ICE CSIC IEEC

Dynamical Casimir Effect with Semi Transparent Mirrors; & the Fate of Gravity Equations as Equations of State

EMILIO ELIZALDE

ICE/CSIC & IEEC, UAB, Barcelona

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Outline of the contribution

- On the Casimir Effect

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- The Dynamical Casimir Effect (Davies-Fulling)

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- Semitransparent Mirrors
- Moving Mirrors and the Black Body Spectrum

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- Gravity Equations as Equations of State: f(R) Case

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Even then: Has the final value real sense ?

Bohr $\longrightarrow$ Casimir $\longrightarrow$ Pauli ...

The Casimir Effect

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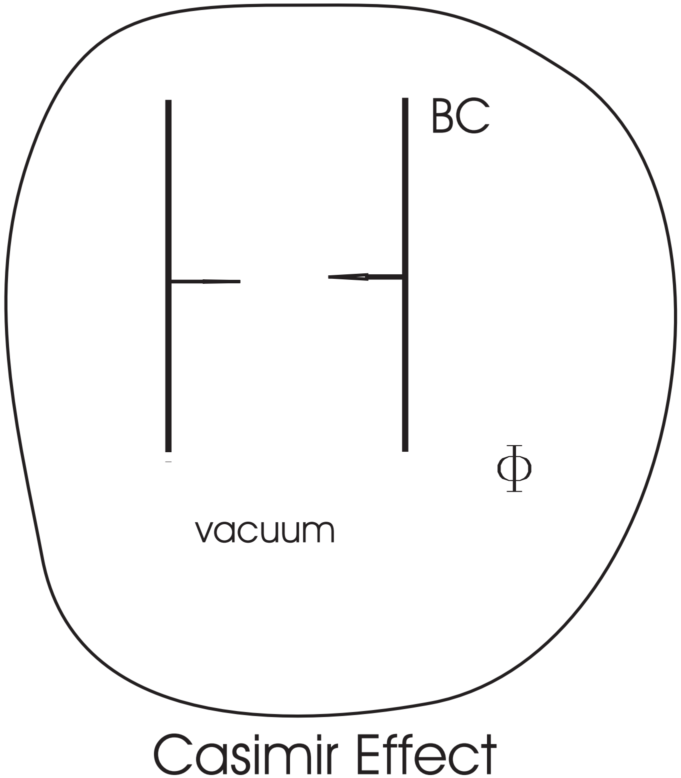
BC e.g. periodic



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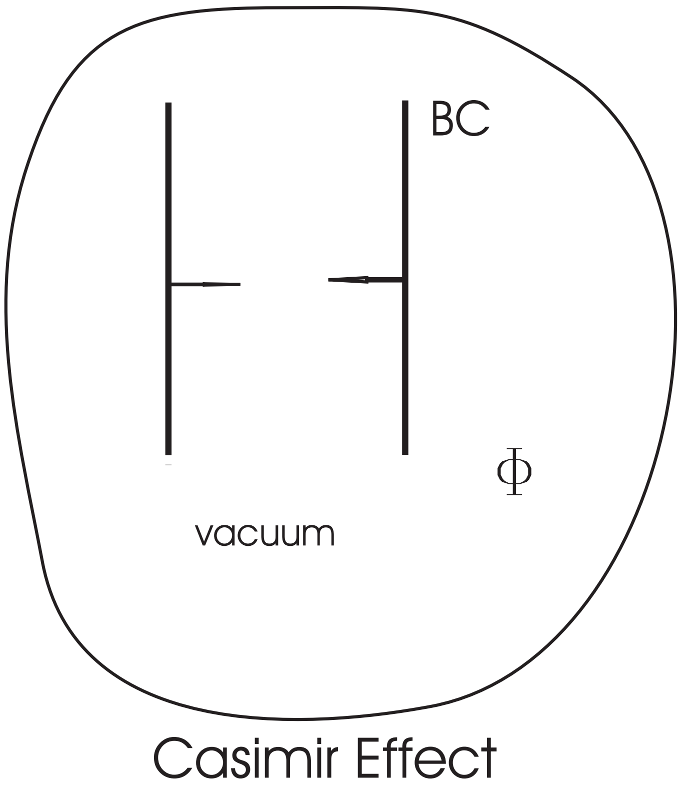

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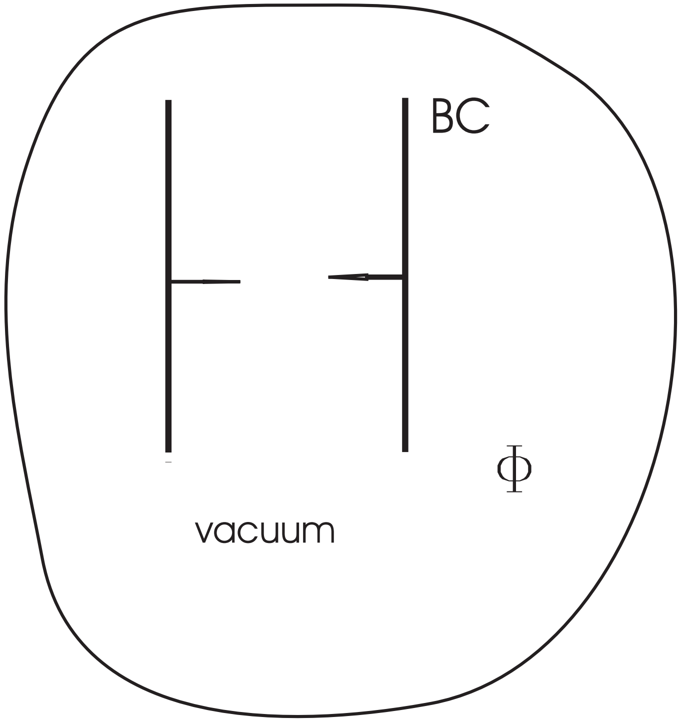

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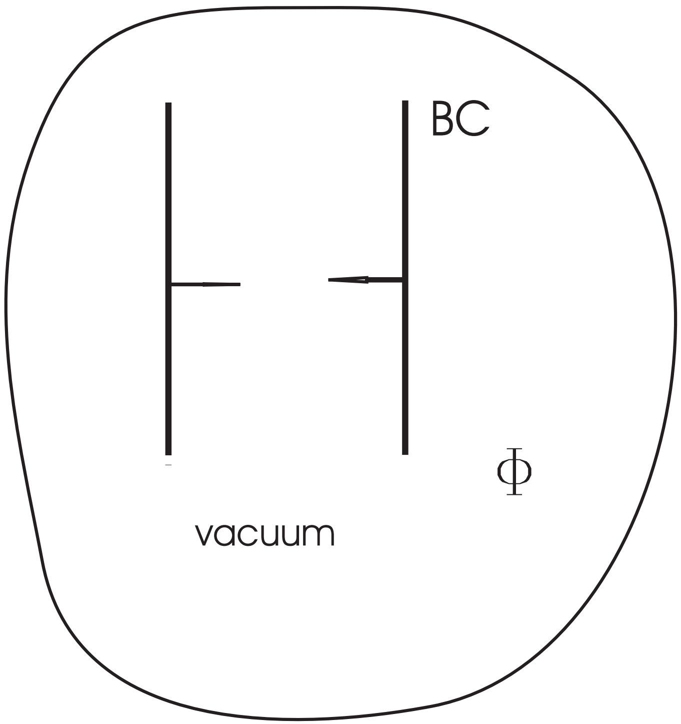

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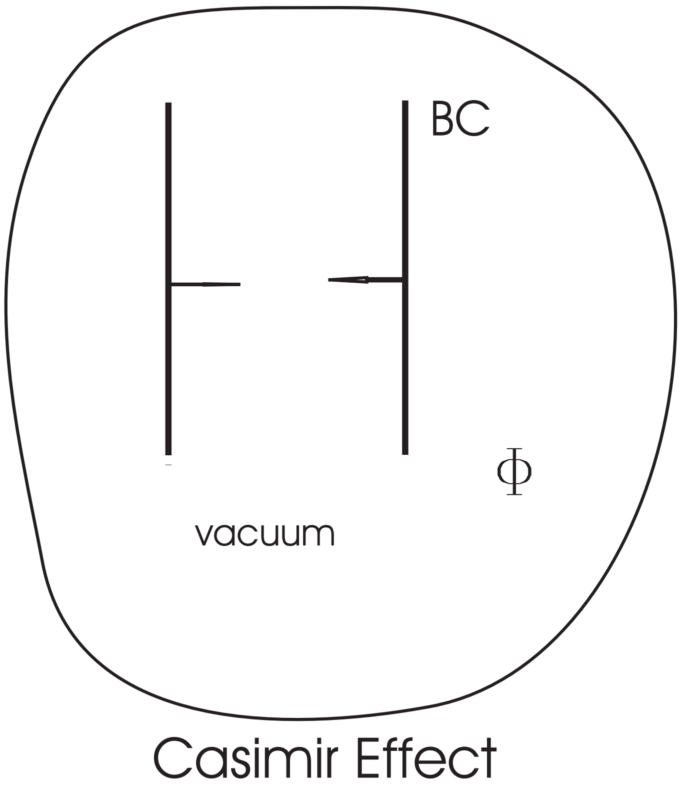
Universal process:

The Casimir Effect



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- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
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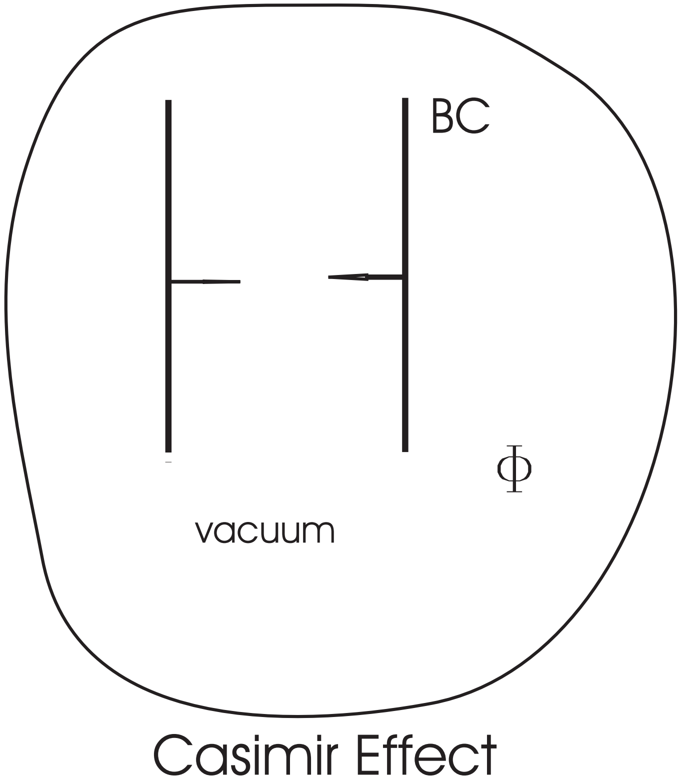

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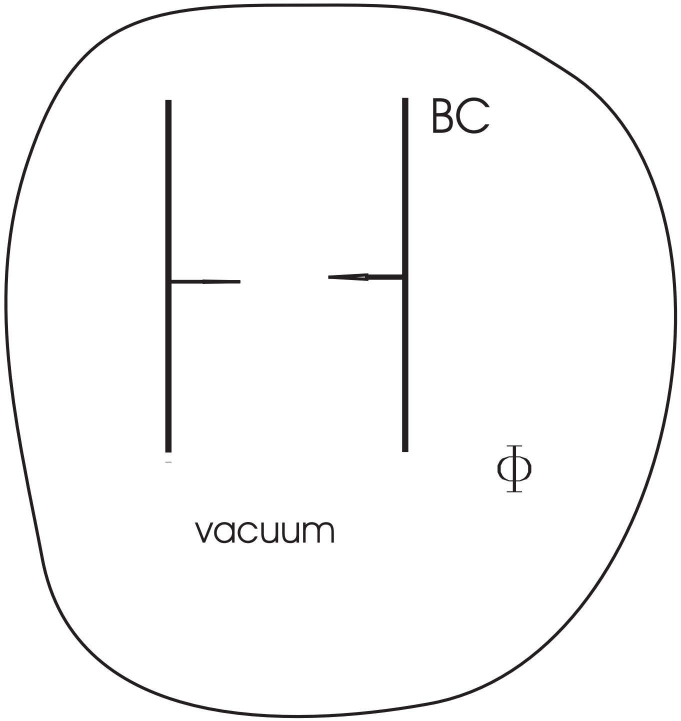

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- Dynamical CE $\Leftarrow$
- Lateral CE, piston, pistol, ...
- Extract energy from vacuum
- CE and the cosmological constant $\Leftarrow$

On the ‘reality’ of zero point fluctuations

- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)
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- Milonni has reformulated all of QED from the point of view of ZPF

The standard approach

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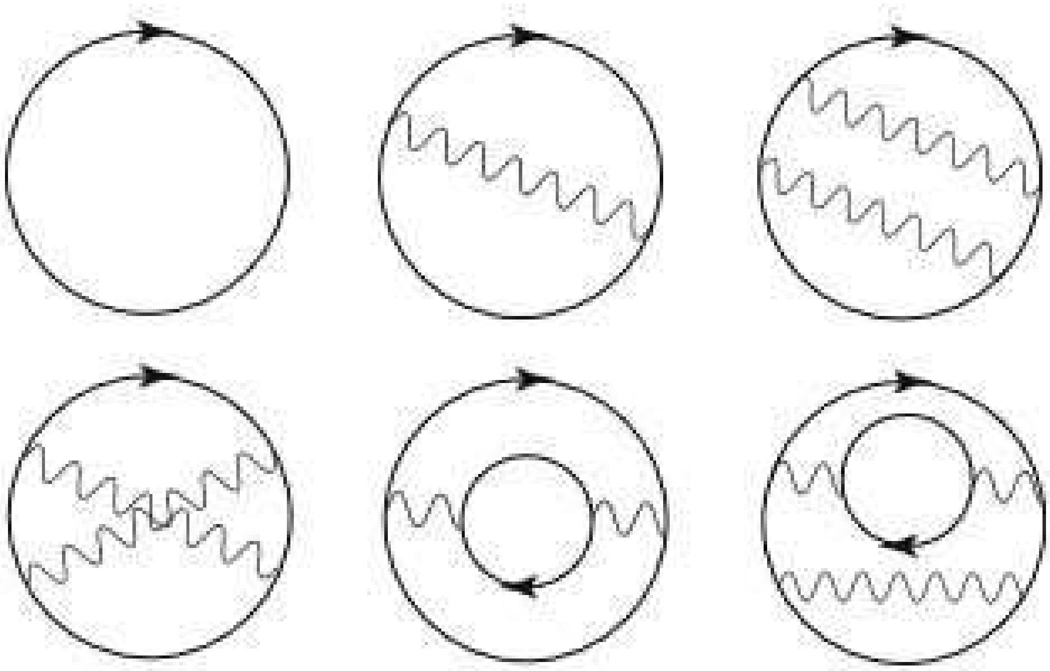

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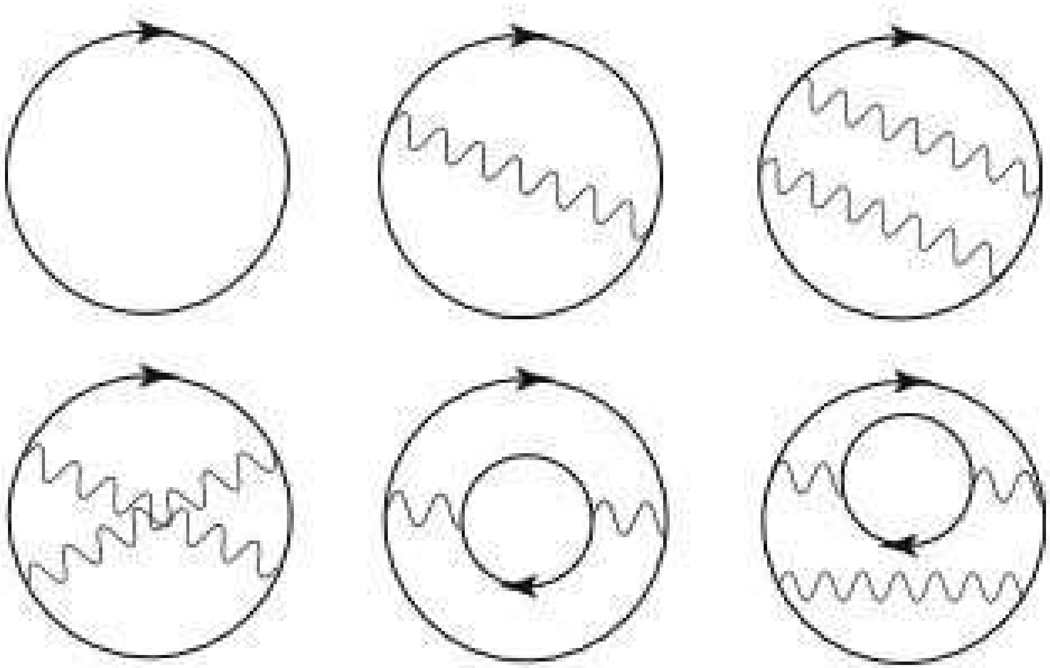

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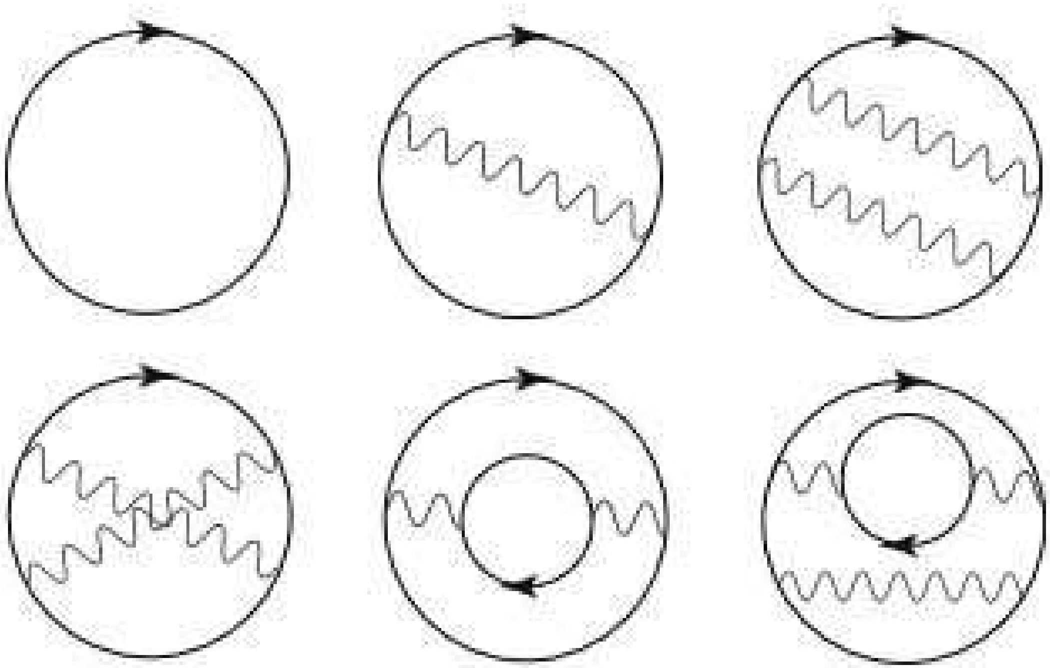

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In modern language the Casimir energy can be expressed in terms of the trace of the Greens function for the fluctuating field in the background of interest (conducting plates)

$$\mathcal{E} = \frac{\hbar}{2\pi}\mathrm{Im}\int d\omega\omega\ \mathrm{Tr}\int d^3x\,[\mathcal{G}(x,x,\omega+i\epsilon)-\mathcal{G}_0(x,x,\omega+i\epsilon)]$$

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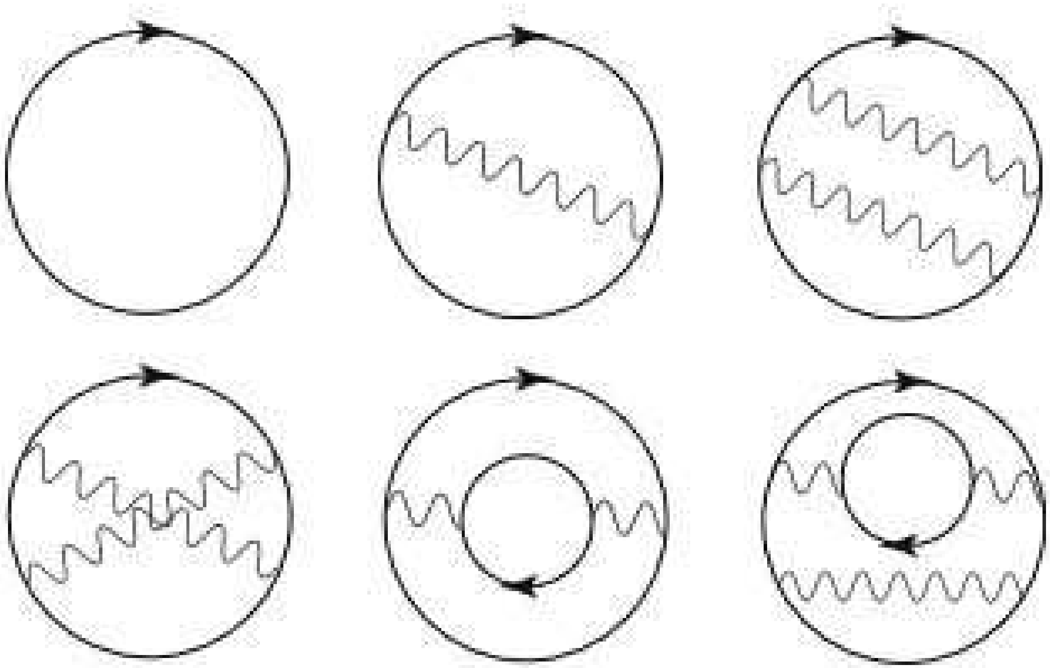

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$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function — Trace is over spin

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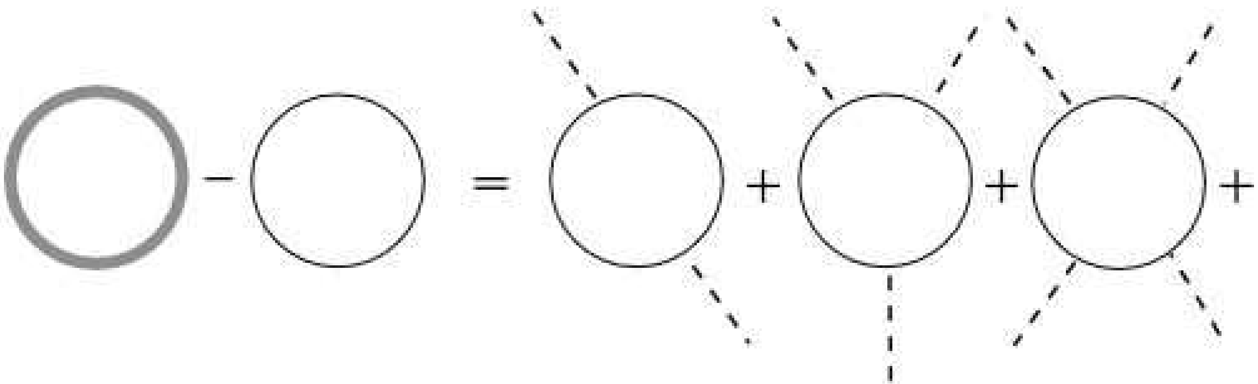

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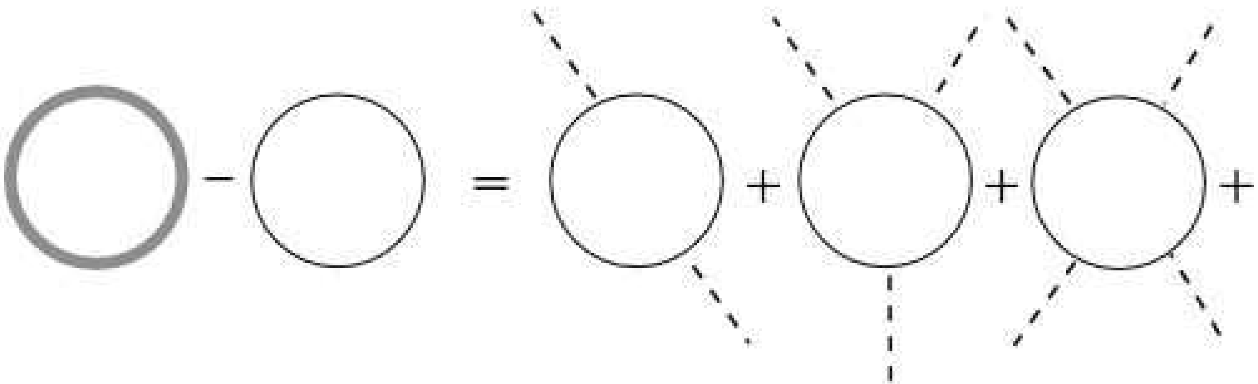

$\Longrightarrow$ Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations

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S.A. Fulling & P.C.W. Davies, Proc Roy Soc A348 (1976)

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Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al; Plunien et al; Barton, Eberlein, Calogeracos; Jaeckel, Reynaud, Lambrecht; Ford, Vilenkin; Brevik, Milton et al; Dalvit, Maia-Neto et al; Law; Parentani, ...

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J. Haro & E.E., PRL 97 (2006); arXiv:0705.0597

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- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

Some details of the method (1)

- Hamiltonian method for neutral Klein-Gordon field in a cavity Ω_t, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$

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$$\mathcal{L}(t, \mathbf{x}) = \frac{1}{2}\left[(\partial_t \phi)^2 - |\nabla_{\mathbf{x}}\phi|^2\right], \quad \forall \mathbf{x} \in \Omega_t \subset \mathbb{R}^n, \; \forall t \in \mathbb{R}$$

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- Hamiltonian. Transform moving boundary into fixed one by (non-conformal) change of coordinates

$$\mathcal{R} : (\bar{t}, \mathbf{y}) \to (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

transform Ω_t into a fixed domain $\widetilde{\Omega}$

$\widetilde{\Omega}$: $(t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$

(with $\bar{t}$ the new time)

SOME DETAILS OF THE METHOD (2)

- Hamiltonian density

$$\widetilde{\mathcal{H}}(\bar{t}, \mathbf{y}) = \frac{1}{2}\left(\widetilde{\xi}^2 + J|\nabla_{\mathbf{x}}\phi|^2\right) + \widetilde{\xi}\left(\partial_{\bar{t}}\widetilde{\phi} - \sqrt{J}\partial_t\phi\right)$$

$\widetilde{\phi}$ field, $\widetilde{\xi}$ conjugate momentum, J Jacobian: $d^3\mathbf{x} \equiv J d^3\mathbf{y}$

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- It turns out that

$$\mathcal{H}(t,\mathbf{x}) = \mathcal{E}(t,\mathbf{x}) + \xi(t,\mathbf{x}) < \partial_s \mathbf{R}(\mathcal{R}^{-1}(t,\mathbf{x})), \nabla_{\mathbf{x}}\phi(t,\mathbf{x}) >$$

$$+\frac{1}{2}\,\xi(t,\mathbf{x})\phi(t,\mathbf{x})\partial_s(\ln J)|_{\mathcal{R}^{-1}(t,\mathbf{x})}$$

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$$+\frac{1}{2}\,\xi(t,\mathbf{x})\phi(t,\mathbf{x})\partial_s(\ln J)|_{\mathcal{R}^{-1}(t,\mathbf{x})}$$

- A simple example:

Single mirror following a prescribed trajectory

$$R(\bar{t},y) = y + \epsilon g(\bar{t})$$

We explicitly get

$$\mathcal{H}(t,x) = \mathcal{E}(t,x) + \epsilon\dot{g}(t)\xi(t,x)\partial_x\phi(t,x)$$

CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

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Seminal Davis-Fulling model [PRSL A348 (1976) 393] renormalized energy is negative: while the mirror moves, the renormalized energy cannot be considered as the energy of the produced particles at time t [cf. paragraph after Eq. (4.5)]

CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

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$\longrightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$

$\longrightarrow$ Unitary: $S(\omega)S^\dagger(\omega) =$ Id

$\longrightarrow$ The identity at high frequencies: $S(\omega) \to$ Id, when $|\omega| \to \infty$

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$$\langle \hat{F}_{Ha}(t) \rangle = -\frac{\epsilon}{2\pi^2} \int_0^\infty \int_0^\infty \frac{d\omega d\omega' \omega\omega'}{\omega + \omega'} \text{Re} \left[e^{-i(\omega+\omega')t} \, \widehat{\dot{g}\theta_t}(\omega + \omega') \right]$$
$$\times \left[|r(\omega) + r^*(\omega')|^2 + |s(\omega) - s^*(\omega')|^2 \right] + \mathcal{O}(\epsilon^2)$$

Note this integral diverges for a perfect mirror ($r \equiv -1$, $s \equiv 0$, ideal case), but nicely converges for our partially transmitting (physical) one where $r(\omega) \to 0$, $s(\omega) \to 1$, as $\omega \to \infty$

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Energy conservation is fulfilled: the dynamical energy at any time t equals, with the opposite sign, the work performed by the reaction force up to that time t

$$\langle \hat{E}(t) \rangle = -\epsilon \int_0^t \langle \hat{F}_{Ha}(\tau) \rangle \dot{g}(\tau) d\tau$$

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- Been able to prove that the force coincides with the radiation-reaction force calculated by Jaekel and Reynaud after renormalization:

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- Barton and Calogeracos [95,00]: two important differences
  – First, to obtain the Schrödinger eq BC make a unitary transformation not easily generalizable to the case of two moving mirrors
  – Second, a mass renormalization is performed to eliminate the reactive part, where the energy of the field is not a positive quantity for all time t (suffic. small)
  Again, concept of particle not well defined while mirror moves

TWO PARTIALLY TRANSMITTING MIRRORS

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- Trajectories $(t, L_j(t;\epsilon))$, where $L_j(t;\epsilon) \equiv L_j + \epsilon g_j(t),\ j = 1, 2$ and assume $L_1(t;\epsilon) < L_2(t;\epsilon), \forall t \in \mathbb{R}$

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- Consider the change

$$R(\bar{t}, y) = \frac{1}{L_2 - L_1} \left[L_2(\bar{t};\epsilon)(y - L_1) + L_1(\bar{t};\epsilon)(L_2 - y) \right]$$

the Hamiltonian density of the field is then

$$\mathcal{H}(t,x) = \mathcal{E}(t,x) + \sum_{j=1,2} \frac{(-1)^j \dot{L}_j(t;\epsilon)\xi(t,x)}{L_2(t;\epsilon) - L_1(t;\epsilon)} \left[\partial_x \phi(t,x)(x - \bar{L}_j(t;\epsilon)) + \frac{1}{2}\phi(t,x) \right]$$

where $\bar{L}_{\binom{1}{2}}(t;\epsilon) \equiv L_{\binom{2}{1}}(t;\epsilon)$

- In the interaction picture, while mirrors move, the full Hamiltonian is

$$\hat{H}_I(t) = -\frac{\epsilon(g_2(t) - g_1(t))}{L_2 - L_1}\left[\int dy\left(\partial_y \widehat{\widetilde{\phi}}_I(y)\right)^2 + \sum_{j=1,2}\alpha_j\left(\widehat{\widetilde{\phi}}_I(L_j)\right)^2\right]$$

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- There are proposals to detect the radiated photons: Kim, Brownell, Onofrio, PRL 96 (2006) 200402

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- Calculate the radiation emitted by the mirror from its back (right) side
- As is well-known, a perfect mirror that follows this kind of trajectory produces a thermal emission of scalar massless particles obeying Bose-Einstein statistics:
  for $1 \ll \omega'/k \ll e^{ku_0}$ and $1 \ll \omega'/\omega \ll e^{ku_0}$, one has

$$\left|\beta^{R,R}_{\omega,\omega'}\right|^2 \equiv \left|(\phi^{out}_{\omega,R}{}^*;\phi^{in}_{\omega',R})\right|^2 \cong \frac{1}{2\pi\omega' k}\left(e^{2\pi\omega/k} - 1\right)^{-1}$$

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- Obtain the 'in' modes on the rhs of the mirror when the reflection and transmission coeffs are

$$r(w) = \frac{-i\alpha}{\omega + i\alpha}, \qquad s(w) = \frac{\omega}{\omega + i\alpha}$$

with $\alpha \geq 0$, that is, when the Lagrangian density is

$$\mathcal{L} = \frac{1}{2}[(\partial_t \phi)^2 - (\partial_x \phi)^2] - \alpha\sqrt{1 - \dot{g}^2(t)}\phi^2 \delta(x - g(t))$$

being $x = g(t)$ the trajectory in the (t, x) coordinates

The Main Results

Some of them quite remarkable indeed

(for $1 \ll \omega'/k \ll e^{ku_0}$ and $1 \ll \omega'/\omega \ll e^{ku_0}$)

- In the perfectly reflecting case, i.e., when $\omega' \ll \alpha$, we obtain

$$\left|\beta^{R,R}_{\omega,\omega'}\right|^2 = \frac{1}{2\pi\omega' k} \cong \left(e^{2\pi\omega/k} - 1\right)^{-1}, \qquad \left|\beta^{R,L}_{\omega,\omega'}\right|^2 \cong 0$$

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- In the physically more realistic case of a partially transmitting mirror (transparent to high enough frequencies, i.e., when $\alpha \ll \omega'$, what we obtain is

$$\left|\beta^{R,R}_{\omega,\omega'}\right|^2 \cong \frac{1}{2\pi\omega k}\left(\frac{\alpha}{\omega'}\right)^2\left(e^{2\pi\omega/k}+1\right)^{-1}$$

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- And, since $\left|\beta^{R,L}_{\omega,\omega'}\right| \ll \left|\beta^{R,R}_{\omega,\omega'}\right|$, we conclude quite surprisingly that a semitransparent mirror emits a thermal radiation of scalar massless particles obeying Fermi-Dirac statistics

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- The physical reason of the remarkable change of statistics that takes place remains, as of now, a mystery. It might well find application in other situations, including perhaps black hole physics

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  T. Padmanabhan; D. Blas, J. Garriga, ...
- Ted Jacobson [PRL1995] obtained Einstein's equations from local thermodynamics arguments only
- By way of generalizing black hole thermodynamics to space-time thermodynamics as seen by a local observer
- This strongly suggests, in a fundamental context:
  Einstein's Eqs are to be viewed as EoS
- Should, probably, not be taken as basic for quantizing gravity
- C. Eling, R. Guedens, T. Jacobson [PRL2006]: extension to polynomial $f(R)$ gravity but as non-equilibrium thermodyn.
  Also Erik Verlinde (personal discussions)

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T \delta S$$

  - entropy proport to variation of the horizon area: $\delta S = \eta \, \delta \mathcal{A}$
  - local temperature T is defined as the Unruh temperature
  - functional dependence of S wrt energy and size of system

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- Key point in our generalization: the definition of the local entropy

$$\delta S = \delta\left(\eta_e A\right)$$

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- Case of $\mathbf{f}(R)$ gravities:

$$\mathbf{L} = \mathbf{f}(R, \nabla^n R)$$

- Also the concept of an effective Newton constant for graviton exchange (effective propagator)

$$\begin{aligned}\frac{1}{8\pi G_{eff}} &= E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs} \\ &= \frac{\partial \mathbf{f}}{\partial R} = \frac{\eta_e}{2\pi}\end{aligned}$$

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- Final result, for $\mathbf{f}(R)$ gravities:
  the local field equations can be thought of as an equation of state of equilibrium thermodynamics (as in the GR case)

- Jacobson's argum non-trivially extended to $f(R)$ gravity field eqs as EoS of local space-time thermodynamics
  EE, P. Silva, Phys Rev D78, 061501(R) (2008), arXiv:0804.3721v2

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Danke Schön!