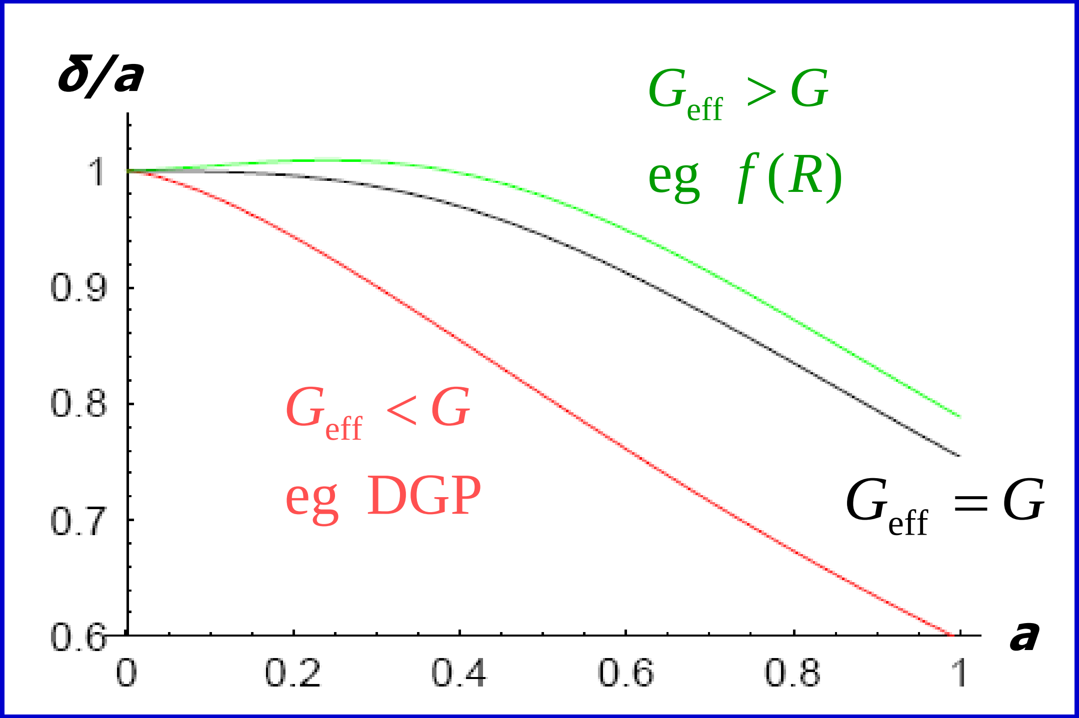
Modified gravity from braneworlds



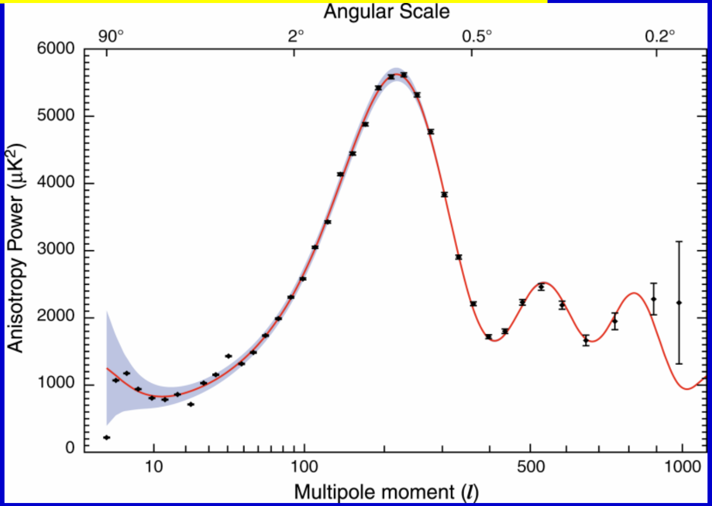
Leopoldina DE

Munich

October 2008

Roy Maartens
ICG
Portsmouth

"minimalist" approach to DE



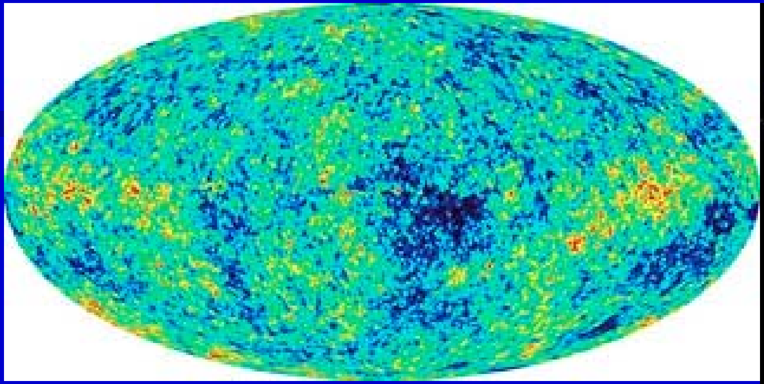
LCDM is the best model

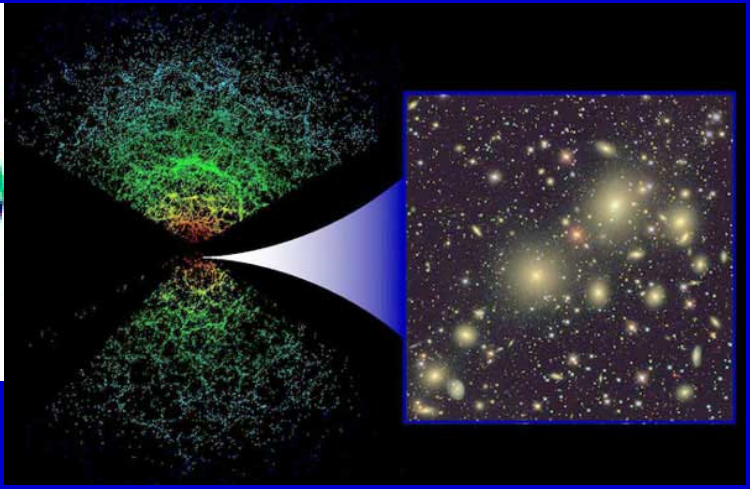
- test this against data
- let Quantum Gravity explain why

$$\rho_{\text{vac}} = (10^{-3}\ \text{eV})^4 << \rho_{\text{new physics}}\ (>1\,\text{TeV})^4$$

- focus on
  * the best tests for w=-1
  * the role of theoretical assumptions
  e.g. w(z) parametrizations,
  curvature=0

… but we can do more with the data: We can test alternatives and test GR





Alternatives to LCDM:

within General Relativity

- Dynamical DE (quintessence, interacting DE-CDM,…)
- Effective 'Dark Energy' via nonlinear effects of structure formation?

modify GR on large scales ("dark gravity")

- 4D: scalar-(vector)-tensor theories [simplest = f(R)]
- higher-D: braneworld models [simplest = DGP]

NB – all these alternatives require that the vacuum energy does not gravitate: $\boxed{\rho_{\text{vac}} \equiv 0}$

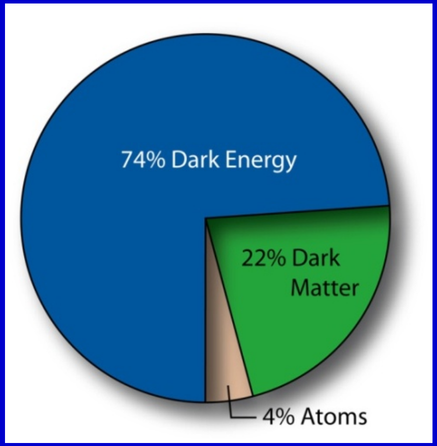
– they do not address the vacuum energy problem

Dark Energy dynamics

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + 8\pi G T_{\mu\nu}^{\text{dark}}$$

$$T_{\mu\nu}^{\text{dark}} = \text{time-varying DE field}$$

$$w = \frac{p_{\text{DE}}}{\rho_{\text{DE}}} < -\frac{1}{3}$$



Modified Gravity dynamics

$$G_{\mu\nu} + G_{\mu\nu}^{\text{dark}} = 8\pi G T_{\mu\nu}$$

$$G_{\mu\nu}^{\text{dark}} = \text{new gravity DOF to induce acceleration}$$

Modified gravity from braneworlds

- new massive graviton modes
- new effects from higher-D fields and other branes
- could these dominate at low energies?

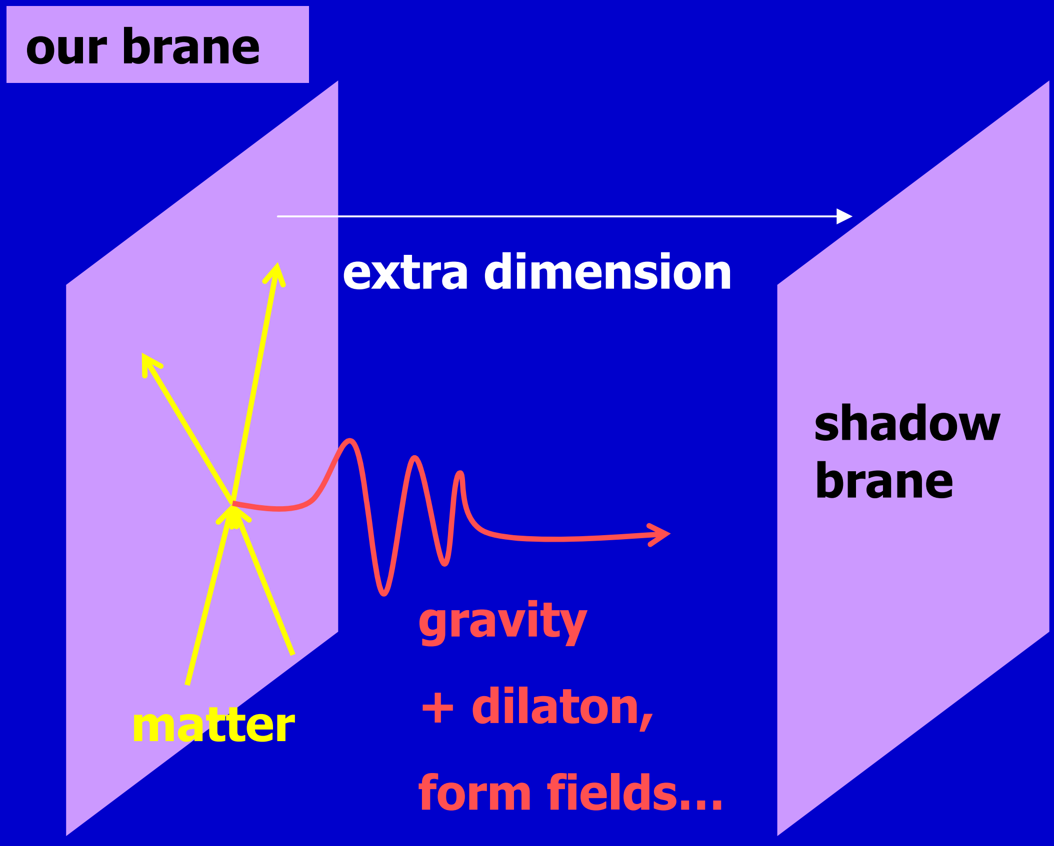
Possibilities

* ‘bulk’ fields as effective DE on the brane

(eg ekpyrotic/ cyclic)

* no bulk fields - effective 4D gravity on the brane modified on large scales

(eg DGP)



DGP – the simplest example

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^{(5)}}\;R^{(5)} + \frac{r_c}{8\pi G_5}\int_{\text{brane}} d^4x\sqrt{-g}\;R$$

(Dvali, Gabadadze, Porrati 2000 – and Deffayet 2001)

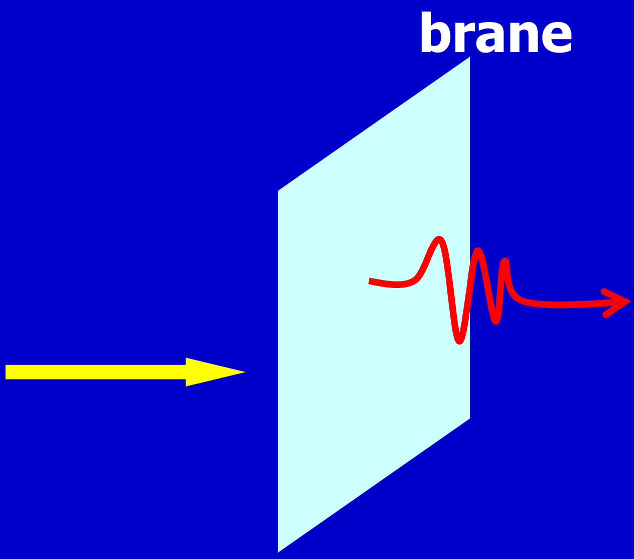
* **DGP was NOT constructed to solve the DE problem**
* **NO free functions, 1 parameter - same as LCDM**

$$r_c = \frac{G_5}{2G}$$ **crossover scale**

Weak field static regime

$$r << r_c \Rightarrow \Phi \propto \frac{1}{r} \;\rightarrow\; 4D$$

$$r >> r_c \Rightarrow \Phi \propto \frac{1}{r^2} \;\rightarrow\; 5D$$



gravity leakage:

gravity on the brane is weaker

DGP self-acceleration

$$H^2 + \frac{K}{a^2} - \frac{1}{r_c}\sqrt{H^2 + \frac{K}{a^2}} = \frac{8\pi G}{3}\rho_m$$

$$\text{late time}: \quad \rho_m \to 0 \Rightarrow H \to \frac{1}{r_c}$$

de Sitter

$$\text{early time}: \quad H^2 + \frac{K}{a^2} >> \frac{1}{r_c^2} \Rightarrow H^2 + \frac{K}{a^2} \approx \frac{8\pi G}{3}\rho_m$$

like GR

early universe – recover GR H(z): 4D gravity dominates

late universe – acceleration without DE: 5D gravity dominates

– gravity “leaks” off the brane

– therefore gravity on the brane weakens

Passes the solar system test: since DGP $\longrightarrow$ GR on small scales

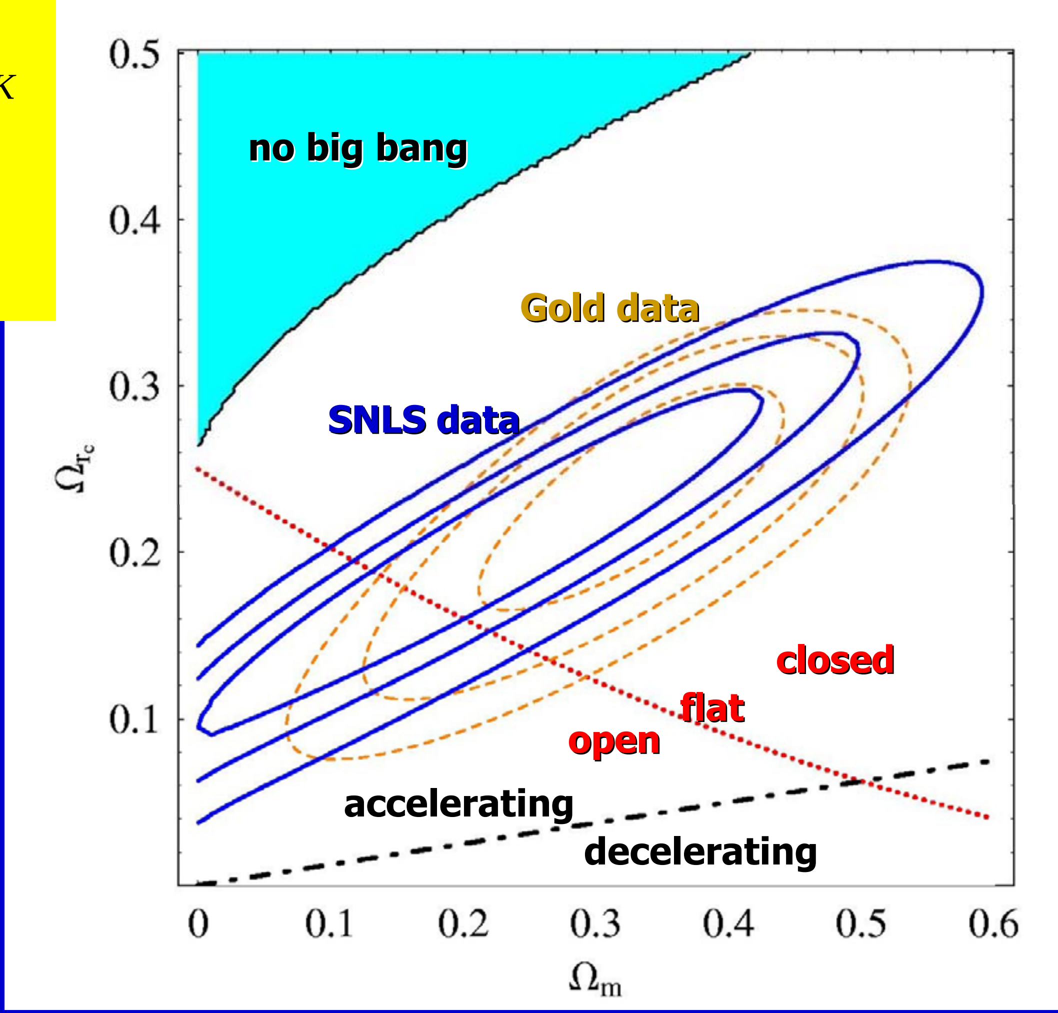
The background is very simple – like LCDM

$$H^2 + \frac{K}{a^2} - \frac{1}{r_c}\sqrt{H^2 + \frac{K}{a^2}} = \frac{8\pi G}{3}\rho_m$$

Modified Friedmann

$$1 = \left(\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_m}\right)^2 + \Omega_K$$

$$\Omega_{r_c} = \frac{H_0^{-2}}{4 r_c^2}$$

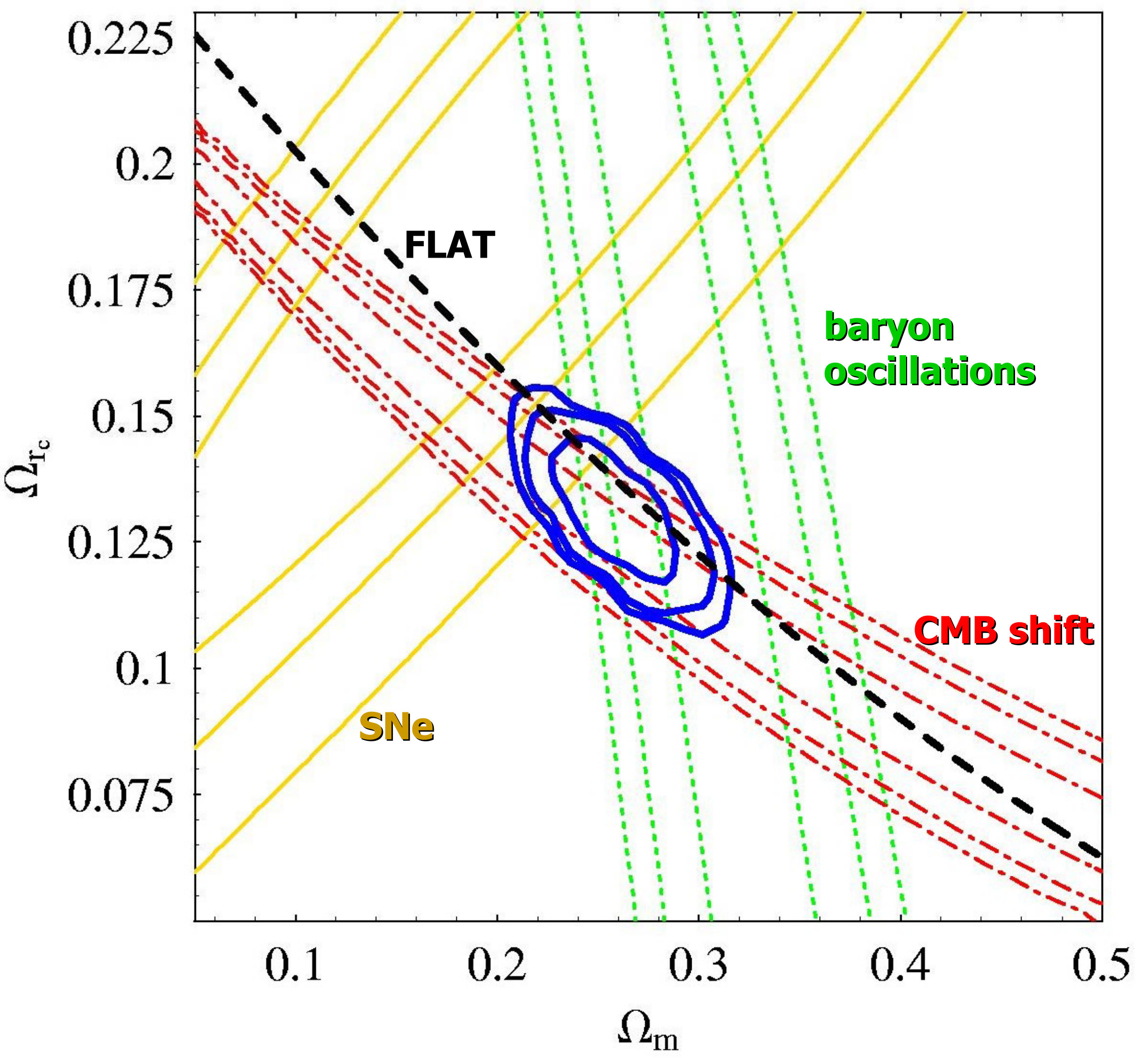


(RM, Majerotto 2006)

Tests of the background expansion history

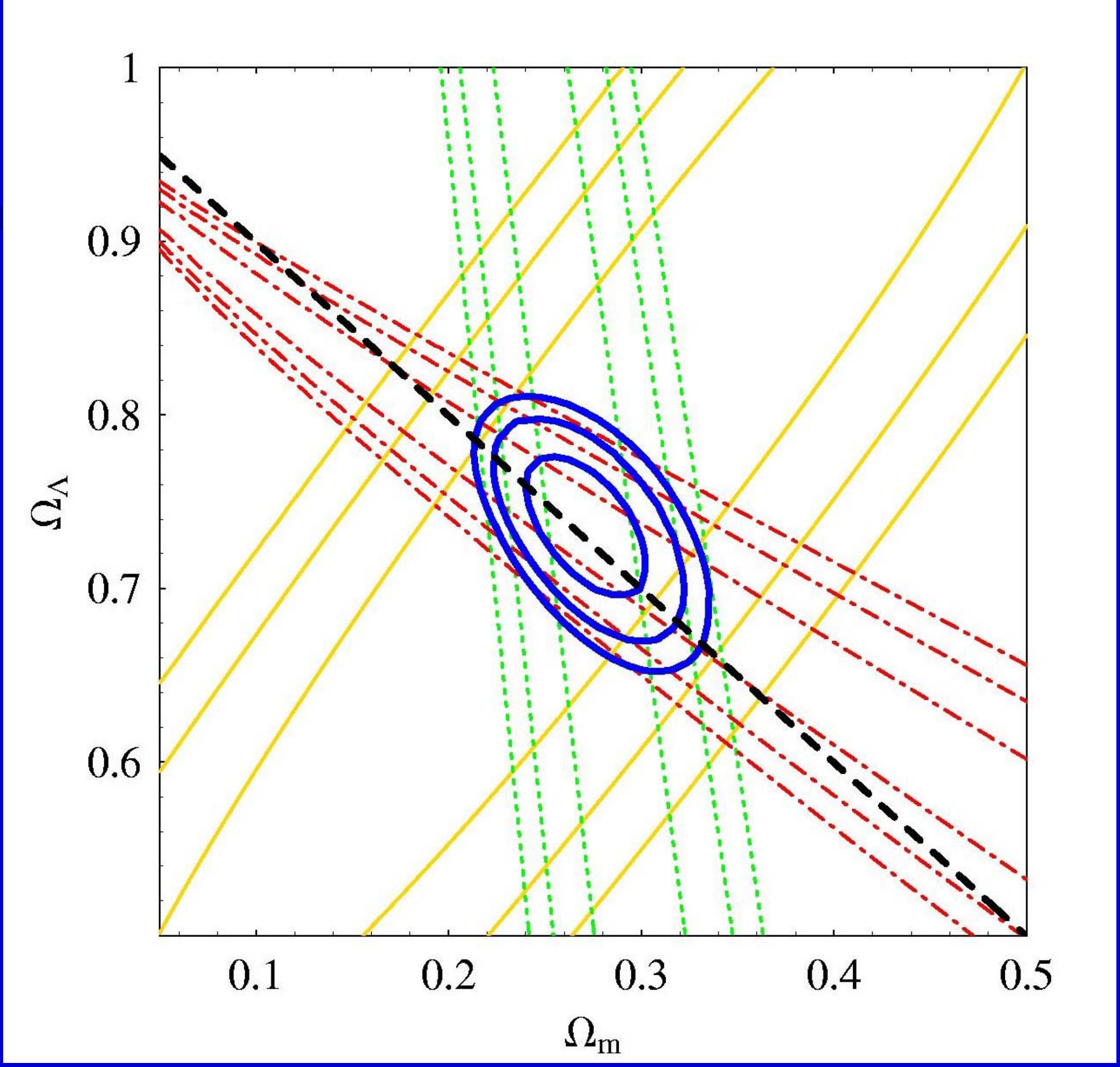
Tension between SNe, BAO and CMB shift

DGP struggles to fit the data



(RM, Majerotto 2006)

Unlike LCDM…



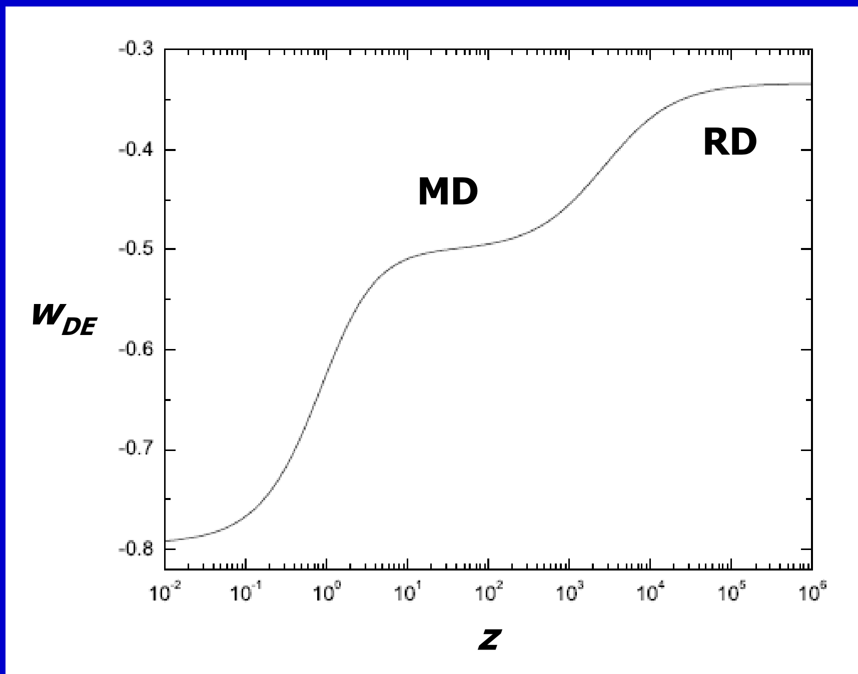
1
0.9
0.8
0.7
0.6
0.1
0.2
0.3
0.4
0.5
Ω_Λ
Ω_m

DGP flat model in trouble

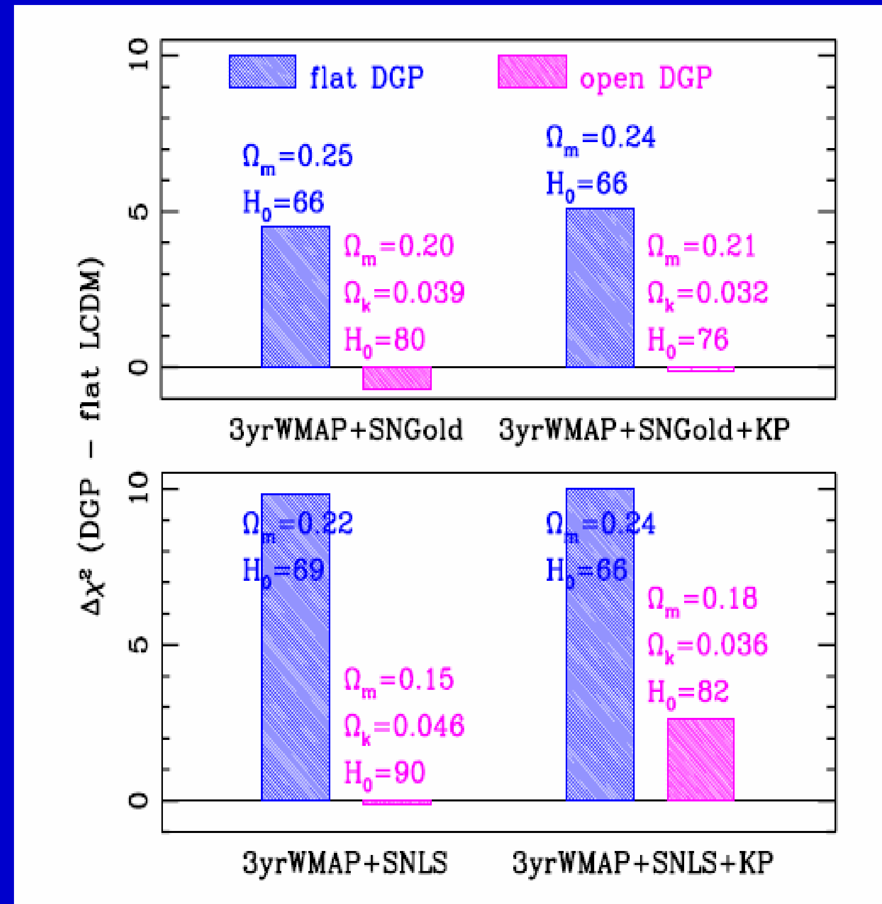
$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{H}{r_c}$$

$$\rho_{DE} = \frac{3}{8\pi G}\frac{H}{r_c}$$

$$w_{DE} = -\frac{1}{1+\Omega_m(a)}$$



Open model is better



(Song, Hu, Sawicki 2007)

Structure formation in DGP

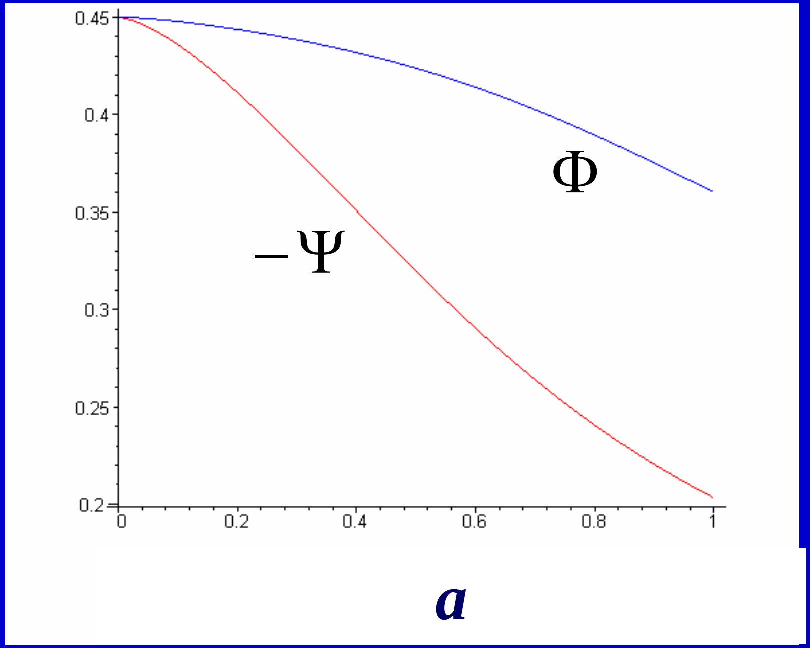
$$ds^2 = -(1+2\Psi)dt^2 + a(t)^2(1+2\Phi)d\vec{x}^2$$

Quasi-static approximation to 5D perturbations gives subhorizon perturbations on the brane:

$$\frac{k^2}{a^2}\Phi = 4\pi G\left(1-\frac{1}{3\beta}\right)\rho\delta,$$

$$\frac{k^2}{a^2}\Psi = -4\pi G\left(1+\frac{1}{3\beta}\right)\rho\delta,$$

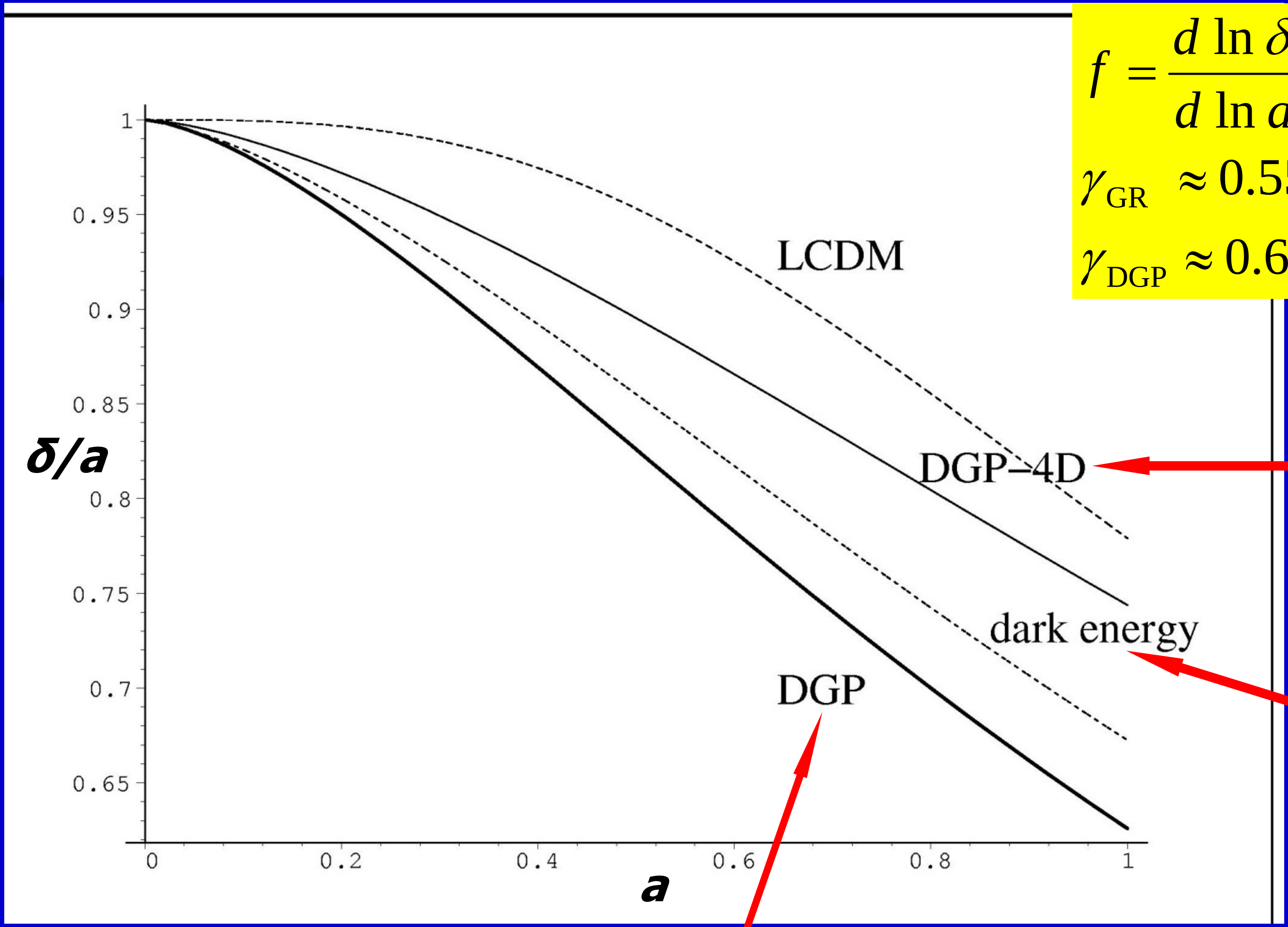
$$\beta = 1-2Hr_c\left(1+\frac{\dot{H}}{3H^2}\right)$$



Like Brans-Dicke with

$$\omega_{BD} = \frac{3}{2}(\beta-1) < 0$$

(Koyama, RM 2006)



$$f = \frac{d\ln\delta}{d\ln a} = \Omega_m^{\gamma}$$

$$\gamma_{\rm GR} \approx 0.55 + 0.05[1 + w(z=1)]$$

$$\gamma_{\rm DGP} \approx 0.68$$

ad hoc 4D treatment of perturbations gives incorrect results – violates 4D Bianchi identity

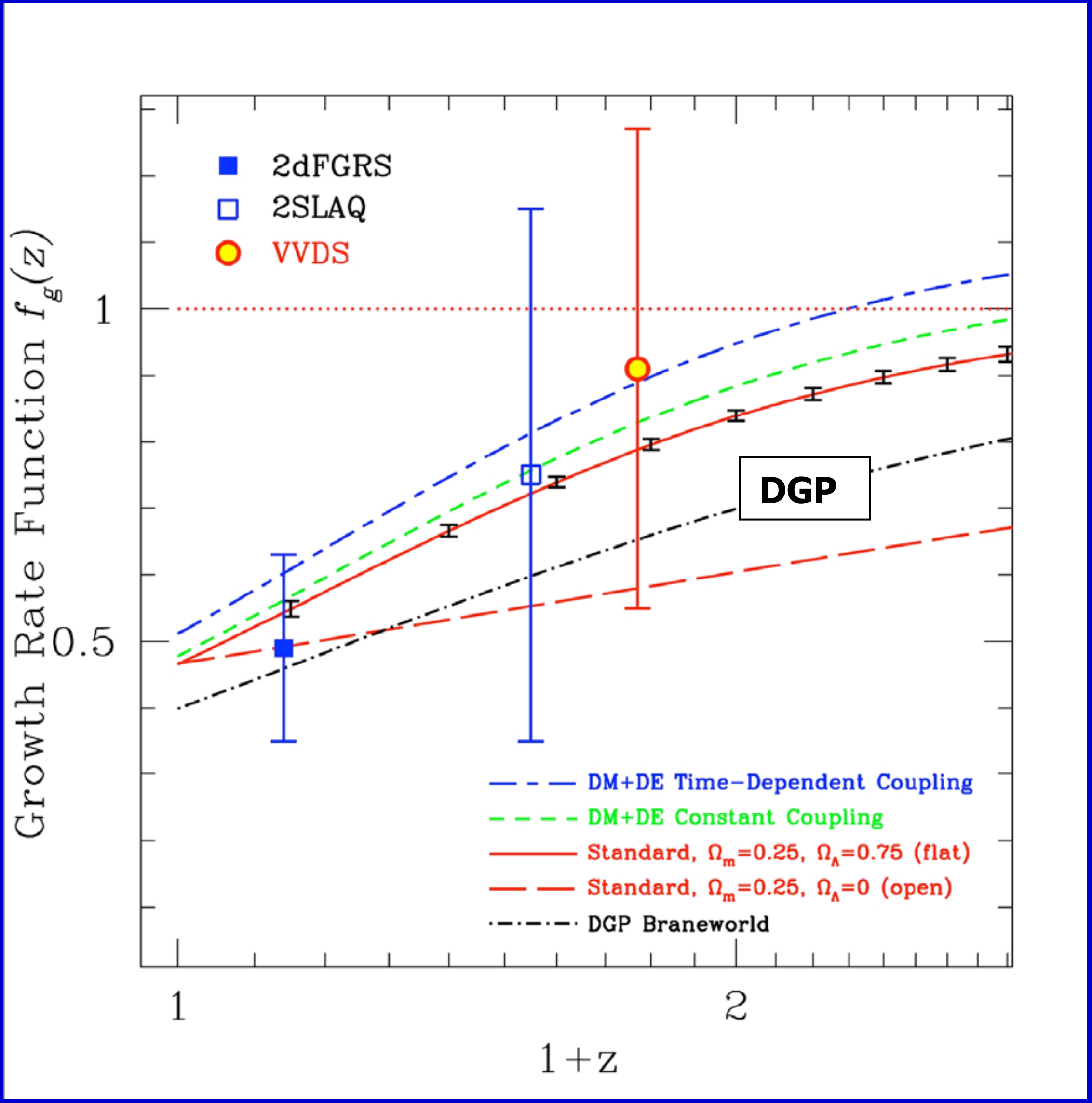
DE model with same expansion history

(Koyama, RM 2006)

Very strong suppression of growth from 5D effects – w_{DE} and Ψ

This could violate observational constraints ...

Growth factor data not yet accurate enough



Need to look at the CMB – and this requires superhorizon perturbations

(Guzzo et al 2008)

Perturbations on all scales: 5D numerical solution

5D metric perturbations described by the 5D "master variable":

$$\frac{\partial^2\Omega}{\partial u \partial v} - \frac{3}{2v}\frac{\partial \Omega}{\partial u} + \frac{k^2 r_c^2}{4v^2}\Omega = 0$$

Density perturbations on the brane

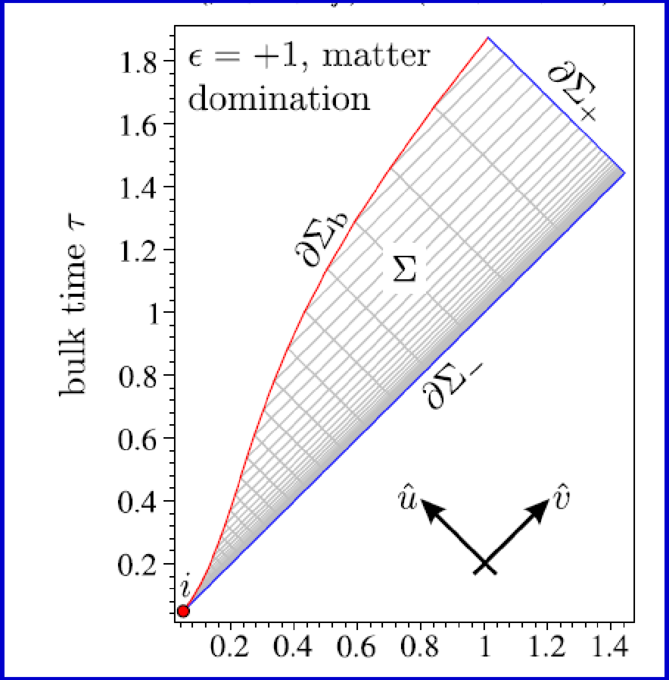
$$\ddot{\Delta} + 2H\dot{\Delta} - 4\pi G\rho F_1 \Delta = F_2 \frac{k^4}{a^5}\Omega_{\rm brane}$$

where

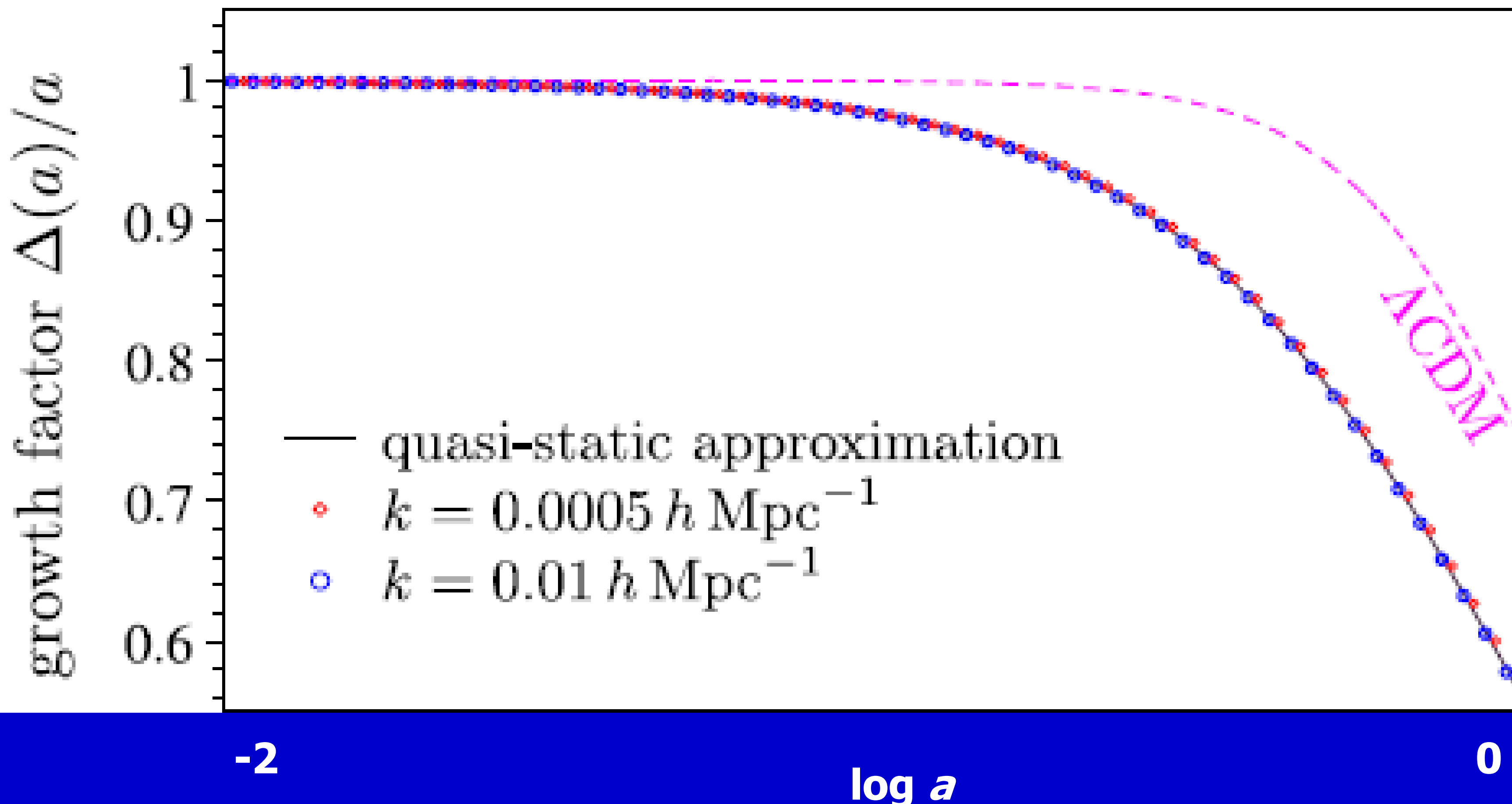
$$F_1 = \frac{2r_c\left(\dot{H} - H^2 + 2r_c H^3\right)}{H\left(2r_c H - 1\right)^2}$$

$$F_2 = -\frac{\left(r_c\dot{H} - H + 2r_c H^2\right)}{3H\left(2r_c H - 1\right)^2}$$

(Cardoso, Koyama et al 2008)



Results:
density perturbations



Results: metric perturbations

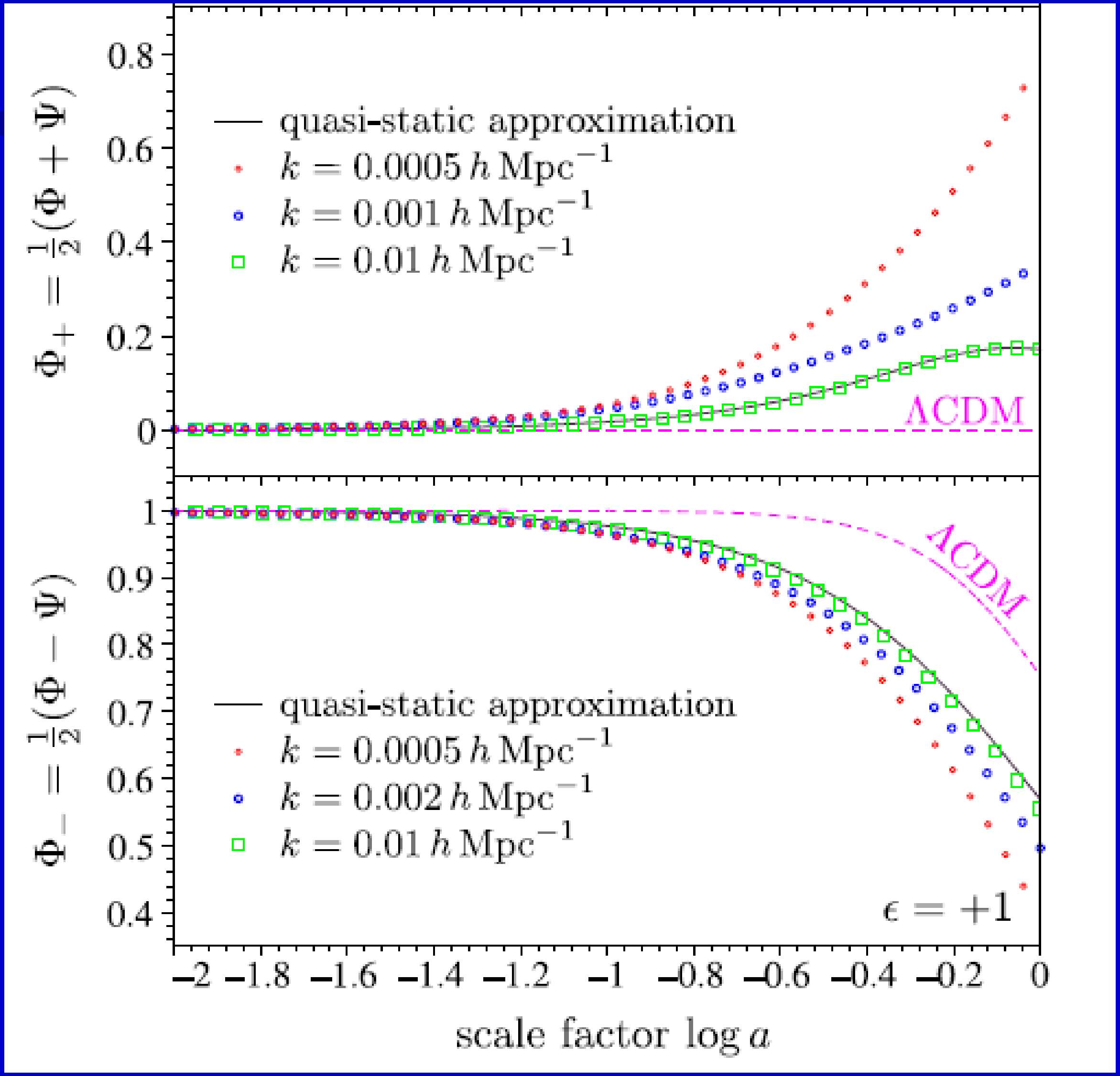
$$ds^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)d\vec{x}^2$$

$$\Phi_{\pm} = \frac{1}{2}(\Phi \pm \Psi)$$

Φ_+ **=0 in GR**

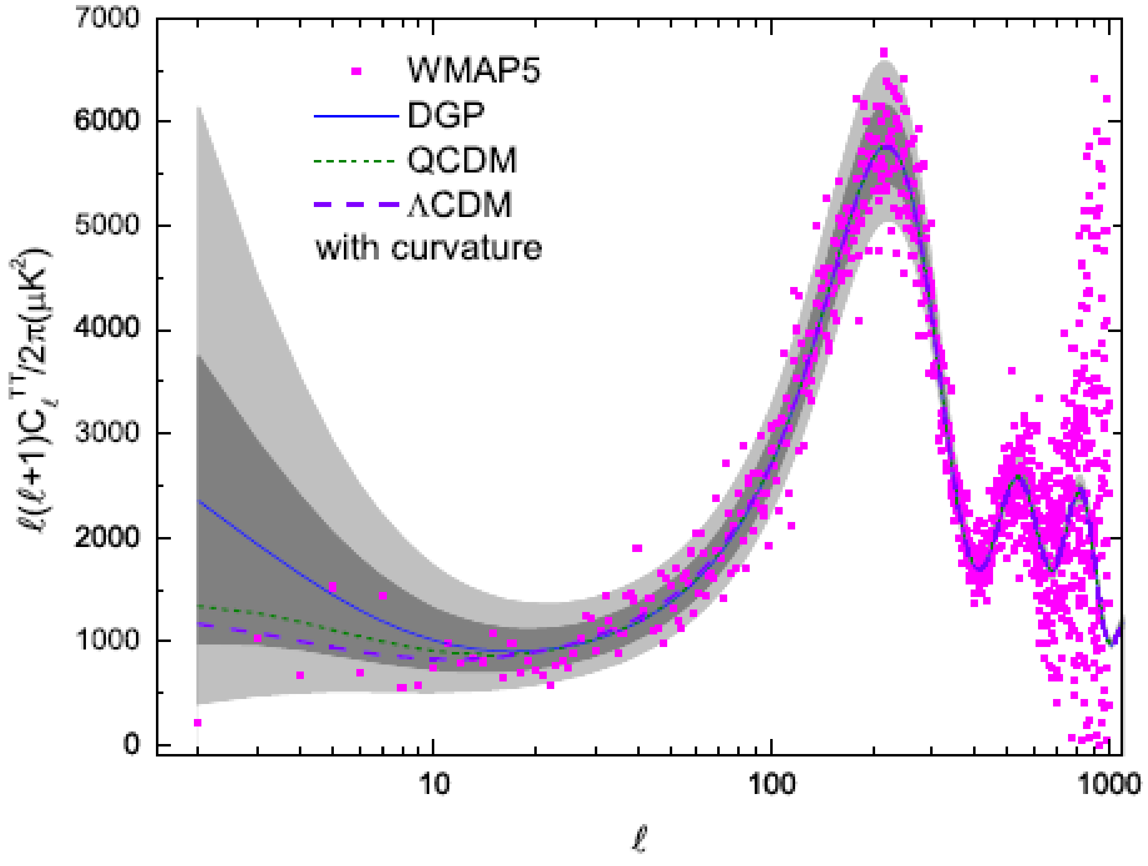
= dark anisotropic stress in DGP

Φ_- **determines ISW, lensing**



CMB: ISW

Steeper Φ_- implies stronger ISW than LCDM



(Fang, Wang, Hu et al 2008)

(also: Song, Sawicki, Hu 2007)

QCDM = DE with the same expansion history as DGP

With geometric data: DGP is a poorer fit than LCDM at ~5 σ

(large-scale CMB has 30% contribution to this conclusion)

DGP seriously challenged

- DGP – simplest MG model from braneworlds
  - – probably the simplest MG model of all
  - – no free functions – same as LCDM
- But it is seriously challenged by data:
  - – both geometric and structure-formation
- Key problem = the scalar degree of freedom:

  DGP like Brans-Dicke with $\omega_{BD} < 0$ on subhorizon scales

  This leads to drastic suppression of growth

Furthermore: $\omega_{BD} < 0$ indicates a ghost: confirmed by detailed analysis

The ghost makes the quantum vacuum unstable

DGP lessons

- Despite the challenge from data and the ghost –
  DGP is a key example of how to combine geometric and structure data to test GR
- Can we avoid the crisis of data and the ghost?
- Ghost-free self-accelerating models:
  * we must go to higher dimensions
  * up to now, no ghost-free cosmological model
  * so we cannot yet test against observations

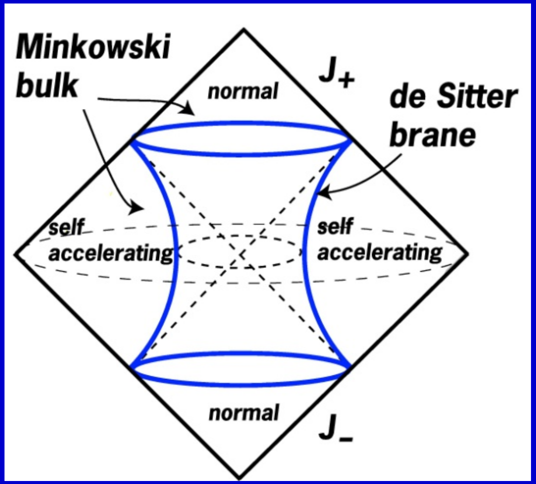
We can find a 5D braneworld cosmology with no ghost problem if we give up self-acceleration – the 'normal' DGP model.

5D gravity screens Λ and gives w<-1 ...

DGP 'normal' branch

Different embedding of the brane in the bulk gives another branch:

- **No self-acceleration: need DE**
- **No ghost** $(\omega_{BD} > 0)$



DGP $\longrightarrow$ nDGP: $r_c \longrightarrow -r_c$ **and** $\rho_m \longrightarrow \rho_m + \Lambda/8\pi G$

$$H^2 + \frac{K}{a^2} + \frac{1}{r_c}\sqrt{H^2 + \frac{K}{a^2}} = \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

$$\text{late time}: \quad \rho_m \to 0 \Rightarrow H \to \sqrt{\frac{\Lambda}{3} + \frac{1}{4r_c^2}} - \frac{1}{2r_c} \quad \left(< \frac{\Lambda}{3}\right)$$

$$\text{early time}: \quad H^2 + \frac{K}{a^2} >> \frac{1}{r_c^2} \Rightarrow H^2 + \frac{K}{a^2} \approx \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

Gravity leakage at late times screens Λ

$$\rho_{DE} = \frac{1}{8\pi G}\left(\Lambda - 3\frac{H}{r_c}\right) \quad < \frac{\Lambda}{8\pi G}$$

- and gives effective 'phantom' behaviour:

$$\dot{\rho}_{DE} \propto -\dot{H} > 0 \;\Rightarrow\; w_{DE} < -1$$

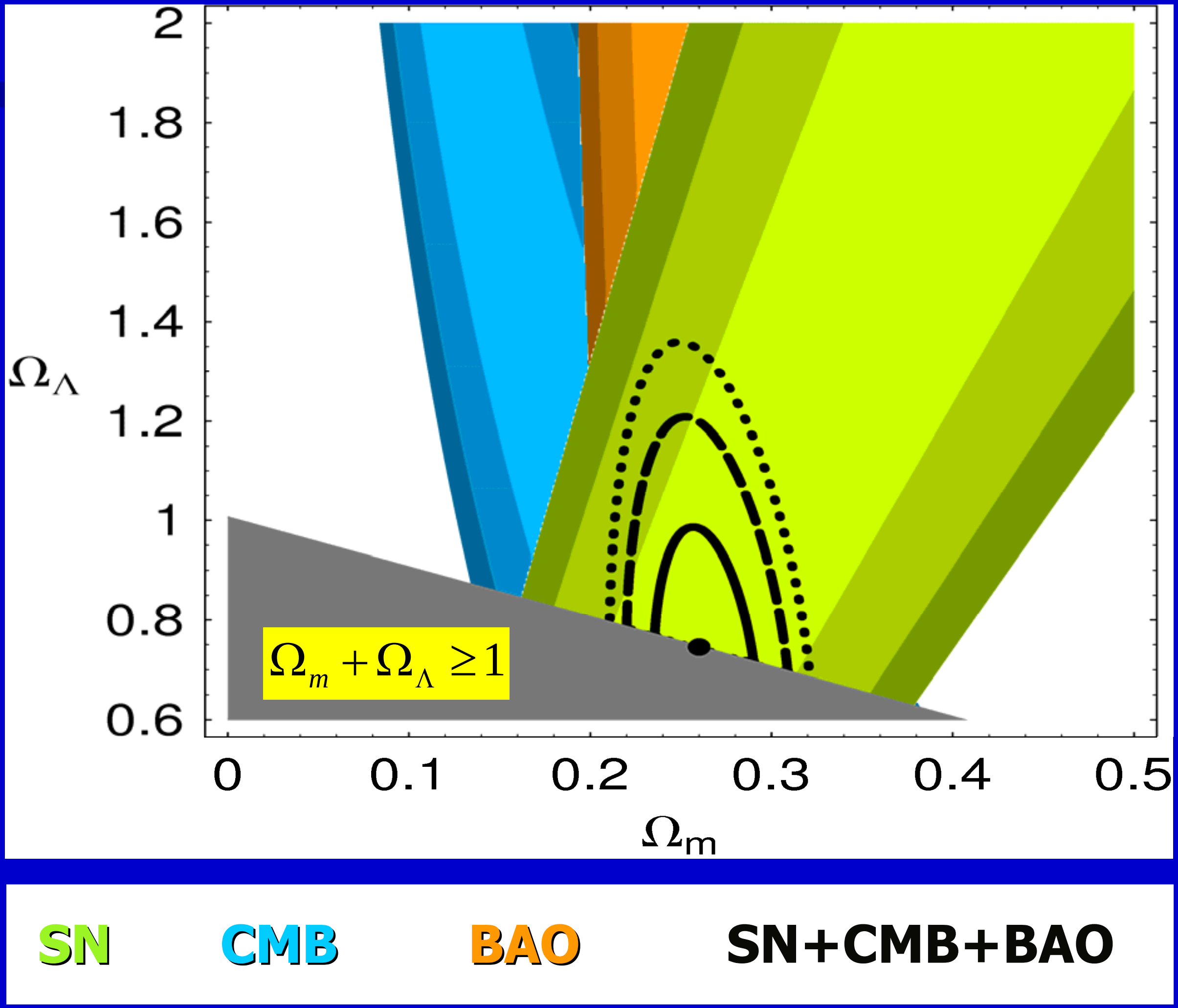
But without any phantom pathologies.

Since $\dot{H} < 0$ **there is no 'big rip' singularity.**

Dimensionless modified Friedmann:

$$1 = \Omega_K + \left(\sqrt{\Omega_m + \Omega_\Lambda + \Omega_{r_c}} - \sqrt{\Omega_{r_c}}\right)^2$$

Flat nDGP – geometric data

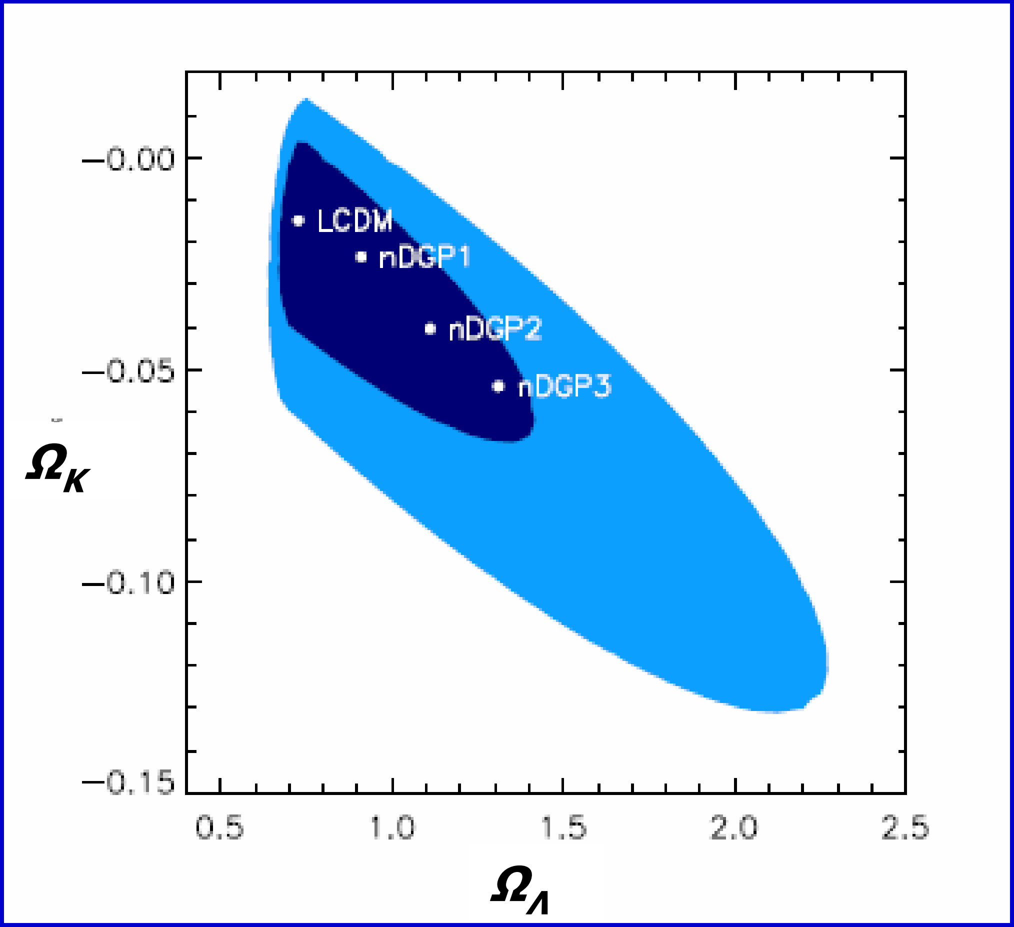


best fit is an LCDM model

(Lazkoz, RM, Majerotto 2006)

Curved nDGP – geometric data

- For significant screening and phantom behaviour – we need a curved model
- Best fit is a closed model



(Giannantonio, Song, Koyama 2008)

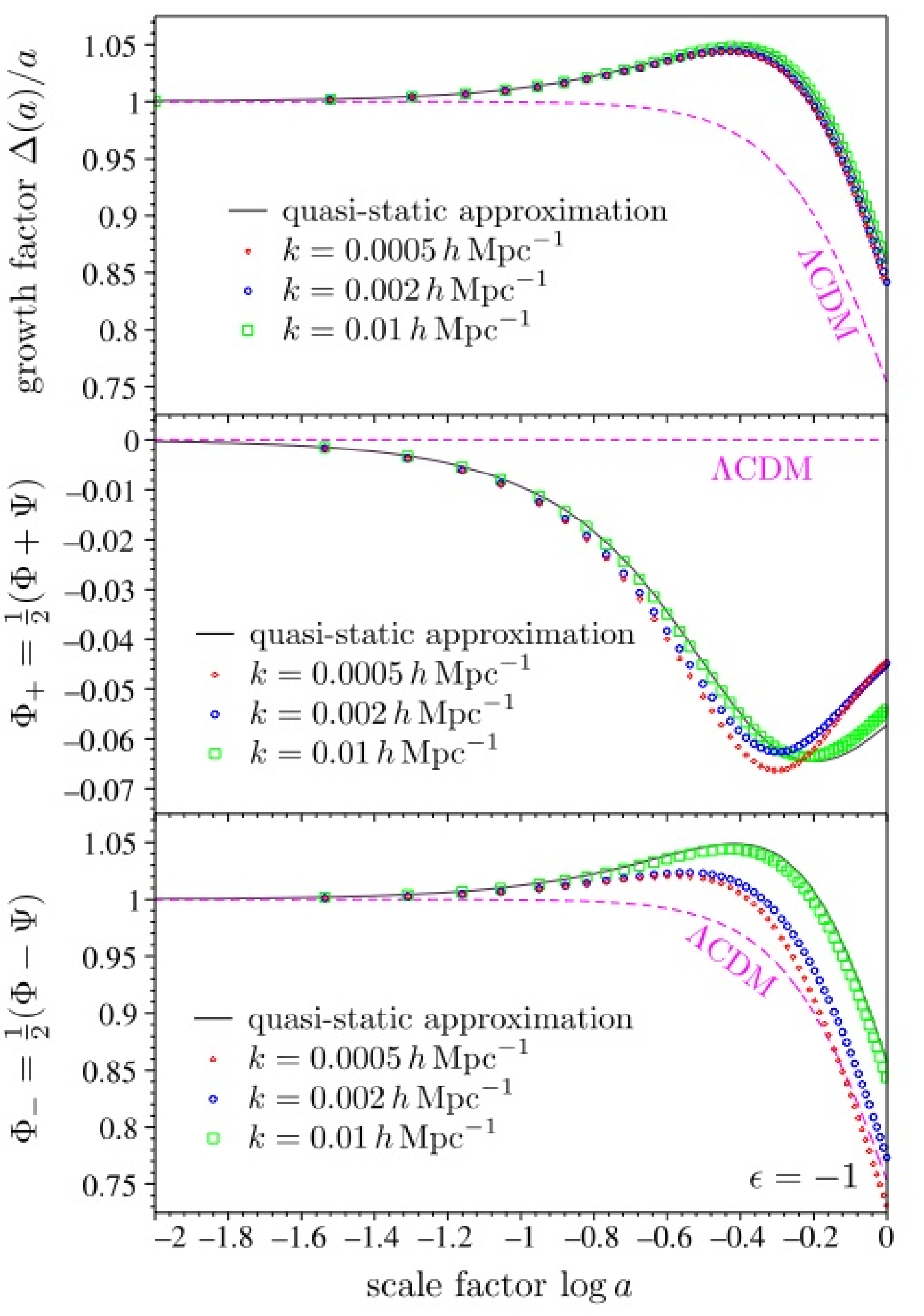
Perturbations

Quasi-static subhorizon approximation and numerical solutions are found by the same approach as in DGP

$r_c \longrightarrow -r_c$

How does nDGP fit to CMB?

Ongoing work



(Cardoso, Koyama et al 2008)