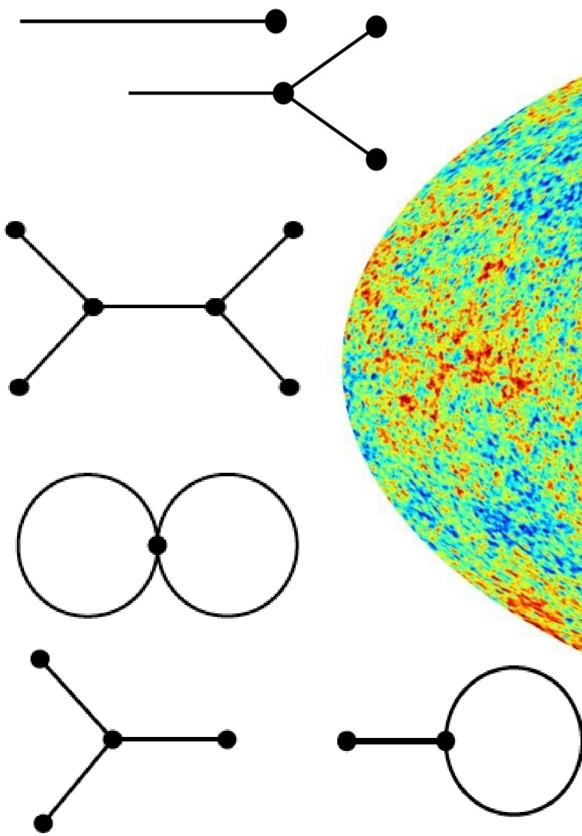
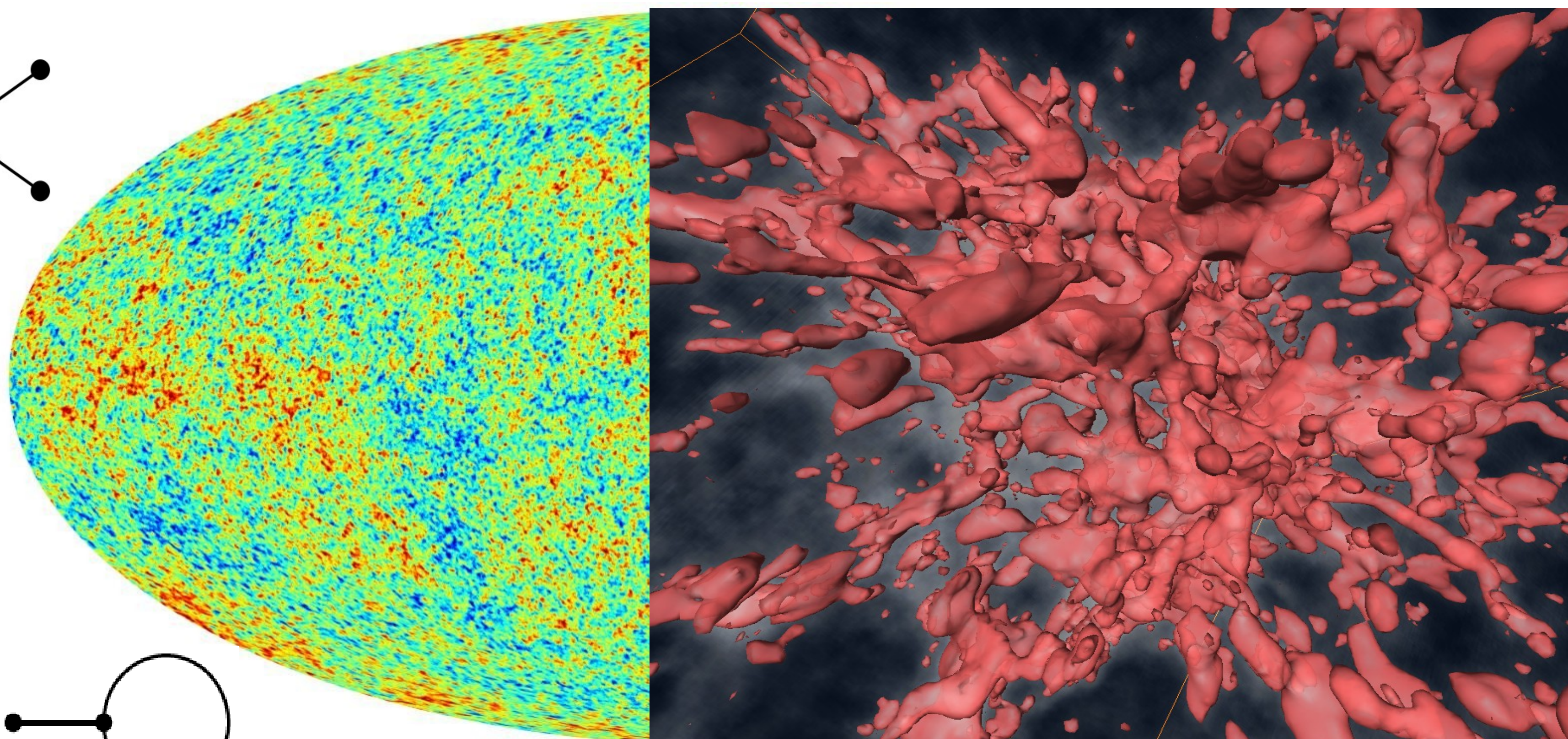
Information Field Theory

for cosmological pertubation reconstructions & signal analysis

Torsten Enßlin, Mona Frommert, Francisco Kitaura, Jens Jasche





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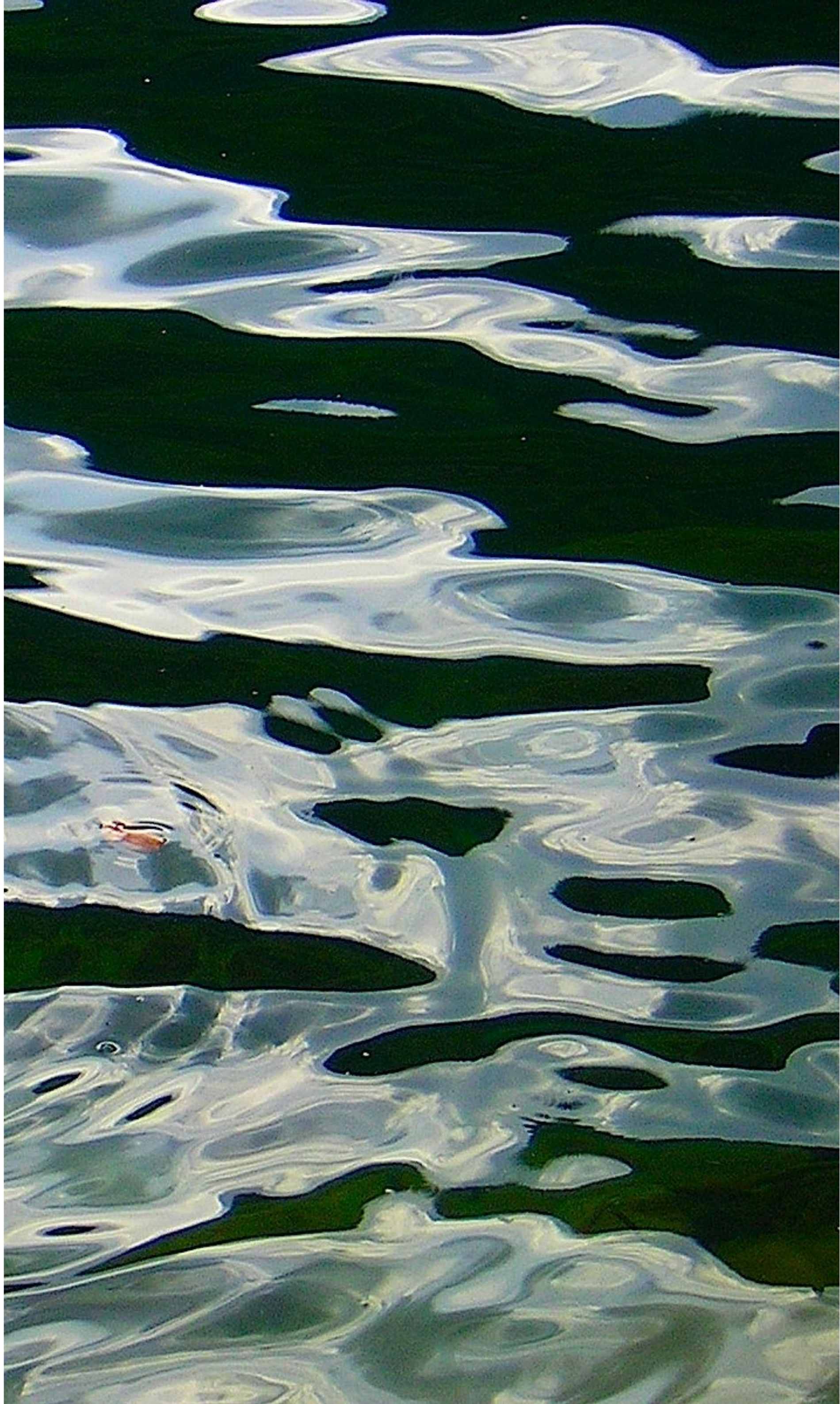
Why Information Field Theory ?

inverse problem => Information Theory

spatially distributed quantity => Field Theory

Information fields of interest:

- primoridial space-time metric
- inflaton quantum fluctuations
- cosmic matter distribution
- power spectra, ...



Information fields of interest:

- primoridial space-time metric
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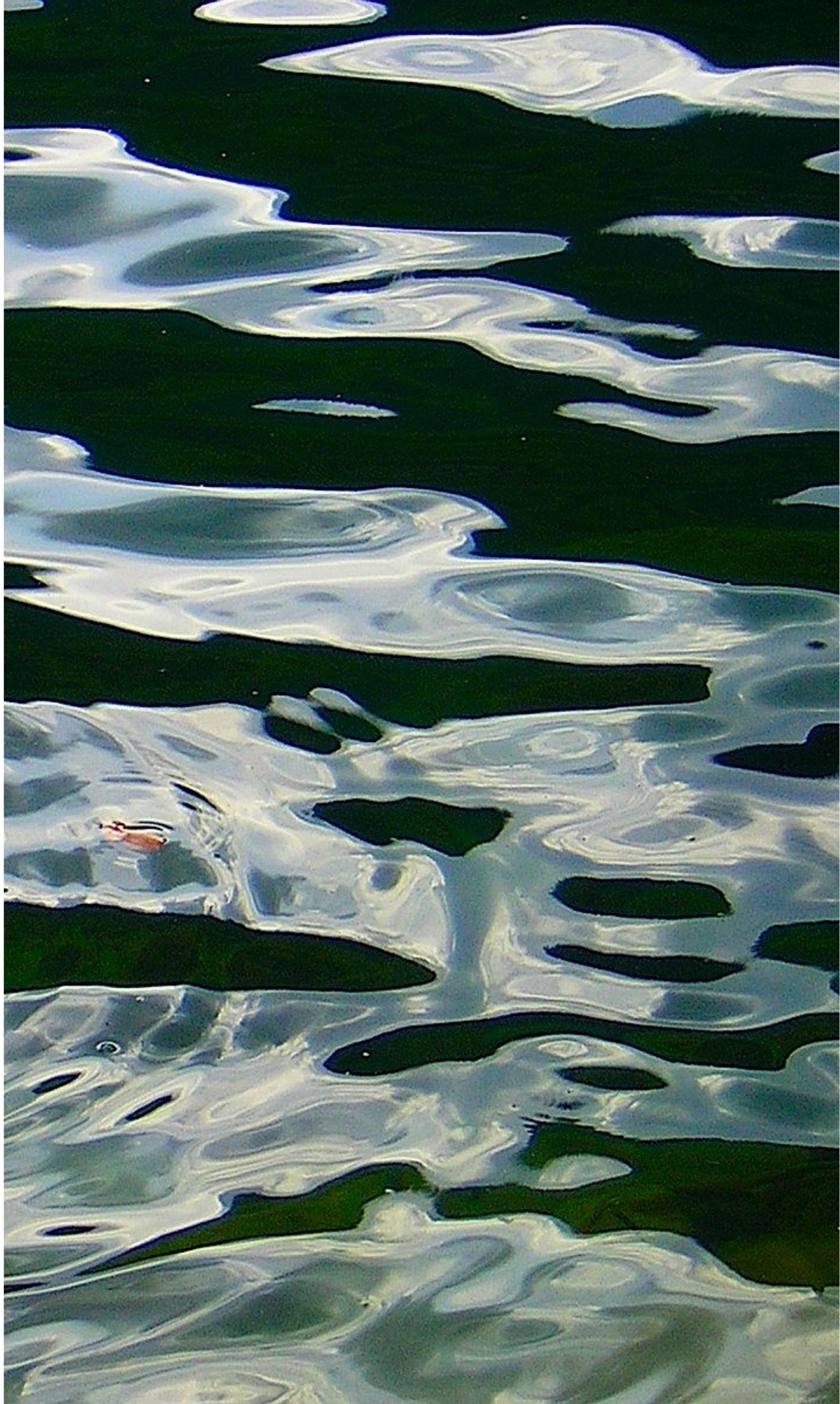
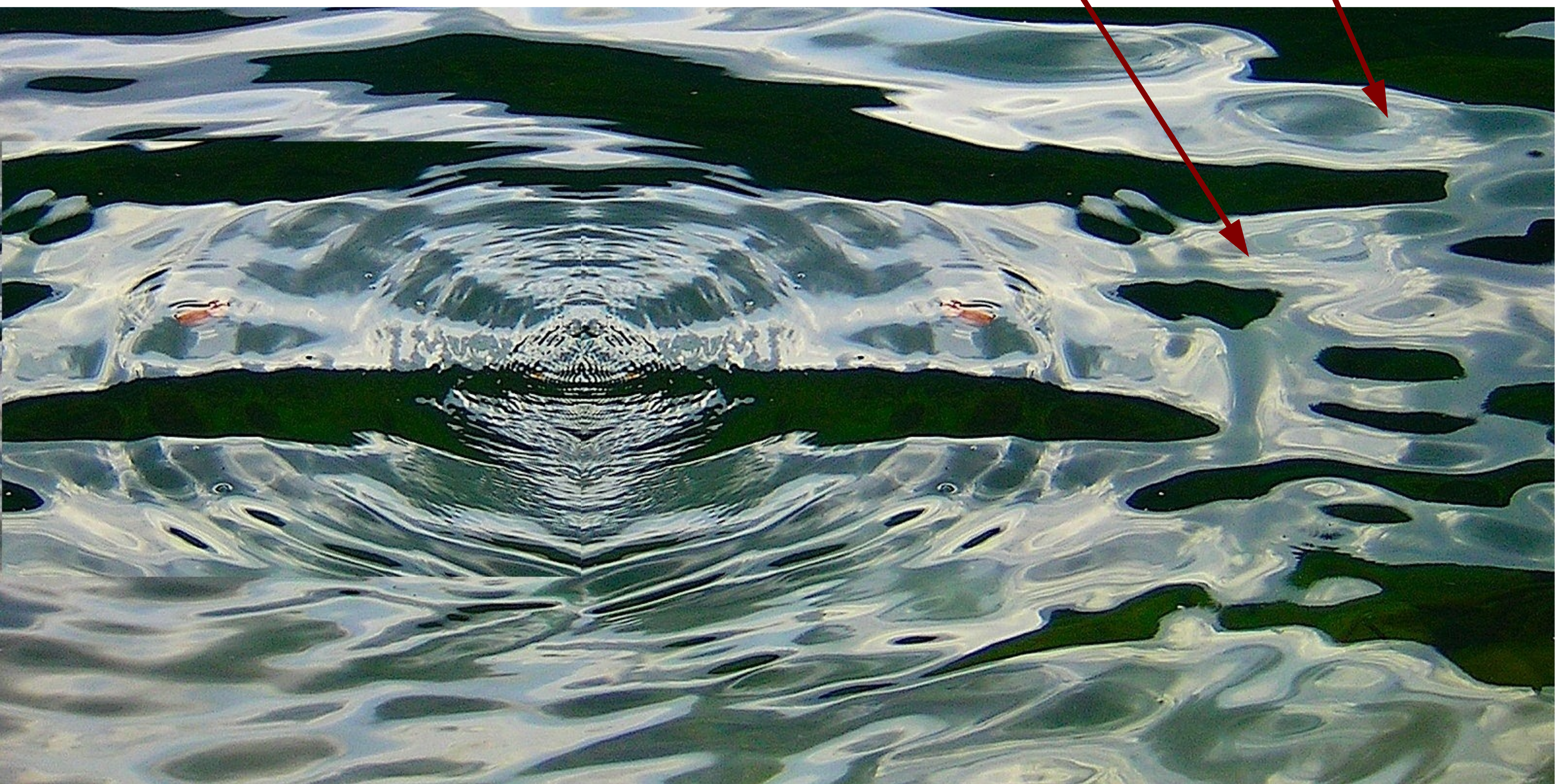
Information sources:

- galaxy surveys
- CMB temperture fluctuations
- gravitaional lensing signal
- intergalactic absorption lines

...



$$
\begin{aligned}
\langle s(x_1)\cdots s(x_n)\rangle_d &\equiv \langle s(x_1)\cdots s(x_n)\rangle_{(s|d)}\\
&\equiv \int \mathcal{D}s\, s(x_1)\cdots s(x_n)\, P(s|d)
\end{aligned}
$$



$$\begin{aligned}\langle s(x_1)\cdots s(x_n)\rangle_d &\equiv \langle s(x_1)\cdots s(x_n)\rangle_{(s|d)} \\ &\equiv \int \mathcal{D}s\, s(x_1)\cdots s(x_n)\, P(s|d)\end{aligned}$$

$$\int \mathcal{D}f\, F[f] \equiv \left(\prod_{i=1}^{N_{\text{pix}}} \int df_i\right) F(f_1, \ldots, f_{N_{\text{pix}}})$$

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$$P(s|d) = \frac{P(d|s)\,P(s)}{P(d)}$$

$$\overbrace{P(s|d)}^{\text{posterior}} = \frac{\overbrace{P(d|s)}^{\text{likelihood}}\,\overbrace{P(s)}^{\text{prior}}}{\underbrace{P(d)}_{\text{evidence}}}$$

posterior likelihood prior

$$P(s|d) = \frac{P(d|s)\,P(s)}{P(d)} \equiv \frac{1}{Z}\,e^{-H[s]}$$

evidence

Information Hamiltonian

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Information Hamiltonian

posterior

likelihood

prior

$$P(s|d) = \frac{P(d|s)\,P(s)}{P(d)} \equiv \frac{1}{Z}\,e^{-H[s]}$$

evidence

Gaussian signal & noise, linear data model: $d = R\,s + n$

Information Hamiltonian

posterior

likelihood

prior

$$P(s|d) = \frac{P(d|s)\,P(s)}{P(d)} \equiv \frac{1}{Z}\,e^{-H[s]}$$

evidence

Gaussian signal & noise, linear data model: $d = R\,s + n$

$$\begin{aligned} H_{\mathrm{G}}[s] &= -\log\left[P(d|s)\,P(s)\right] = \\ &= \frac{1}{2}s^\dagger D^{-1} s - j^\dagger s + H_0^{\mathrm{G}} \end{aligned}$$

$$j = R^\dagger N^{-1} d \qquad\qquad D = \left[S^{-1} + R^\dagger N^{-1} R\right]^{-1}$$

$$m = \langle s \rangle_{(s|d)} = D\, j$$

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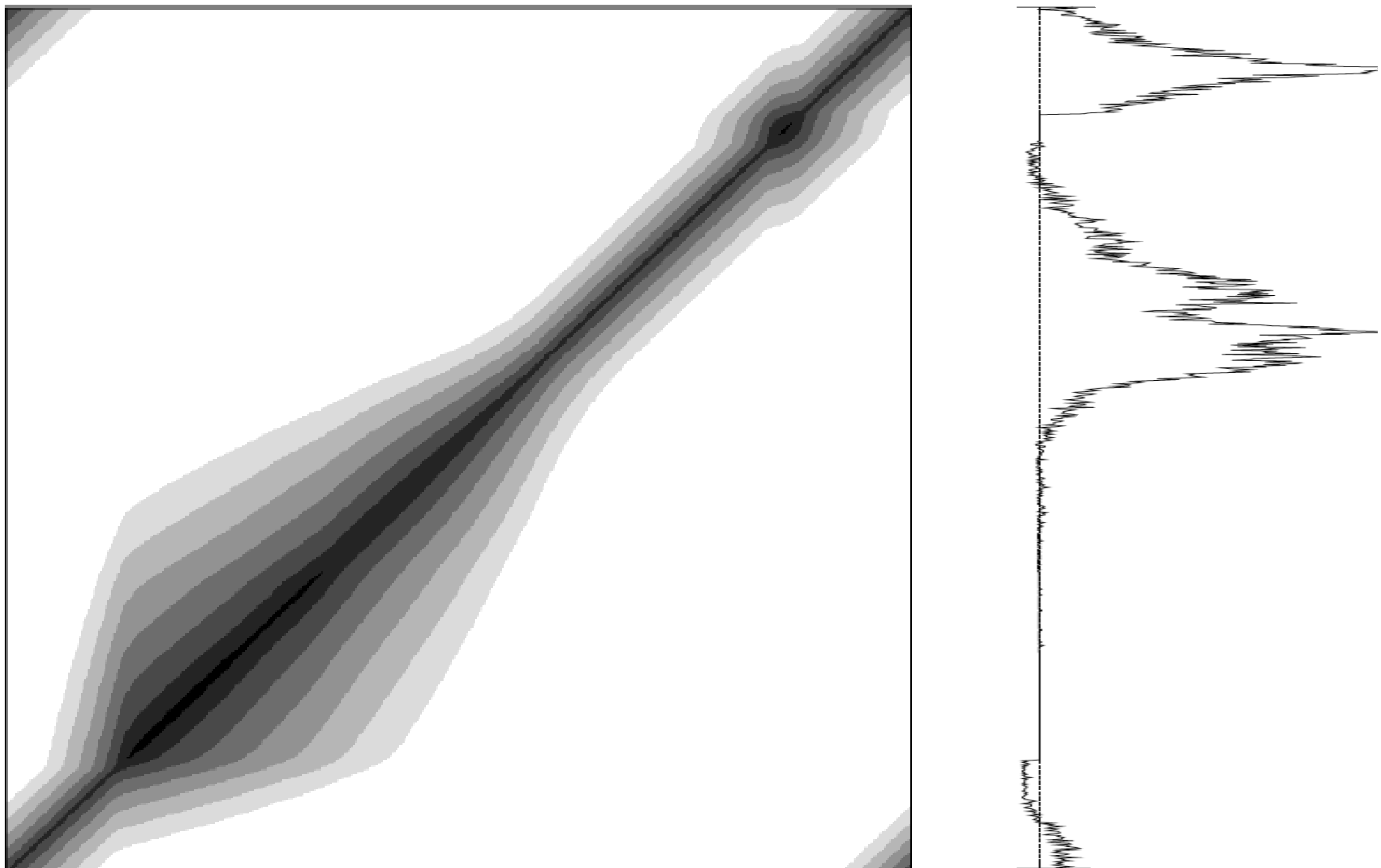

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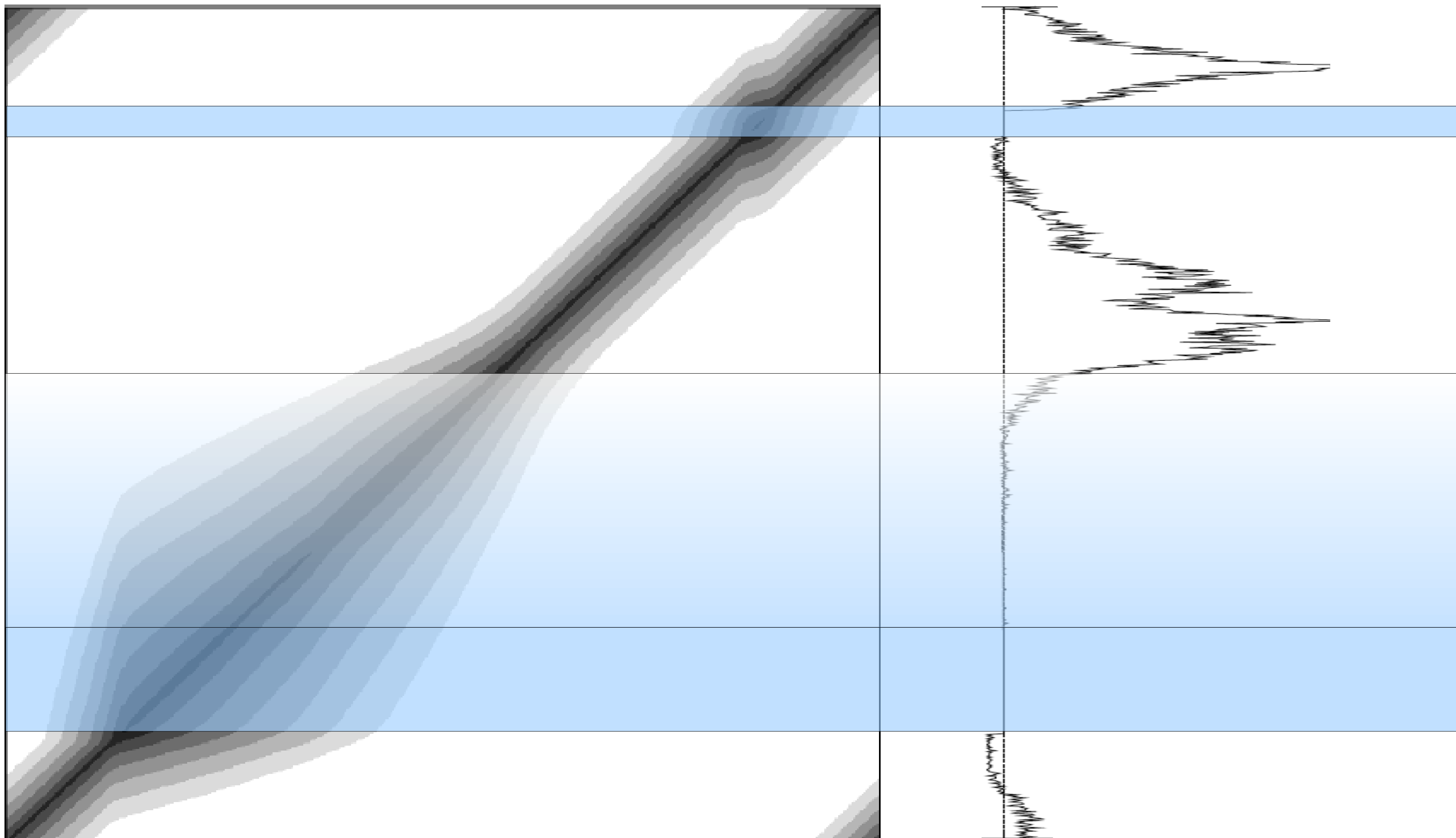

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$$
\begin{aligned}
m = \langle s \rangle_{(s|d)} &= \; D\, j = D_{xy}\, j_y \\
&\equiv \; \int dy\, D(x,y)\, j(y)
\end{aligned}
$$

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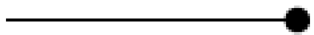 $$\equiv \int dy\, D(x,y)\, j(y)$$

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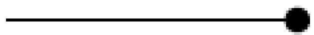

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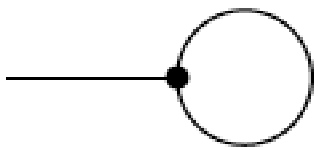

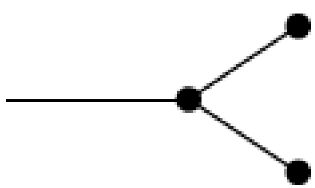

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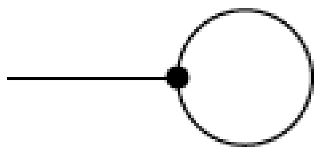

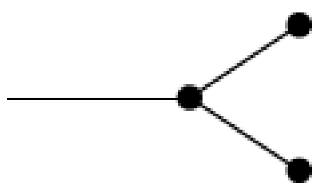

$$= -\frac{1}{2} D_{xy}\,\Lambda^{(3)}_{yuz}\,D_{zz'}\,j_{z'}\,D_{uu'}\,j_{u'}$$

$$H[s] = \frac{1}{2}\,s^{\dagger} D^{-1}\,s - j^{\dagger} s + H_0 + \sum_{n=3}^{\infty} \frac{1}{n!}\,\Lambda^{(n)}_{x_1 \ldots x_n}\,s_{x_1} \cdots s_{x_n}$$

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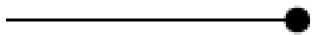

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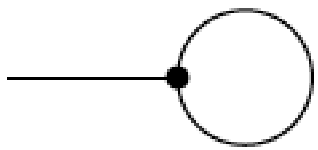

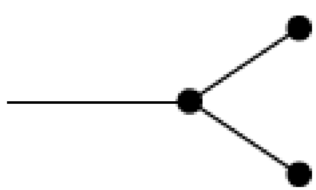

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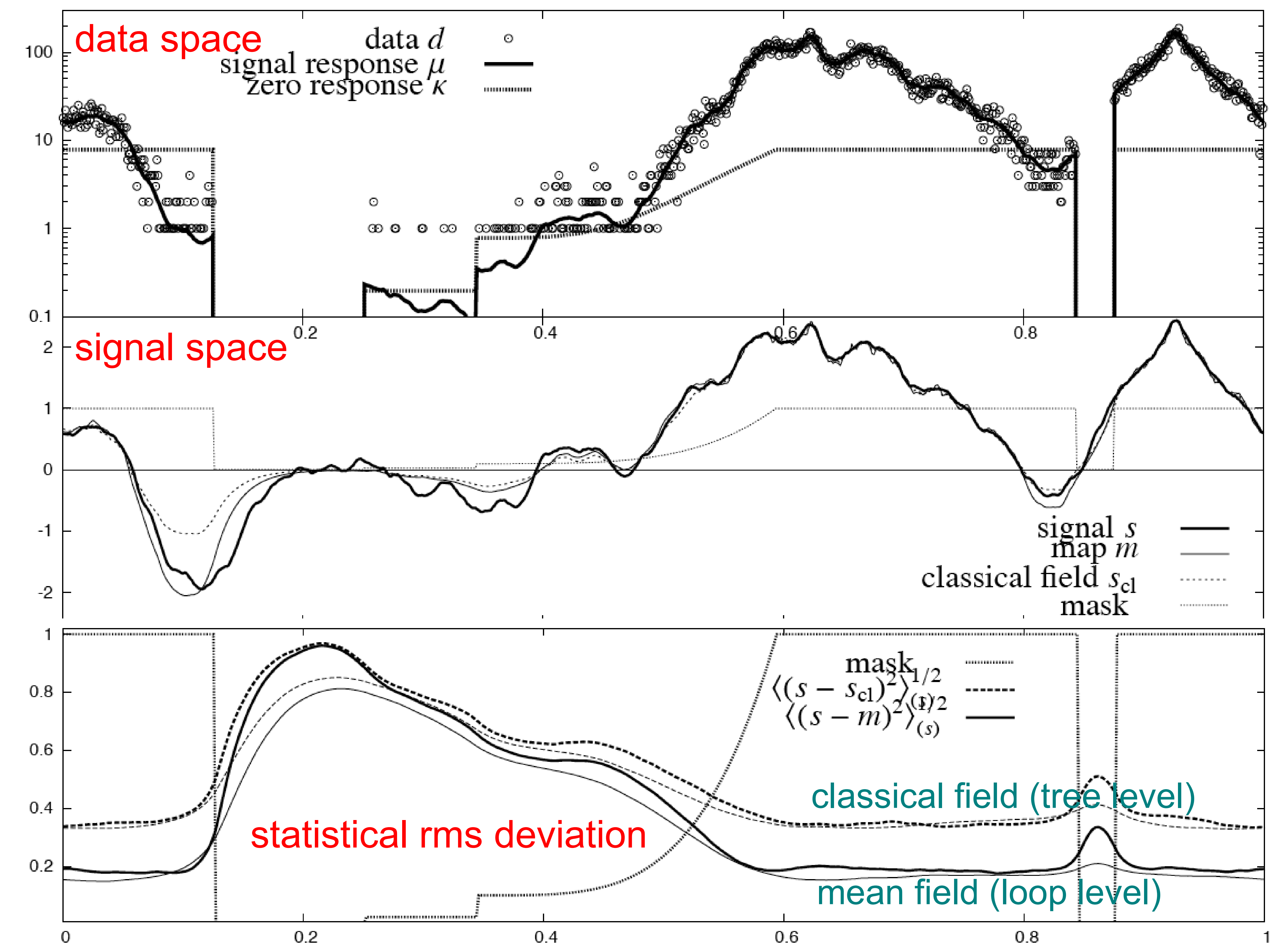
Optimal non-linear inversion algorithms from linear filtering !!!



data space
data d
signal response μ
zero response κ
100
10
1
0.1
signal space
signal s
map m
classical field $s_{\rm cl}$
mask
2
1
0
-1
-2
mask
$\langle (s - s_{\rm cl})^2 \rangle^{1/2}_{(s)}$
$\langle (s - m)^2 \rangle^{1/2}_{(s)}$
classical field (tree level)
statistical rms deviation
mean field (loop level)
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1

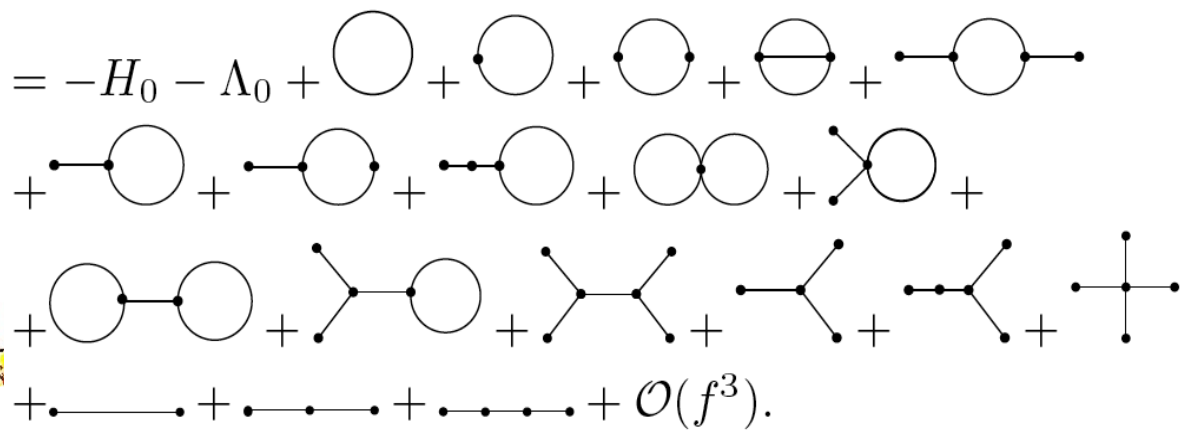
Applications:

Applications:

- CMB non-Gaussianity likelihood & optimal estimator
  for f_{nl}-theory, Enßlin et al. (arXiv:0806.3474)
- ISW likelihood (CMB & galaxies)& optimal estimator
  S/N improvement by being conditional on the known LSS
  Frommert et al. (arXiv:0807.0464)
- Wiener reconstruction of large-scale structure (LSS)
  as traced by the 6^{th} SDSS galaxy catalog, Kitaura et al. (2008, & in prep.)
- Joined LSS-map and power-spectrum reconstruction
  for BAO measurement without geometry assumptions, Jasche et al. (in prep.)

Applications:

$$\log Z_f[d] = \log \int \mathcal{D}\phi \, P(\phi, f|d)$$
$$= \log \int \mathcal{D}\phi \, e^{-H_f[d,\phi]}$$
$$= -H_0 - \Lambda_0 + \ldots$$



$$+ \mathcal{O}(f^3).$$

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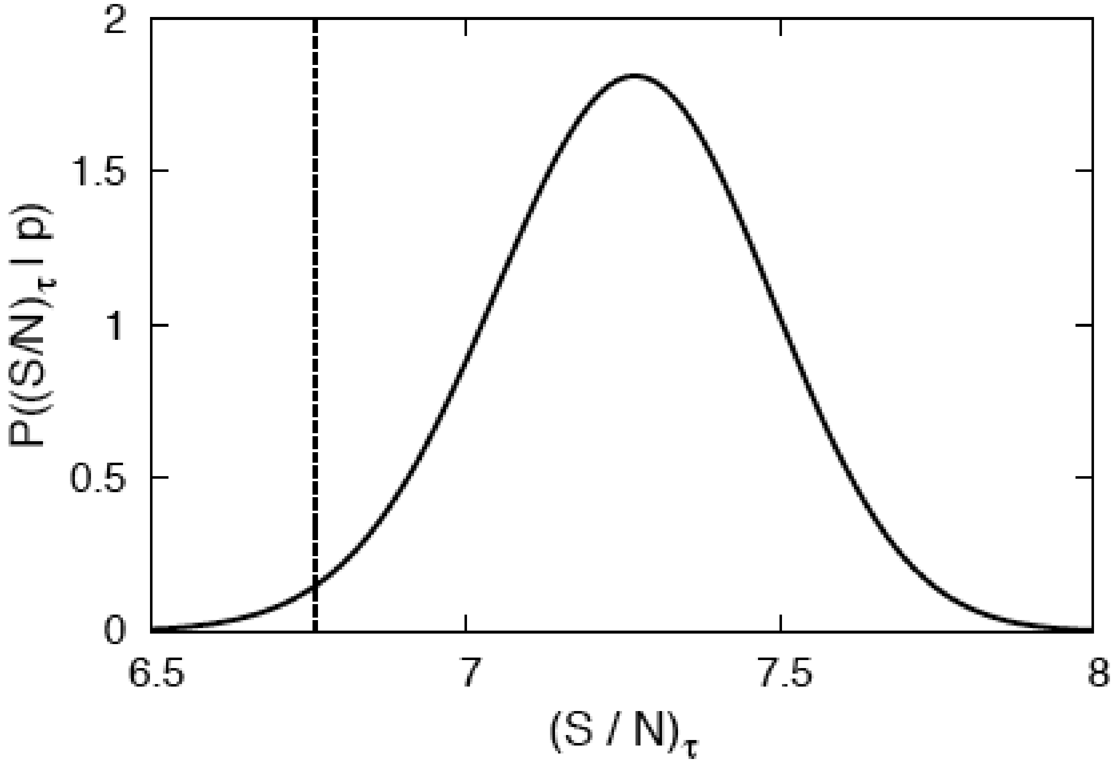

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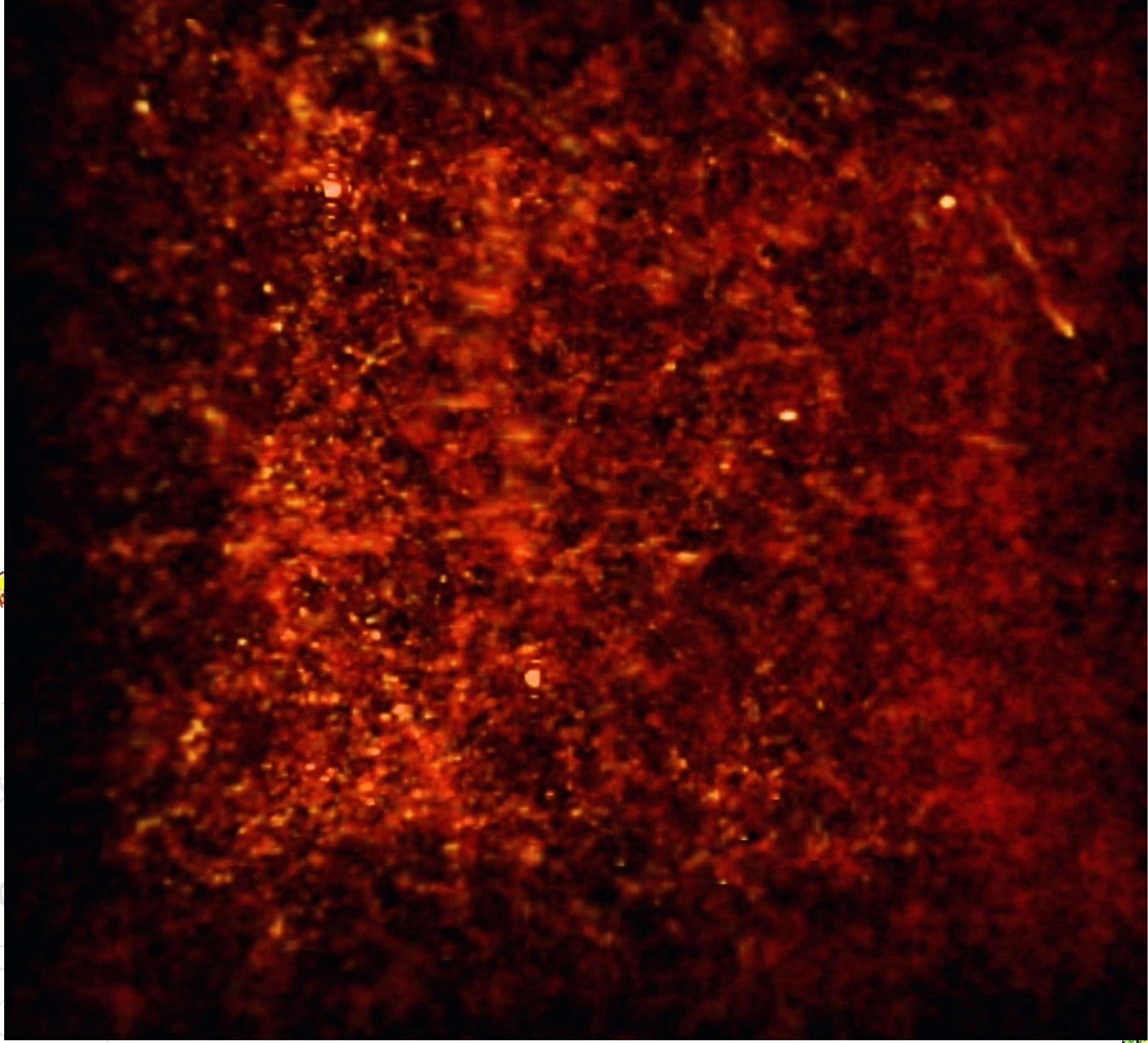

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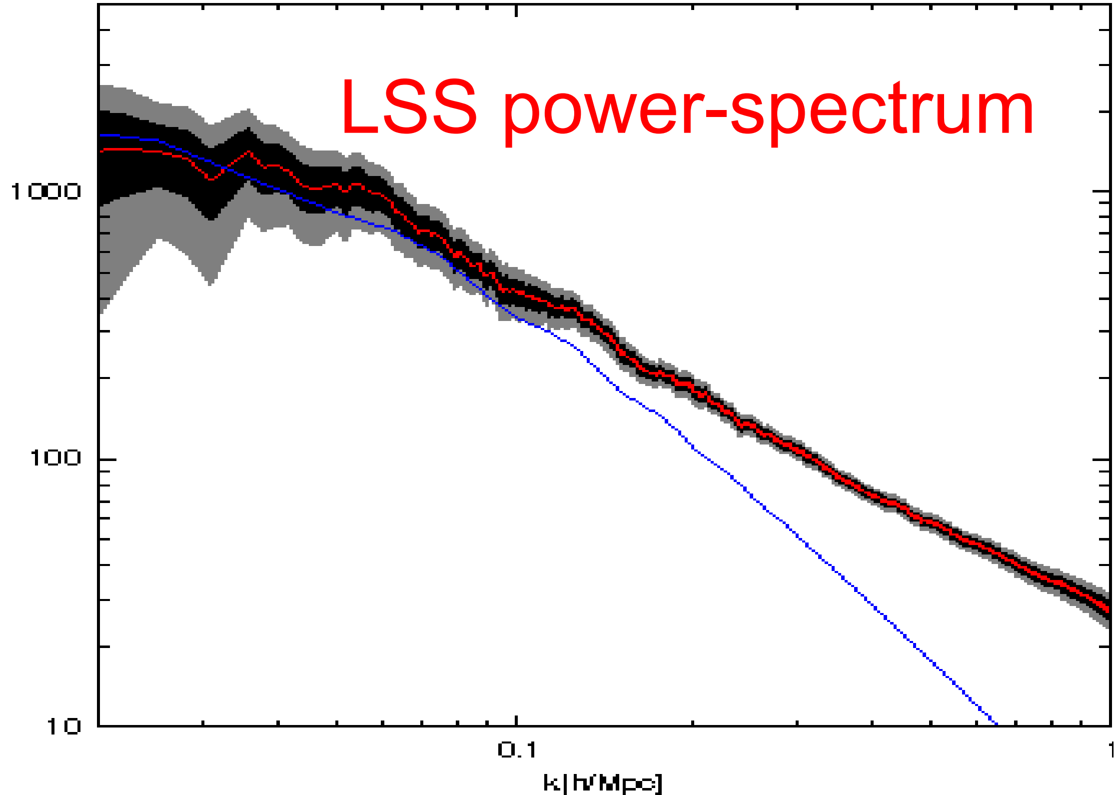

- CMB non-Gaussianity likelihood & optimal estimator
  for f_{nl}-theory, Enßlin et al. (arXiv:0806.3474)
- ISW likelihood (CMB & galaxies)& optimal estimator
  S/N improvement due to benefiting from cosmic variance
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- Wiener reconstruction of large-scale structure (LSS)
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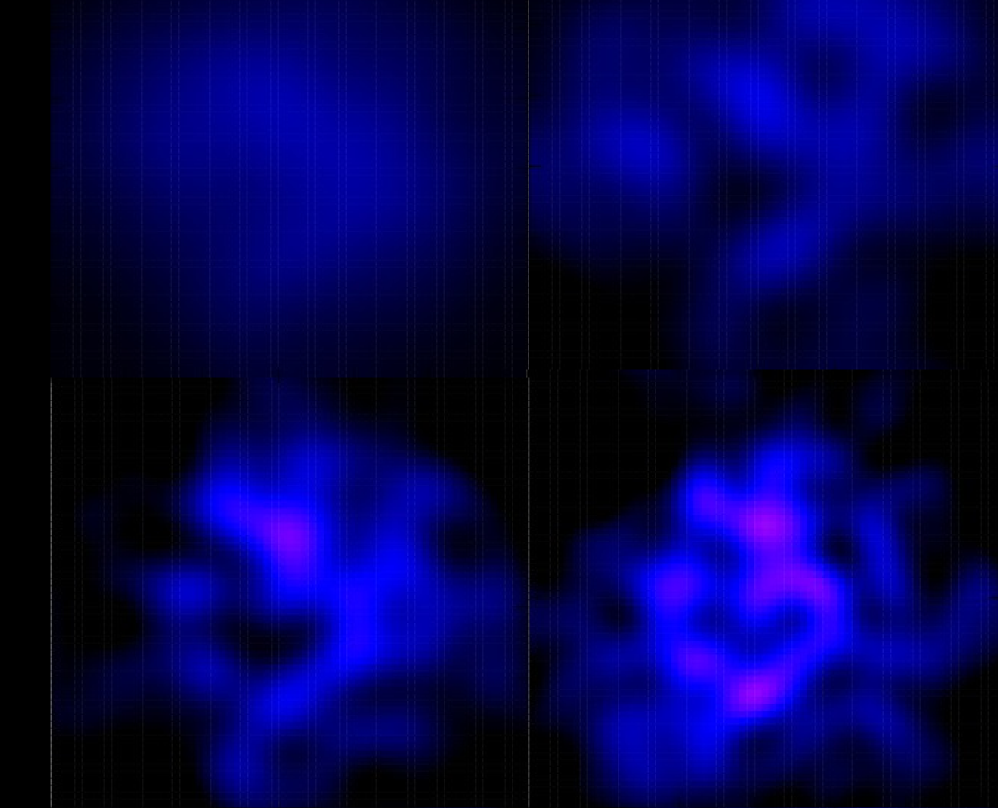

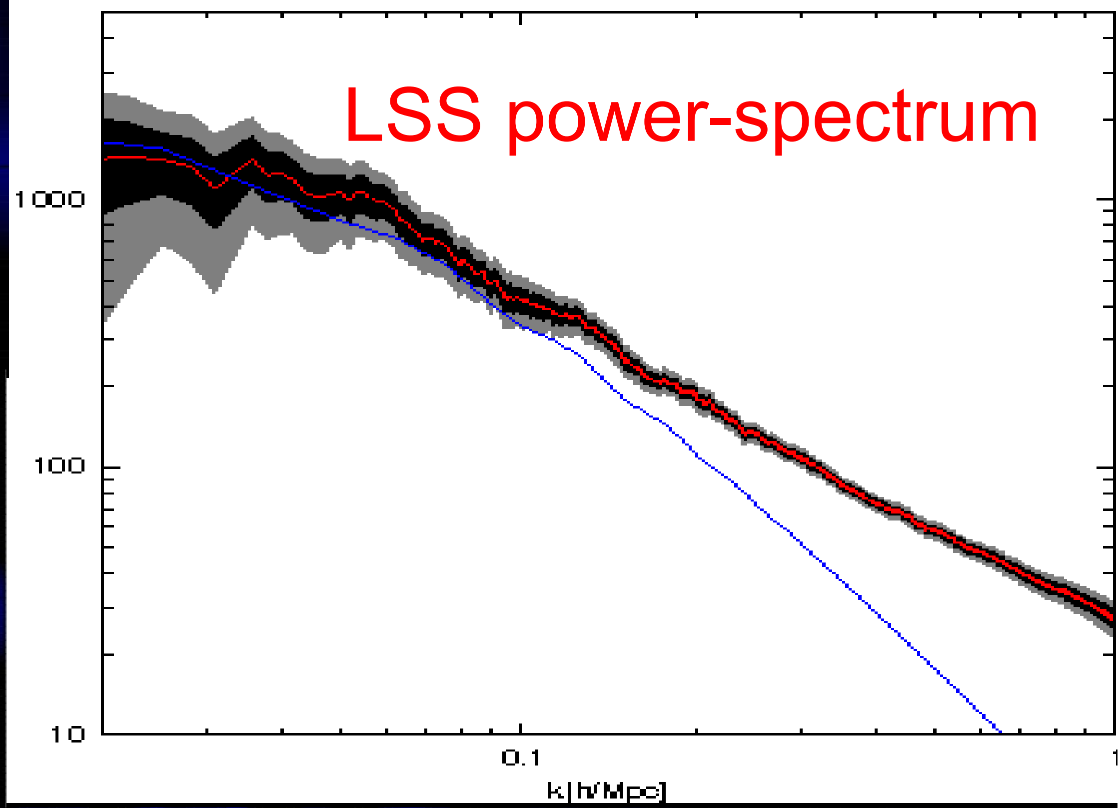

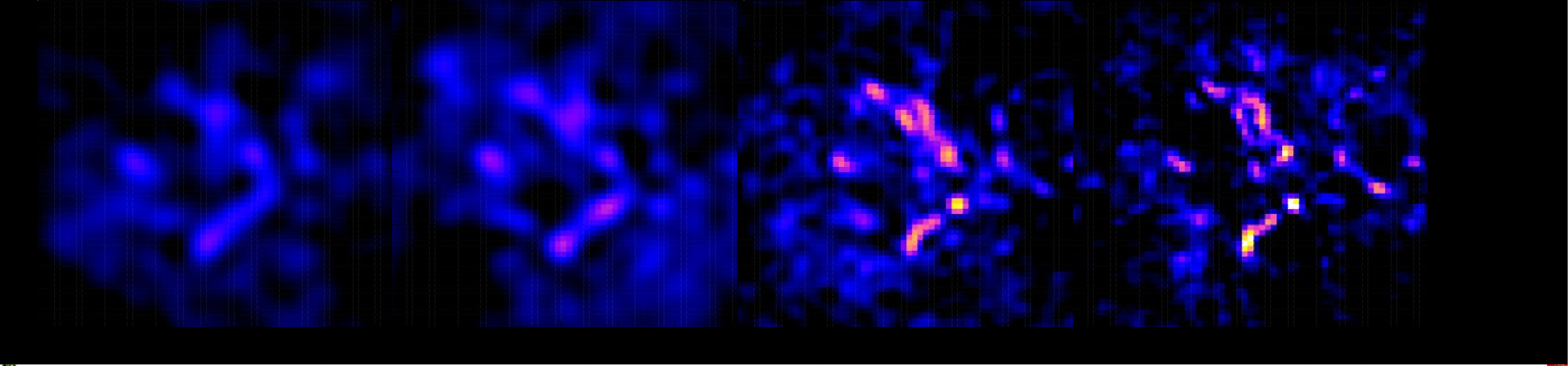

→ Joined LSS-map and power-spectrum reconstruction

for BAO measurement without geometry assumptions, Jasche et al. (in prep.)

Conclusions:

- inverse problems for physical fields => Information Field Theory
- IFT is mathematically a statistical field theory
  peculiarities: non-locality, high-order interactions, well normalised
- free IFT is equivalent to Wiener filtering
  => numerical implementation of propagators are available
- non-linear problems can be treated by diagrammatic perturbations
  => mean signal, n-point correlation functions, evidence obtainable
- cosmological applications:
  - CMB non-Gaussianities
  - optimal signal detection (ISW, ...)
  - cosmography
  - power spectrum reconstruction (BAOs, ...)
  - data fusion in signal space
  - your signal of choice

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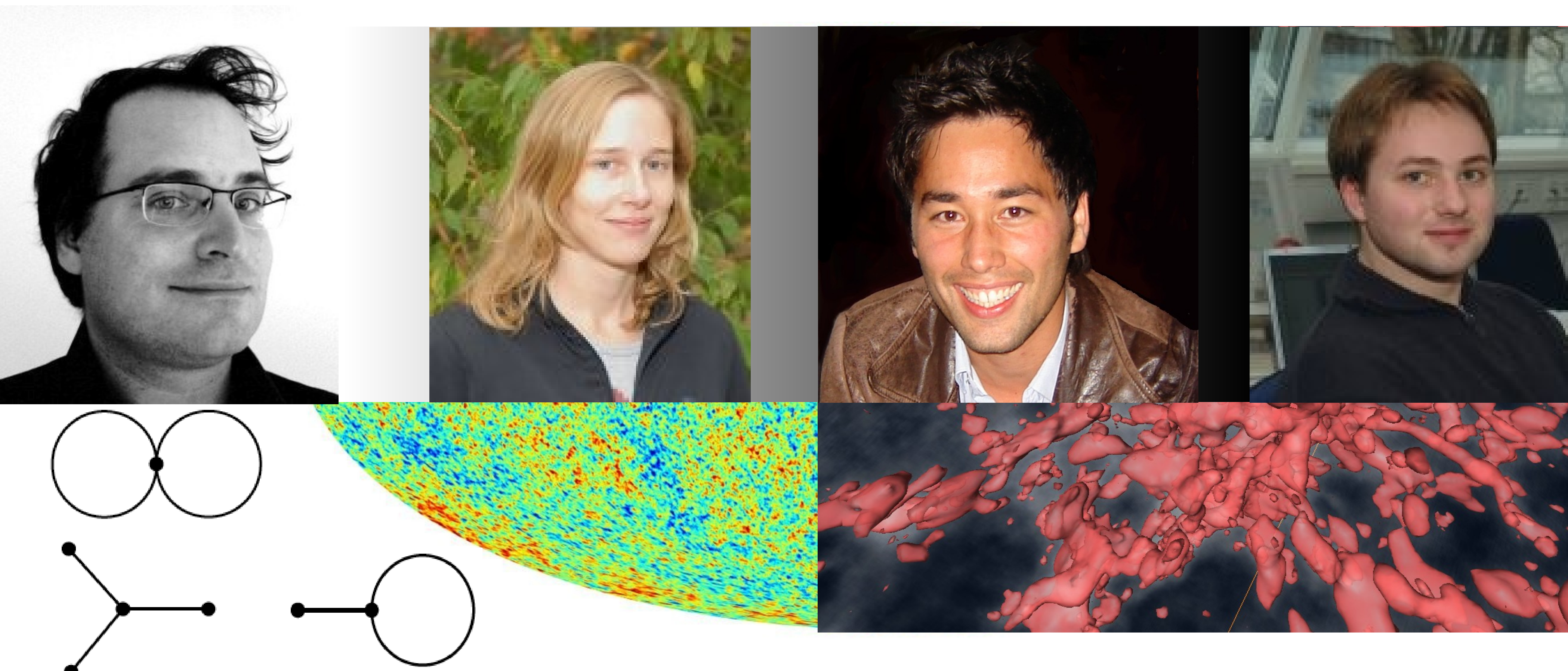

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