Weak Gravitational Lensing Flexion

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ABSTRACT

Gravitational lensing provides a powerful tool for studying the mass distribution of clusters of galaxies as well as large-scale structure in the universe. It has led to constraints on cosmological parameters, such as the mass density of the Universe. Lensing flexion is the gradient of gravitational lensing shear. It describes higher-order effects (Bacon et al. 2006), turning a round source into a banana-shape image, called arclet. It is expected to be sensitive to small-scale structure. We derive the brightness moments relation between sources and images of background galaxies; this approach is similar to the HOLICs by Okura et al.(2006). The measurement accuracy of flexion depends on the product of flexion and the source size. Two applications of the flexion are address, mass reconstruction and galaxy-galaxy lensing.

1 Flexion Notation in New Lensing Equ.

We expand the lensing equation to second order,

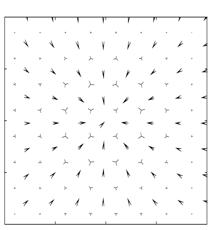
$$\beta_i = \theta_i - \psi_{,ij}\theta_j - \psi_{,ijk}\theta_j\theta_k/2 \qquad (1)$$

β, θ are the sources, image coordinate, ψ is the deflection potential, indices separated by a comma denote partial derivatives with respect to θ_i. Due to the mass-sheet degeneracy, only the reduced shear $g = \gamma/(1-\kappa)$ is observable. The same holds for the gradients of shear –the **reduced flexion** $G_1 = \nabla^* g$, $\quad G_3 = \nabla g$; here $1, 3$ indicate the spin of the quantities, and we define the differential operator $\nabla = \frac{\partial}{\partial\theta_1} + \mathrm{i}\frac{\partial}{\partial\theta_2}$. Then the lens equation reads

$$\hat\beta \equiv \frac{\beta}{(1-\kappa)} = \theta - g\theta^* - \Psi_1^*(G_1)\theta^2 - 2\Psi_1(G_1)\theta\theta^* - \Psi_3(G_1,G_3)(\theta^*)^2, \quad (2)$$

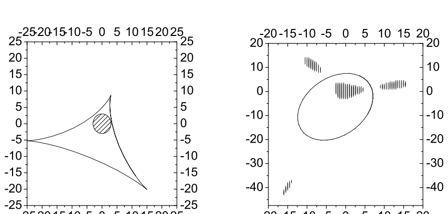
and Ψ_1, Ψ_3 are functions of the reduced shear and reduced flexion.

- **Flexion field**



This panel shows the flexion corresponding to an axially-symmetric density field, where arrows indicate the spin-1 flexion and the skeletons the spin-3 flexion component.

- **Critical curve and multiple images**
  Since flexion becomes important in strong lensing regimes, we must be careful about critical curves. Up to four images of a source can be obtained, due to the non-linearity of equ.(2)



Examples of critical curves (left) and caustics (right) of the lens equation (2). A circular source is mapped onto four images, as indicated. If the source size were increased, it would hit the caustic, three images would merge, and the flexion concept would break down.

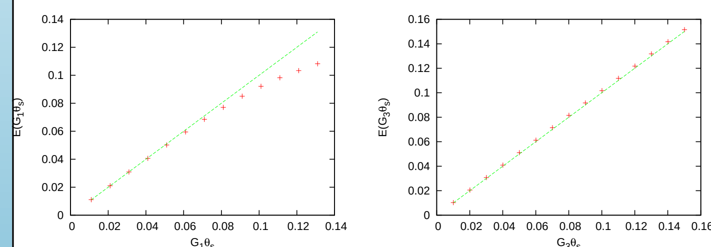
2 Flexion Measurement

Like the ellipticity being an estimator of reduced shear g, we found an approximate relation between reduced flexion and higher-order brightness moments.

- **Flexion Estimators to lowest order**

$$G_1 \simeq \frac{4}{9F_0 - 12Q_0^2}T_1$$
$$G_3 \simeq \frac{4}{3F_0}T_3 \qquad (3)$$

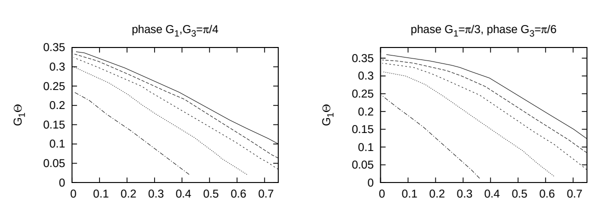
where the T_1,T_3 are third-order spin-1, spin-3 brightness moments and F_0, Q_0 are fourth and second-order spin-0 moments. Here we assume that the expectation value of non-zero spin source brightness moments vanish due to phase averaging, which is very small even for individual source galaxy images.



Comparison of the reduced flexion estimators (3) with the input value. The horizontal and vertical axis show $G_i\theta_s, i = 1,3$. For both panels, we take $g = 0.05$, and $G_3 = 0(G_1 = 0)$ for the left(right) panel. The line indicates the input value, the plus symbols show the estimate (3).

3 Breakdown of flexion

As we mentioned before, the lens equ.(2 can give rise to multiple images. And if the flexion becomes sufficiently large - or the source is large enough, the multiple images of an extended source will merge, then the whole method of determining shear and flexion will break down. This can be seen by considering the caustic curve cutting the source. We check that in our simulations, by controlling the sign of the Jacobian determinant of equ.(2). If the source size becomes too large, some points in the image will have a negative Jacobian.



Constraints on the combination of source size and reduced flexion for the validity of concept of flexion. Each curve shows the dividing line between a circular source of limiting isophote Θ being cut by a caustic (above the curve). The different curves in each panel are for different values of g, chosen as $g = 0.4, 0.2, 0.1, 0.05, 0$, as indicated by different line tpyes. The two panels differ in the phase of the reduced flexion, as indicated.

4 Combine Flexion in Mass Reconstruction

One application of flexion is mapping the dark matter density. Since the signal-to-noise ratio will be larger for flexion than for the shear for strong field gradients, it is better to combine shear and flexion to improve the mass reconstruction. The main idea is to describe the cluster mass-distribution by the value of the deflection potential ψ on a regular grid, then define a function χ^2 and minimize it with respect to ψ,

$$\chi^2(\psi) = \chi^2_s(\psi) + \chi^2_f(\psi) + R. \qquad (4)$$

$\chi^2_s(\psi)$ and $\chi^2_f(\psi)$ contain information from shear and flexion, R is a regularization term.

$$\chi^2_s = \sum_1^{N_g}\frac{|\epsilon - g|^2}{\sigma_s^2}, \quad \chi^2_f = \sum_1^{N_g}\frac{|G_i - \hat G_i|^2}{\sigma_f^2} \ (i = 1,3) \qquad (5)$$

ϵ is ellipticity of galaxy, G_i is the measured reduced flexion, g and $\hat G_i$ are expectation value of the shear and flexion calculated from the field ψ by finite differencing. In order to find the minimum χ^2 solution, we solve

$$\frac{\partial\chi^2(\psi)}{\partial\psi} = 0 \qquad (6)$$

which is a non-linear set of equations. We linearize this system and solve it in an iterative way. In addition, we can also combine the strong lens information, the multiple image systems to improve the result.

5 Conclusion & Outlook

- Flexion is sensitive to the small-scale structure (shear derivatives) and is estimated from higher-order brightness moments of galaxy images.
- Only the *reduced shear* and *reduced flexion* is measurable, and the accuracy of our estimates depends on the dimensionless quantities $G_1\theta_s$, $G_3\theta_s$ and g.

  Since flexion is relerant in rather strong lensing regines (e.g. arclets), the difference between flexion ($\partial\gamma$) and reduced flexion (∂g) is expected to be significant.
- Flexion can probably be used for determining galaxy mass profiles by galaxy-galaxy lensing, and substructure of clusters in mass reconstructions.

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