Optimal Binning

Kaastra & Bleeker, 2016, A&A 587, A151

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Introduction

We had

$$n_{ph}(c) = \int_0^\infty R(c,E) \cdot A(E) \cdot F(E)\, dE + n_{background}(c) \tag{1}$$

and discretized to

$$N_{ph}(c) = \Delta T \cdot \sum_{i=0}^{n_{ch}} A(E_i) \cdot R(c,i) \cdot F(E_i) \cdot \Delta E_i + N_{background}(c)$$

$$\forall c \in \{1, 2, \ldots, n_{en}\} \tag{2}$$

but

- What value should we choose for the number of energy channels, n_{en}?
- What value should we choose for the number of spectral (PHA/PI) channels, n_{ch}?

$\Longrightarrow$ Optimal Binning

Introduction

Implementation in FITS:

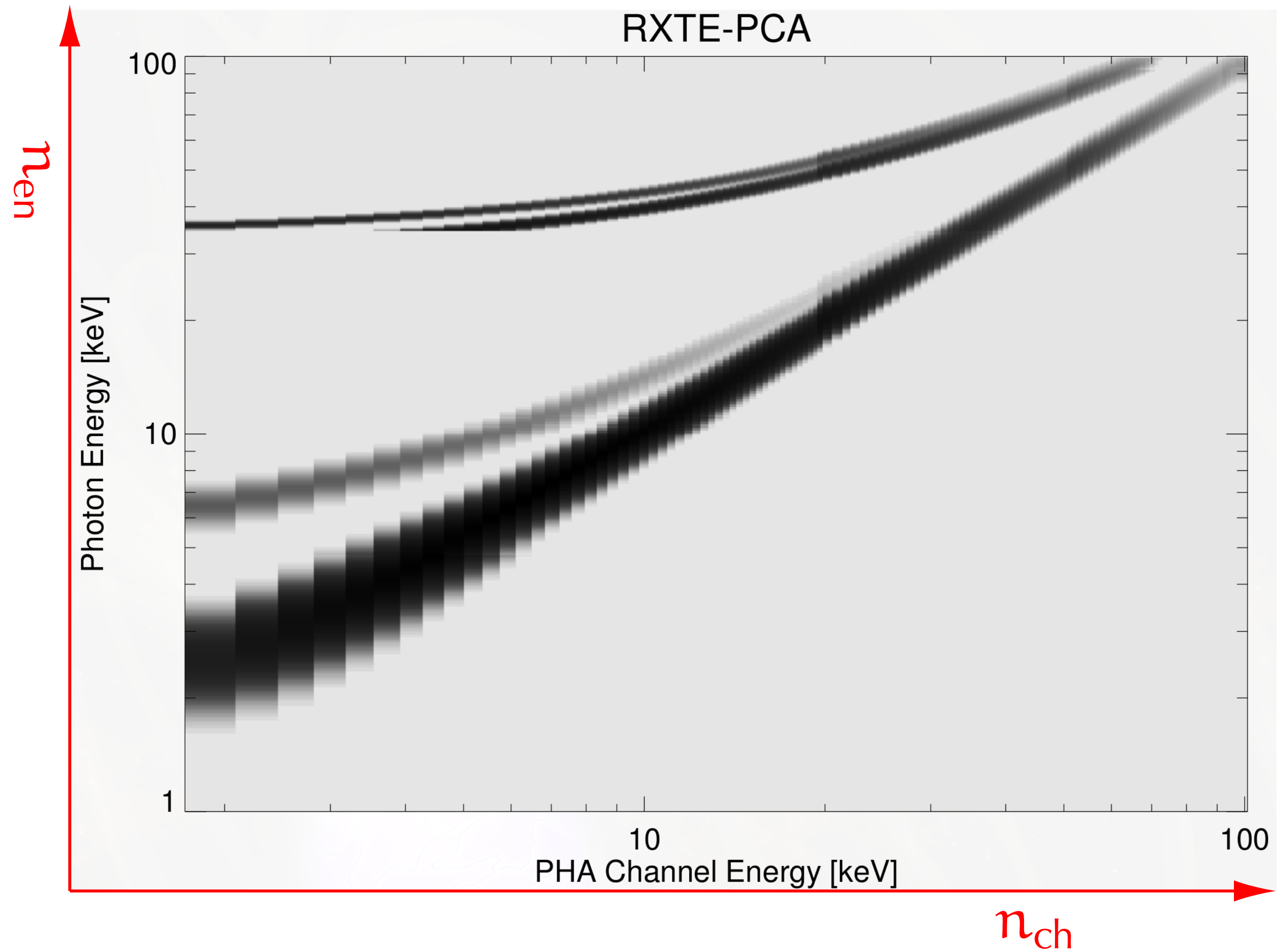
```
wilms@leo:~/data/lmcx3/0671420501/pn_timing/lmcx3_all/spectrum> fstruct src_sd.rmf
  No. Type     EXTNAME      BITPIX Dimensions(columns)      PCOUNT  GCOUNT

   0  PRIMARY                 8     0                           0     1
   1  BINTABLE MATRIX         8     38(6) 2067            3647984     1

      Column Name                Format     Dims       Units     TLMIN  TLMAX
      1 ENERG_LO                   E                   keV
      2 ENERG_HI                   E                   keV
      3 N_GRP                      I
      4 F_CHAN                     5I                                0     4095
      5 N_CHAN                     5I
      6 MATRIX                     1PE(1110

   2  BINTABLE EBOUNDS        8     10(3) 4096                  0     1

      Column Name                Format     Dims       Units     TLMIN  TLMAX
      1 CHANNEL                    I                                 0     4095
      2 E_MIN                      E                   keV
      3 E_MAX                      E                   keV
```

n_{en}

n_{ch}

Note: RMF does not have to be square, i.e., $n_{\mathrm{ch}} \neq n_{\mathrm{en}}$

Introduction



Optimum choice of n_{ch} and n_{en}: Kaastra & Bleeker, 2016, A&A 587, A151

this presentation follows the paper very closely

Number of energy channels

In simplified notation, ignoring ARF for the moment: measured flux

$$s(E') = \int_0^\infty R(E', E) f(E) dE \tag{3}$$

Analytical integration impossible $\Longrightarrow$ discretize ("model energy grid" or "model grid", boundaries E_{1j}, E_{2j}, center $E_j = (E_{1j} + E_{2j})/2$, width $\Delta E_j = E_{2j} - E_{1j}$

Then:

$$S_i = \sum_j R_{ij} \cdot F_j \quad \text{where} \quad F_j = \int_{E_{1j}}^{E_{2j}} f(E) dE \tag{4}$$

To evaluate, however, most analysis packages use

$$F_j = f(E_j) \Delta E_j \tag{5}$$

$\Longrightarrow$ implies $f(E) \sim$ const. over ΔE_j.

$\Longrightarrow$ use "sufficiently large number of bins"

(e.g., Suzaku RMF: 1 eV, XMM-EPIC-pn: 5 eV)

Implies $\mathcal{O}(10000)$ bins for typical Si-based detectors today

Number of energy channels

What is "sufficiently large"?

Determination of narrow line centroid:

$$\text{accuracy of } E_{\text{line}}: \quad \sigma E \propto \frac{\sigma}{\sqrt{N}} \tag{6}$$

where σ: resolution, N: photon number

Number of energy channels

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- Athena-WFI ($\sigma \sim 20\,\text{eV}$ $1\,\text{keV}$), line w/20000 counts: $\sigma E = 0.1\,\text{eV}$
  $\Longrightarrow$ 100 000 bins

Number of energy channels

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- Athena-X-IFU ($\sigma \sim 1\,\text{eV}$ @ $1\,\text{keV}$), line w/20000 counts: $\sigma E = 0.005\,\text{eV}$

  $\Longrightarrow$ 2 000 000 bins

Just increasing number of n_{ch} will not help with future detectors.

Already now model evaluations are extremely expensive, e.g., NuSTAR and Suzaku require rebinning of matrix.

Number of energy channels

K&B: Optimize evaluation of F_j

Assuming Gaussian redistribution

- 0th order (...the XSPEC way)

$$f_{1,0}(E) = N\delta(E - E_j) \tag{7}$$

where N: number of photons in bin

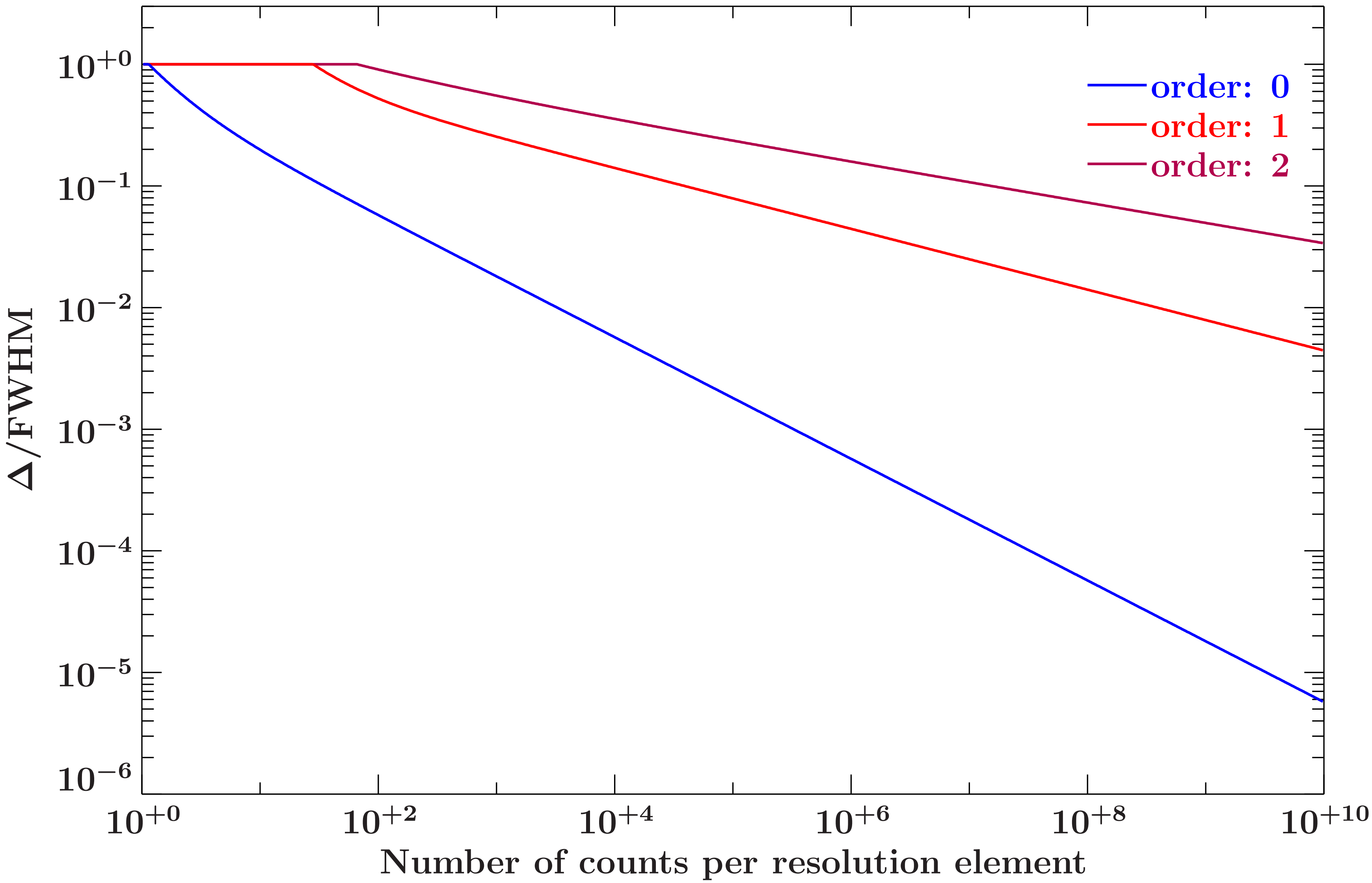
- 1th order: use average photon energy:

$$f_{1,1}(E) = N\delta(E - E_a) \quad \text{with} \quad E_a = \int_{E_{1j}}^{E_{2j}} f(E) E \, dE \tag{8}$$

- 2nd order: consider average and variance of photon energy (1st and 2nd moments):

$$f_{1,2}(E) = N \exp\left((E - E_a)/2\tau^2\right) \quad \text{with} \quad \tau^2 = \int_{E_{1j}}^{E_{2j}} f(E)(E - E_a)^2 dE \tag{9}$$

Number of energy channels



after K&B, Fig. 1

Already using 1st order gives order of magnitude improvement in number of channels!

Number of energy channels

K&B:

- recommend 1st order: good compromise between numerical complexity and improvement
- They also present equations to take into account ARF effects

implemented in SPEX and ISIS, but in principle also doable for XSPEC with a binfile for `ftrbnrmf`

Number of PI channels

Number of PI channels: How should I bin my data?

Information theory: Shannon information theorem:

for a band limited function

$$f(x) = \sum_{n=-\infty}^{+\infty} f(n\Delta) \frac{\sin \pi(x/\Delta - n)}{\pi(x/\Delta - n)} \tag{10}$$

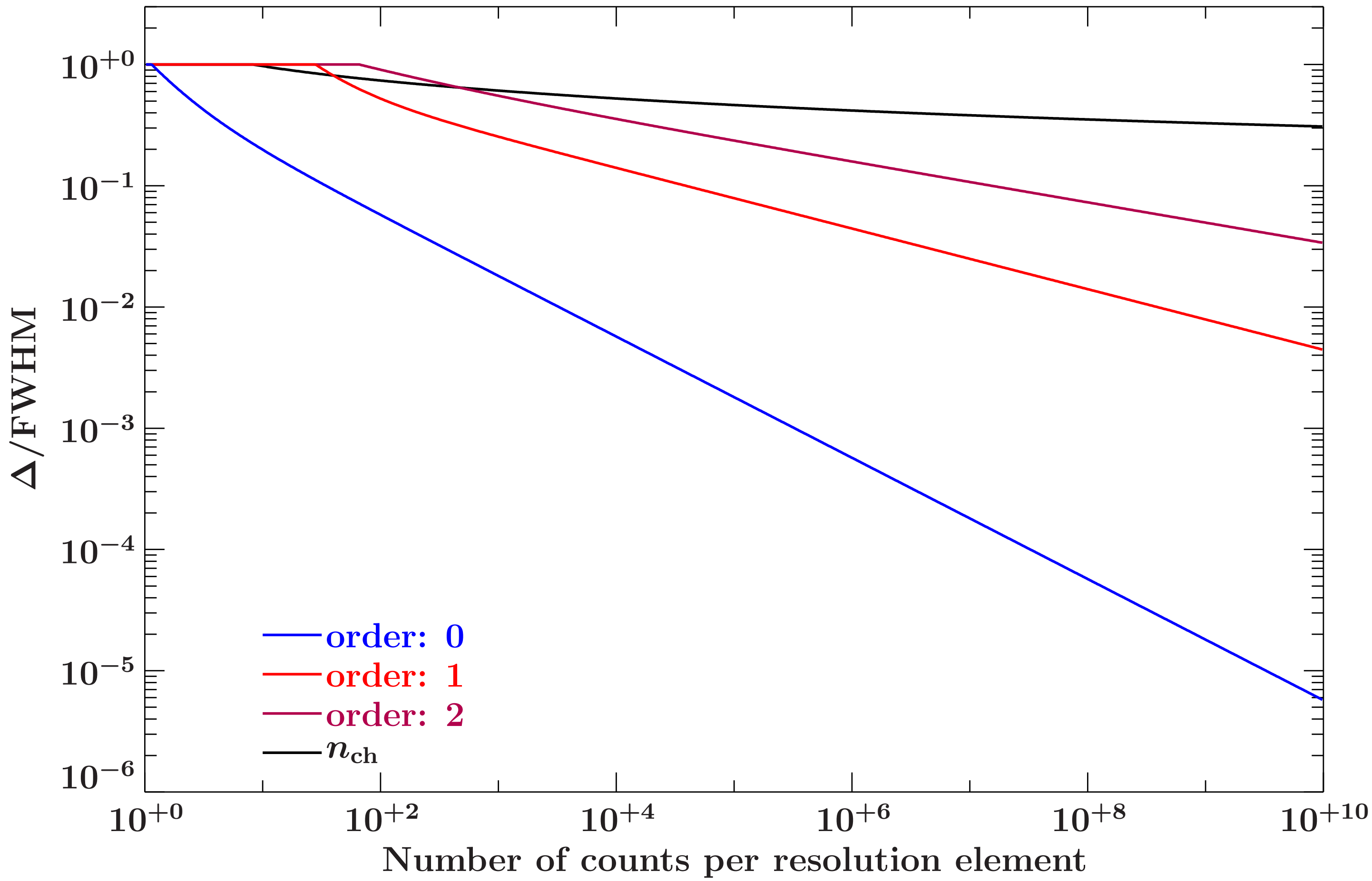
where $\Delta = 1/2W$ and where W: frequency above which Fourier transform of $f(x)$ is 0.

i.e., can describe $f(x)$ fully using its values at discretely sampled values

X-rays: measure channel integrated quantities at boundaries $m\Delta \Longrightarrow$ Integrate the above:

$$F(m\Delta) = \frac{\Delta}{\pi} \sum_{n=-\infty}^{\infty} f(n\Delta) \left(\frac{\pi}{2} + \mathrm{Si}[\pi(m-n)]\right) \tag{11}$$

Number of PI channels



after K&B, Fig. 1

Determine Δ optimal for N counts per resolution element and R resolution elements:

$$\frac{\Delta}{\mathrm{FWHM}} = \begin{cases} 1 & \text{if } x \leqslant 2.119 \\ \frac{0.08+7/x+1.8/x^2}{1+5.9/x} & \text{otherwise} \end{cases} \tag{12}$$

where $x = \ln\left(N(1 + 0.2 \ln R)\right)$.