Constraining the Dark Energy Potential

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ABSTRACT

We generalize to non-flat geometries the formalism of [1] to reconstruct the dark energy potential. Since present and forthcoming data do not allow an exact non-parametric reconstruction of the potential, we consider a general parametric description in term of Chebyshev polynomials. We consider present and future measurements of *H(z),* Baryon Acoustic Oscillations and Supernovae type IA surveys and investigate their constraints on the dark energy potential. We find that, relaxing the flatness assumption increases the errors on the reconstructed dark energy evolution but does not open up significant degeneracies, provided that a prior on geometry is imposed. Direct measurements of *H(z),* such as those provided by BAO surveys, are crucially important to constrain the evolution of the dark energy equation of state, especially for non-trivial deviations from the standard ΛCDM model.

1. How to Reconstruct the Dark Energy Potential

Recent observations indicate that the present-day energy density of the Universe is dominated by a "dark energy" component, responsible for its current accelerated expansion. Even if the observations are compatible with a cosmological constant, dark energy can also be explained by a slowly rolling scalar field. To improve our understanding of dark energy it is important to test its dynamical nature and to try to reconstruct the possible shapes of the potential so as to discriminate among different physical models.

A scalar field ϕ contributes to the pressure and energy as $\rho = K(\phi) + V(\phi)$ and $p = K(\phi) - V(\phi)$ with $K(\phi) = \dot{\phi}^2/2$, the Friedmann's equations read:

$$H^2 = \frac{\kappa}{3}(\rho + V + K) - k\frac{c^2}{a^2} \qquad \frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho + 3p + 4K - 2V(\phi))$$

Thus:

$$3H^2(z) - \frac{1}{2}(1+z)\frac{dH^2(z)}{dz} = \kappa\left(V(z) + \frac{1}{2}\rho_m(z) + \frac{2}{3}\rho_k(z)\right)$$

In order to reconstruct *V(z),* both *H(z)* and its derivative are then needed. We instead expand the potential in Chebyshev polynomials:

$$V(z) \approx \sum_{n=0}^{N} \lambda_n T_n\left(2\frac{z}{z_{\max}} - 1\right)$$

So that the potential can be recovered from *H(z):*

$$H^2(z) = (1+z)^6 H_0^2\left[1 - 3z_{\max}\sum_{n=0}^{N}\frac{\lambda_n}{\rho_c}F_n(z) - \Omega_{m,0}\left(1 - \frac{1}{(1+z)^3}\right) - \Omega_{k,0}\left(1 - \frac{1}{(1+z)^4}\right)\right]$$

with $\quad F_n(z) \approx \int T_n(z)(1+z)^{-7}$

3. Results

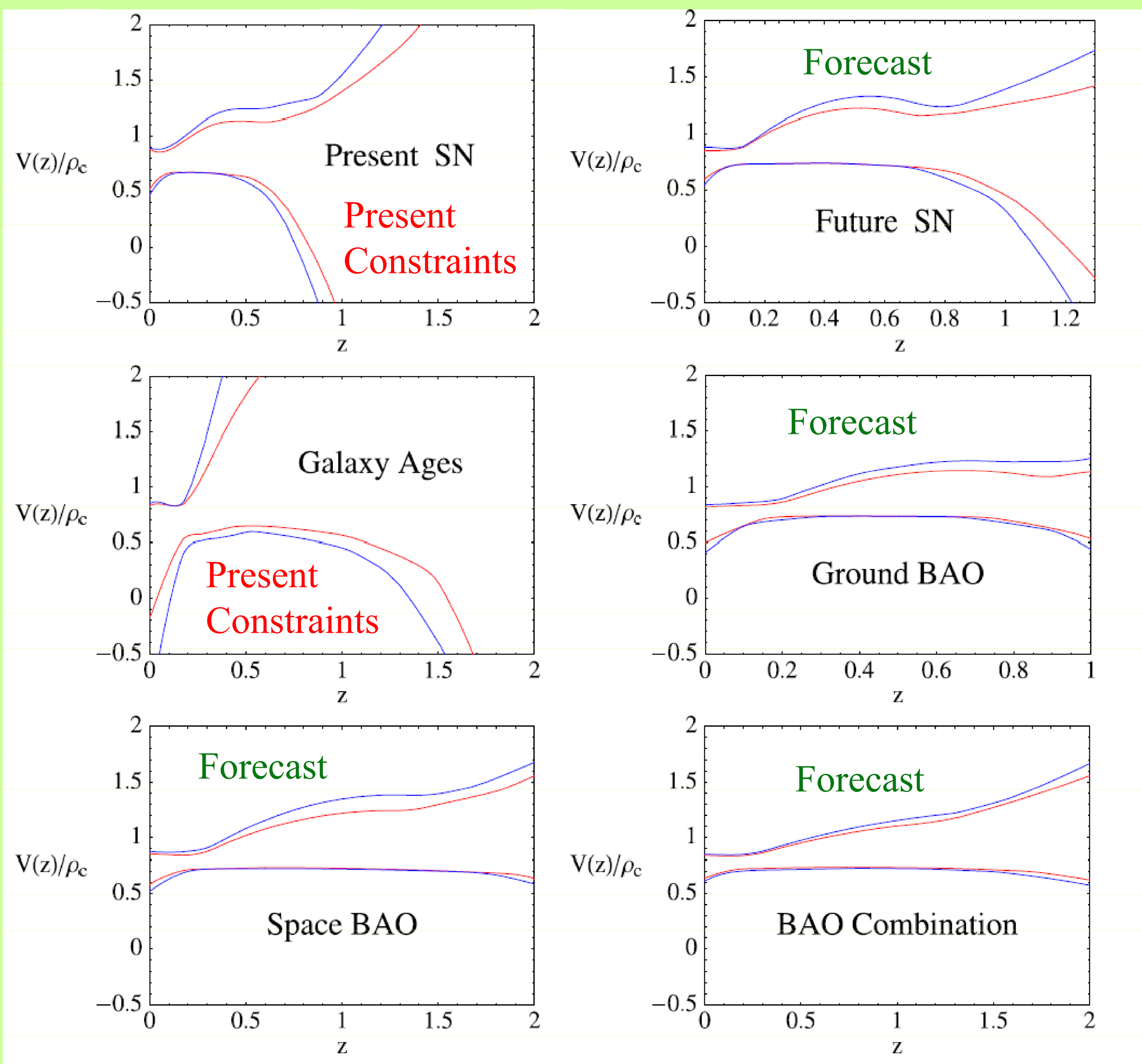

Figure 1. 1 and 2 σ constraints on the reconstructed potential as a function of the redshift *V(z)*, from present SN data (top left), SN data forecasted from a future space-based survey (top right), galaxy ages (middle left), a "ground" based BAO survey (middle left) and a"space" based BAO survey (bottom left).

For the different datasets we compute the constraints on the first three coefficients of the Chebyshev expansion. This constraints are derived exploring the likelihood surface via a Markov Chain Monte Carlo. The constraints on the λ_i derived can then be translated into bounds on *V(z).* Fig 1 shows the results of this reconstruction.

Notice that for all the datasets the bounds are strongest for $z \sim 0.1$ - 0.3 since at larger redshifts dark energy becomes subdominant. This tightening of the priors at low redshifts translates into a strong linear degeneracy between the first two coefficients of the Chebyshev expansion, present for all the datasets considered. Indeed, such a degeneracy implies:

$$\lambda_0 = \alpha\lambda_1 + \beta$$

In all cases we found $\beta \sim \Omega_\Lambda$, and $\alpha < 1$. Thus

$$V(z) = (\alpha-1)\lambda_1 + \Omega_\Lambda + \lambda_1 \cdot 2z/z_{\max}$$

So that, $V = \Omega_\Lambda$ for $z = z_{\max}(1-\alpha)/2$ regardless of λ_1.

In Fig. 2 we show an example of the bounds derived for the first three coefficients of the Chebyshev expansion of the potential with forecasted BAO data. The left plots correspond to bounds derived with information on d_A alone (extracted from the measurement of the BAO scale across the line sight) and the right plots to information on *H(z)* (from line of sight measurements of the BAO scale). Notice the much tighter constraints derived from *H(z)* due to its more direct relation to *V(z),* which in $d_A(z)$ is diluted in an integral.

Notice also the strong linear degeneracy among the two first coefficients due to the tighter bounds achievable at lower redshifts.

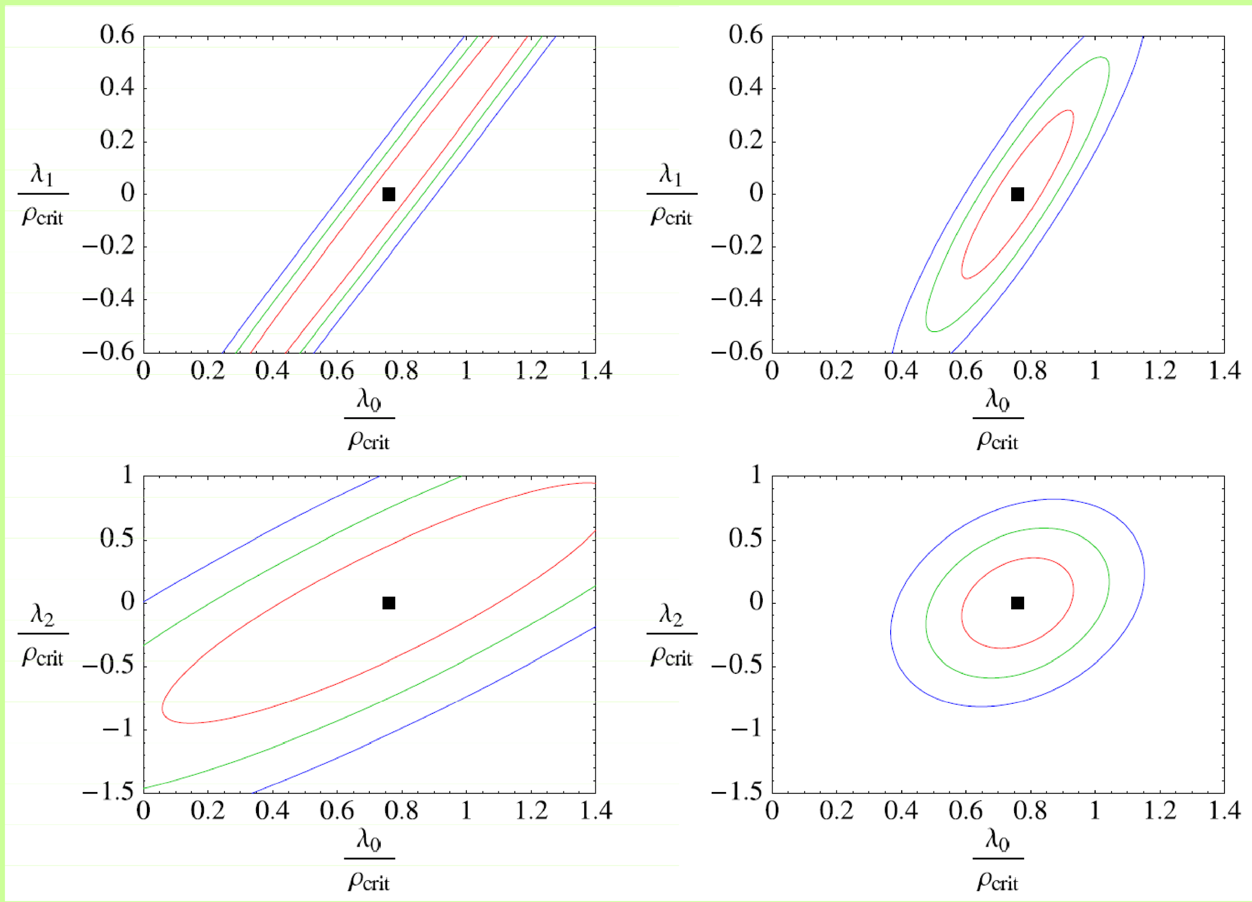

Figure 2. 1, 2 and 3 σ constraints on the first three coefficients of the Chebyshev expansion of V(z) from measurements of d_A (left) and *H(z)* (right) alone at a "ground" BAO survey. Notice the much tighter bounds places by the constraints on *H(z)* compared to the $d_A(z)$ ones.

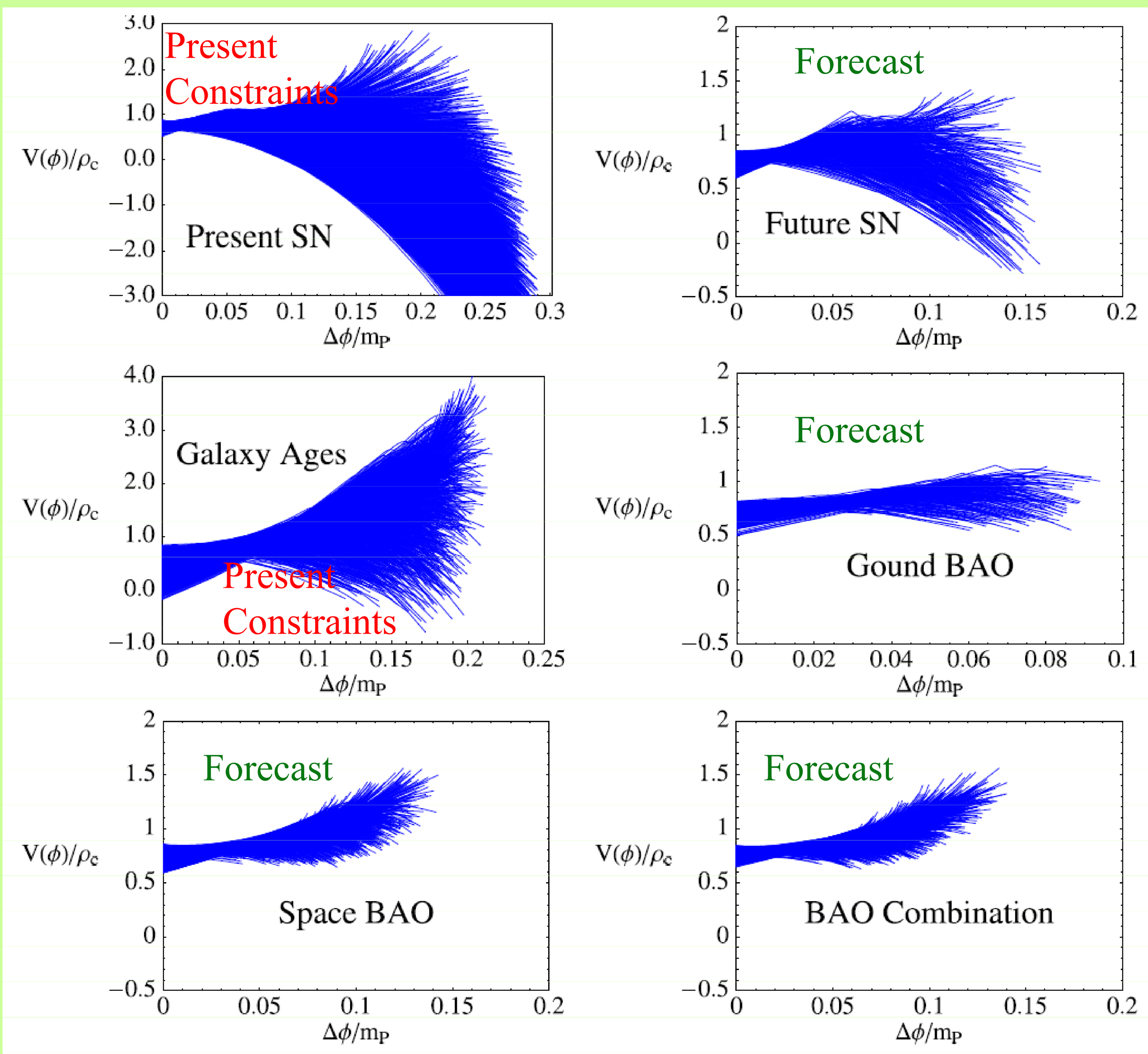

Figure 3. Reconstructed $V(\phi)$ for the 68% best models for the different datasets.

The constraints derived on the Chebyshev coefficients allow to reconstruct $V(\phi)$ in addition to *V(z)* by reconstructing $\phi(z)$. From the first Friedmann equation:

$$K(z) = \frac{1}{2}\left(\frac{d\phi}{dz}\right)^2(1+z)^2 H(z)^2 = 3\kappa^{-1}H^2(z) - \rho_m(z) - \rho_k(z) - V(z)$$

Which can be integrated to obtain:

$$\phi(z) - \phi(0) = \int_0^{z_{\max}} \frac{\sqrt{6\kappa^{-1}H^2(z) - 2\rho_m(z) - 2\rho_k(z) - 2V(z)}}{(1+z)H(z)}$$

Fig 3 shows the results of this reconstruction for the 68% best models for the different datasets. Note that, upon integration up to $z = z_{max}$, only a limited $\Delta\phi(z)$ can be recovered. This maximum value will strongly depend on the actual cosmological model. For the ΛCDM, $\Delta\phi(z) = 0$, thus, the tighter the constraints that a given dataset places around the ΛCDM the smaller the interval in $\Delta\phi(z)$ that will be recovered.

2. Datasets and Priors

Baryon Acoustic Oscillations: We consider measurements of the BAO scale both along and across the line of sight, from which *H(z)* and $d_A(z)$ are respectively extracted. To estimate the errors with which these magnitudes will be recovered we use the fitting formulas of Ref. [2]. We consider future "ground" and "space" based surveys with the following parameters:

Survey	Area (dg²)	z_{min}	z_{max}	bins in z
ground	10000	0.1	1	9
space	30000	1	2	10

Supernovae type IA: We consider the measurement of $d_L(z)$ from present and future Supernovae surveys. For the present sample we consider that of [3]. For the forecast we consider 500 near and 1000 far supernovae distributed in the following redshift bins [4]:

Mean z	0.1	0.85	0.95	1.05	1.15	1.25
SN	500	231	219	200	183	167

Galaxy Ages: The age of passively evolving galaxies determined from high signal-to-noise spectra of massive luminous galaxies can be accurately determined [1]. This provides "standard clocks" to measure *dt/dt* and thus *H(z)* through $H(z) = dz/dt(1+z)^{-1}$. We use the data from [5].

Priors: In all cases we consider a fiducial LCDM model with Gaussian priors of σ_H = 8 Km s^{-1} Mpc^{-1} for H_0, σ_m = 0.01 for $\Omega_m h^2$ and σ_k = 0.03 for Ω_k.

4. Reconstructing w(z)

It is widespread to characterize dark Energy by the evolution of its equation of state *w(z).* If *w(z)*>-1, this description is equivalent to considering the redshift evolution of a scalar potential. But, allowing *w(z)* < -1, more exotic explanations of the DE can be accommodated. Expanding *w(z)* in Chebyshev polynomials:

$$w(z) \approx \sum_{n=0}^{N} w_n T_n\left(2\frac{z}{z_{\max}} - 1\right)$$

A reconstruction procedure analogous to the one for *V(z)* can then be performed. The results of this reconstruction can be found in [6]. We found that the bounds derived with information on the integrated *H(z),* such as d_L from SN or d_A from BAO surveys are much weaker than those stemming from direct measurements of *H(z)* such as in radial BAO measurements.

As an example Fig.4 shows $d_A(z)$ and *H(z)* for a model that fitted well measurements of d_L or d_A but not direct measurements of *H(z).* The black lines stand for the LCDM behavior. In this model *w* decreases very steeply with *z* reaching *w* = -168 for *z*=2. However d_A measurements alone will have a difficult time ruling out this model. This manifests the more direct dependence of *H(z)* on dark energy properties that provides stronger constraints for non trivial parameterizations of the dynamics of dark energy.

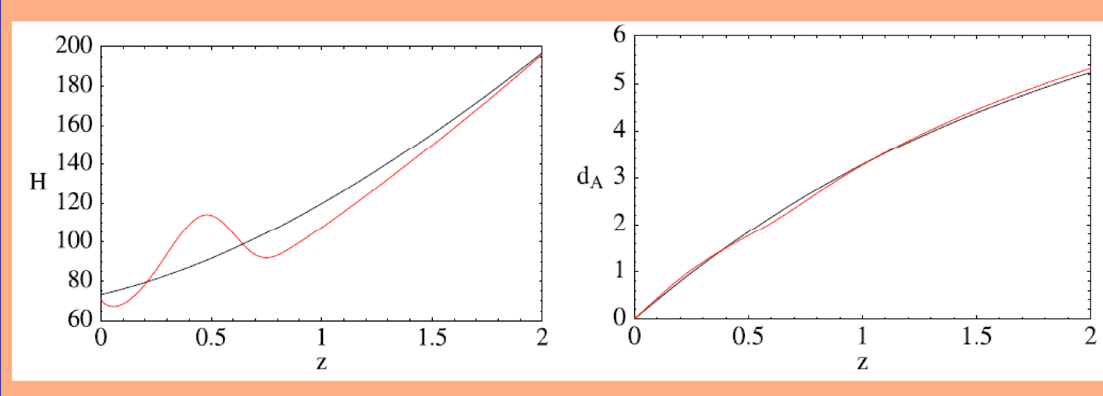

Figure 4. Comparison of an example model (red lines) with the ΛCDM (black lines). Notice the stronger sensitivity of *H(z)* measurements (left) compared to d_A or d_L measurements (right) to the dynamics of dark energy.

5. References

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