the potential slope have to be tuned (both!) in order to get the observed cosmological parameters
- an additional (unknown) DM particle has to be introduced
idea: coupled neutrinos (with a negative coupling)

During the matter dominated scaling:

\[ \rho_g \propto a^3(\gamma - 1), \quad \gamma \equiv - \frac{\beta}{\alpha} > 0 \]

\( \rho_g \) triggers the final scaling attractor:

\[ \ddot{\phi} + 3H \dot{\phi} = - \frac{dV}{d\phi} + \frac{\beta}{M} \rho_g \]

✔ no new unknown particle has to be introduced
✔ neutrino mass increases (cosmological bounds on neutrino mass do not apply)
✔ the features of the final scaling attractor can be related to the measured neutrino (average) mass
growing matter – a coupled neutrino scenario III


$$\Omega_{DE} = 1 - \Omega_g = 1 - \frac{1}{\gamma + 1} + \frac{3}{\alpha^2(\gamma + 1)^2}$$

$$w = -1 + \frac{1}{(\gamma + 1)}$$

For a large value of $\gamma$ we get for the final scaling solution:

$$\Omega_g(t_0) = \frac{m_\nu(t_0)}{16eV}$$

$$\Omega_h(t_0) \approx \frac{\gamma m_\nu(t_0)}{16eV} \quad \Rightarrow [\rho_h(t_0)]^{1/4} = 1.07 \left( \frac{\gamma m_\nu(t_0)}{eV} \right)^{1/4} 10^{-3}eV$$

$$w = -1 + \frac{m_\nu(t_0)}{12eV} \quad \Rightarrow m_\nu < 2.4eV \quad\text{for}\quad w < -0.8$$
Summary:

- The nature and the phenomenological behavior of dark energy constitute a puzzle for cosmology that a simple cosmological constant doesn’t address in a fully satisfactory way;

- Standard scalar field models of dynamic dark energy can ease the fine tuning of the dark energy density but don’t solve the coincidence problem;

- The introduction of a coupling for the scalar field could be a solution;

- The fifth-force arising from the coupling would likely affect both the linear and nonlinear regimes of structure formation, therefore requiring an appropriate treatment in N-body codes;

- A possible coupling to neutrinos would realize the optimum scenario of two subsequent scaling regimes, and would relate the properties of the final attractor to the neutrino mass, then solving the coincidence problem.
implementation of CDE in GADGET

open issues:

the effect of the coupling on the transfer function of the power spectrum might be relevant [Mainini & Bonometto (2007)] and ICs might have to be modified accordingly
Introducing dark energy...

95% of the Universe is made of dark components!!!