Dynamic system classifier

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OUTLINE

MOTIVATION- COMPLEX SYSTEMS

To classify complex dynamical systems

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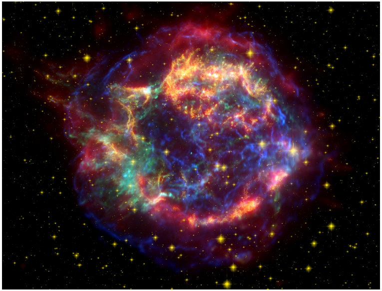
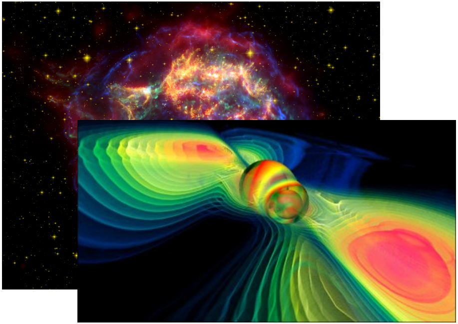


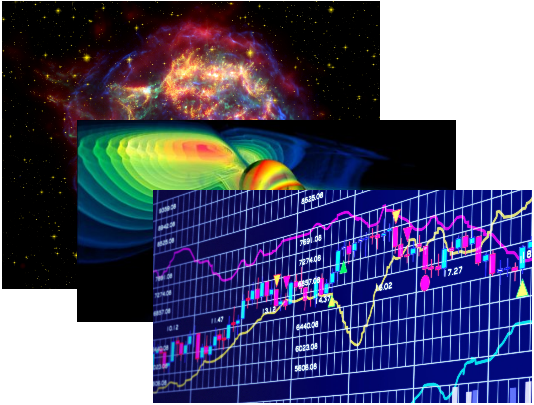
MOTIVATION- COMPLEX SYSTEMS

To classify complex dynamical systems



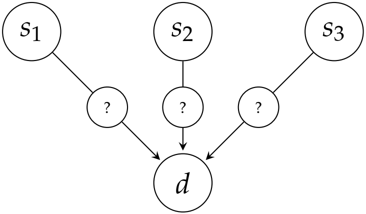
MOTIVATION- COMPLEX SYSTEMS

To classify complex dynamical systems



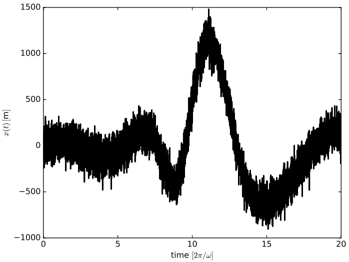
THE GOAL

To classify complex dynamical systems



system classes

data



Bayes Theorem

"Information is what forces a change in belief" *by Caticha*

$$\mathcal{P}(s \mid d) = \frac{\mathcal{P}(d \mid s)\mathcal{P}(s)}{\mathcal{P}(d)}$$

STOCHASTIC DIFFERENTIAL EQUATION (SDE)

oscillating dynamical systems

$$\frac{d^2x(t)}{dt^2} + \gamma\frac{dx(t)}{dt} + \omega^2 x(t) = F(t)$$

STOCHASTIC DIFFERENTIAL EQUATION (SDE)

complex dynamical systems

$$\frac{d^2x(t)}{dt^2} + \underline{\gamma(t)}\frac{dx(t)}{dt} + \underline{\omega_0\, e^{\beta(t)}}x(t) = \underline{\xi(t)}$$

STOCHASTIC DIFFERENTIAL EQUATION (SDE)

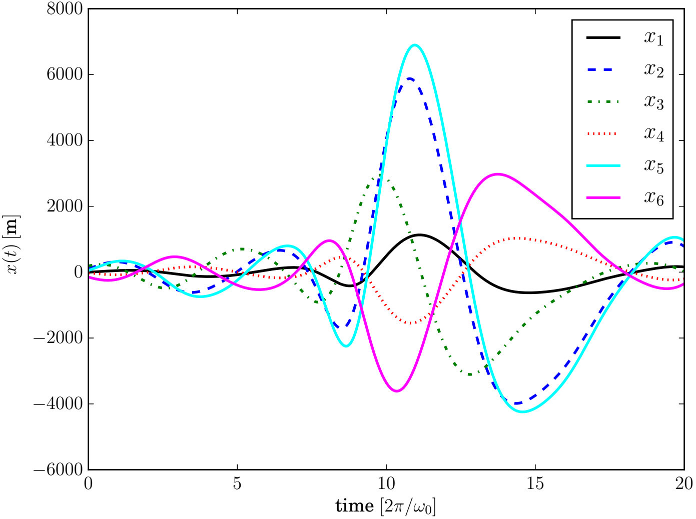
complex dynamical systems

$$\frac{d^2x(t)}{dt^2} + \underline{\gamma(t)}\frac{dx(t)}{dt} + \underline{\omega_0\, e^{\beta(t)}}x(t) = \underline{\xi(t)}$$

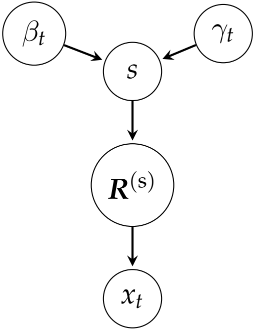
Operator form

$$\begin{aligned} x_t &= R^{(s)}_{tt'}\xi_{t'} \\ \left(\boldsymbol{R}^{(\mathrm{s})}_{\mathrm{tt'}}\right)^{-1} &= \delta^{(2)}\left(t-t'\right) - \gamma_t\delta^{(1)}\left(t-t'\right) + \omega_0\, e^{\beta_t}\delta\left(t-t'\right) \end{aligned}$$

TRAINING DATA



CONSTRUCTION OF THE LIKELIHOOD

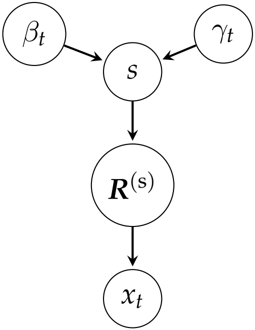


signal field

Response operator

training data

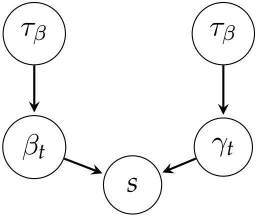
THE LIKELIHOOD OF A SDE



$$\mathcal{P}(x|s) = \mathscr{G}\left(x, \boldsymbol{R}^{(\mathrm{s})\dagger}\, \boldsymbol{\Xi}\, \boldsymbol{R}^{(\mathrm{s})}\right)$$

- temporarily structured covariance
- characterizes a non-stationary processes

THE PRIOR



$$\mathcal{P}(\beta_t|\mathbf{\Omega}) = \mathscr{G}\left(\beta_t, \mathbf{\Omega}\right)$$

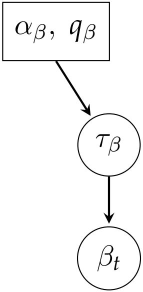
assuming statistical stationarity:

$$\mathbf{\Omega} = \sum_k e^{\tau_k} \mathbf{\Omega}_k$$

A HIERARCHICAL PRIOR MODEL

inverse Gamma Distribution

$\mathcal{P}(e^{\tau}|\alpha, q)$



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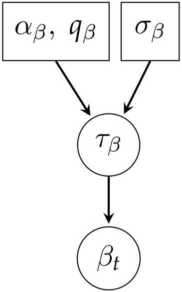
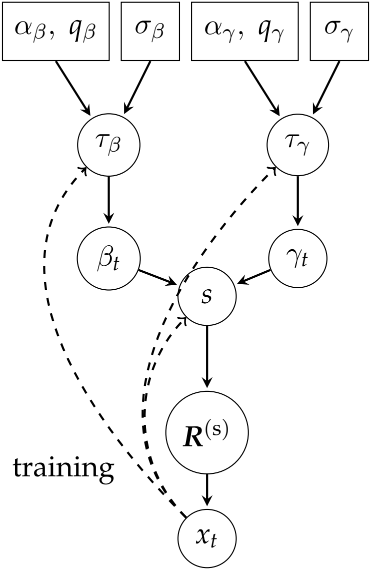


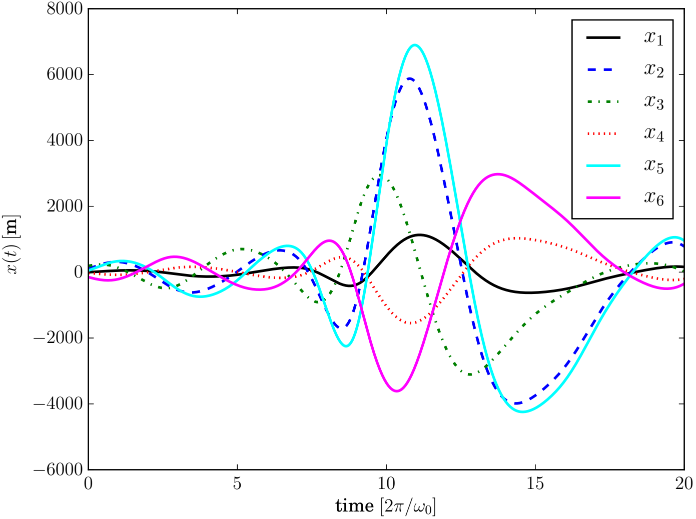
smoothness enforcing

$\mathcal{P}(\tau|\sigma)$

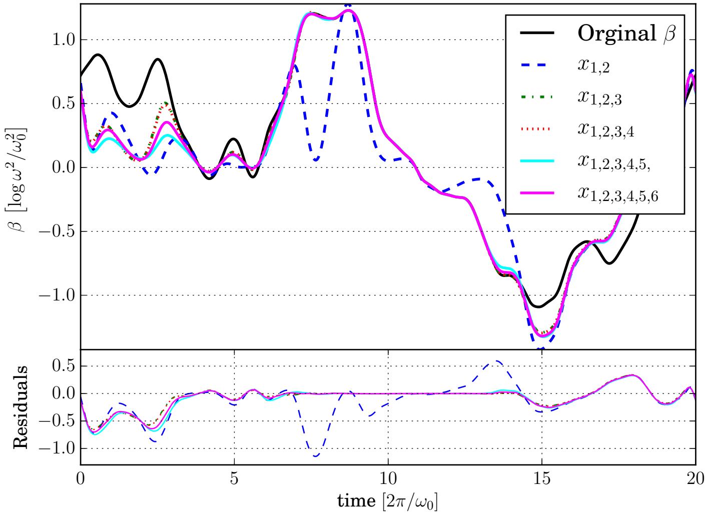
THE MODEL TRAINING



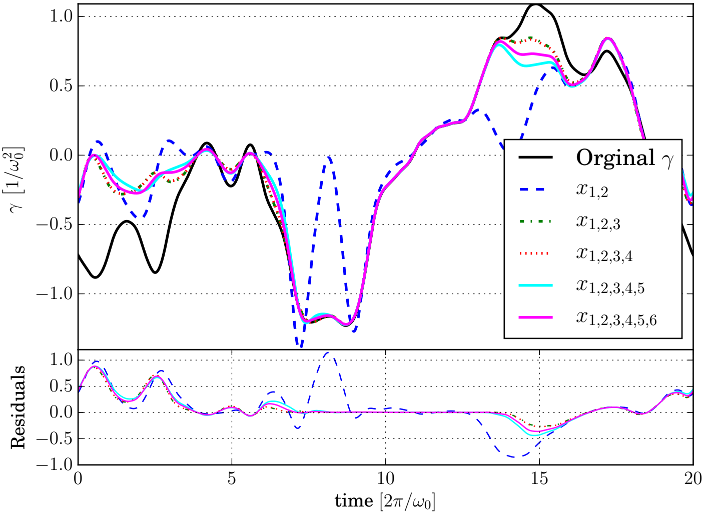
TRAINING DATA



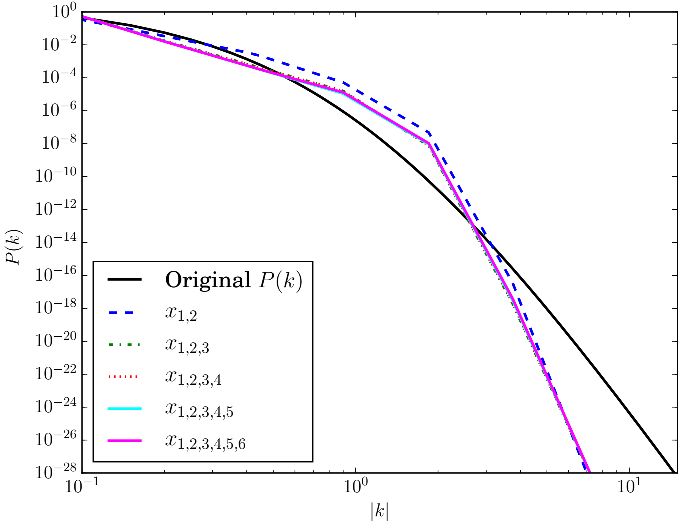
RECONSTRUCTED β_{REC}



RECONSTRUCTED γ_{REC}



RECONSTRUCTED $P(k)$



MODEL SELECTION

$$\boldsymbol{d} = \boldsymbol{R}_{\text{OBS}} x + n = \boldsymbol{R}_{\text{OBS}} \, \boldsymbol{R}^{(\text{s})} \, \xi + n$$

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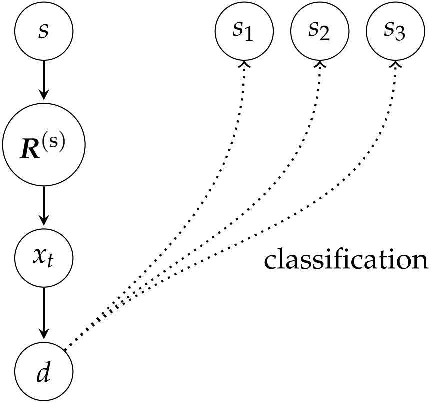

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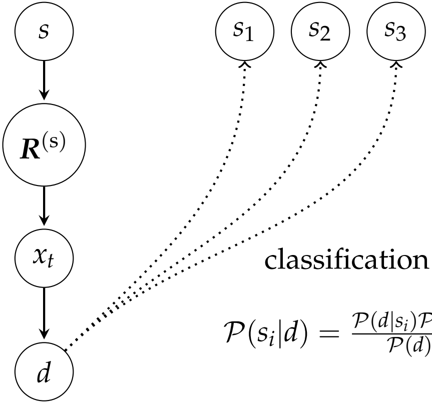
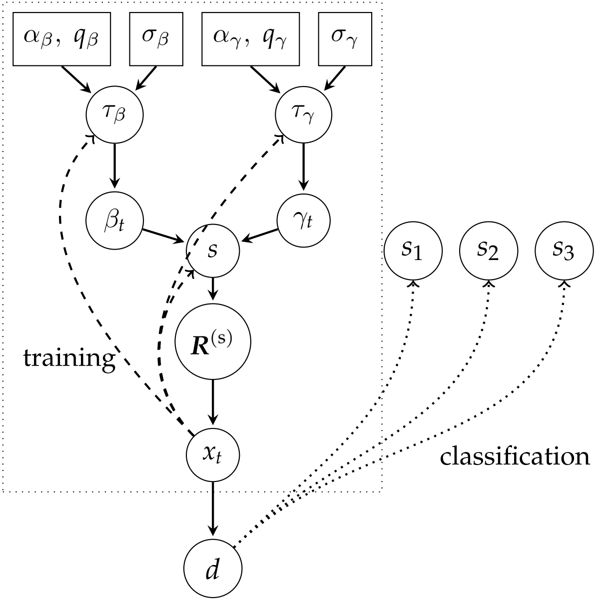
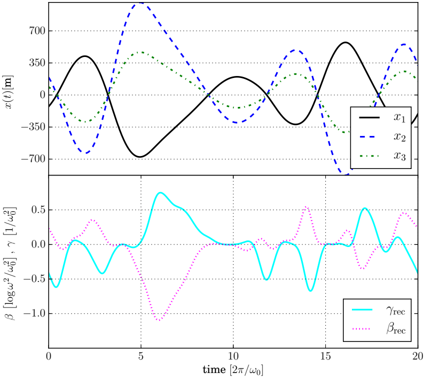


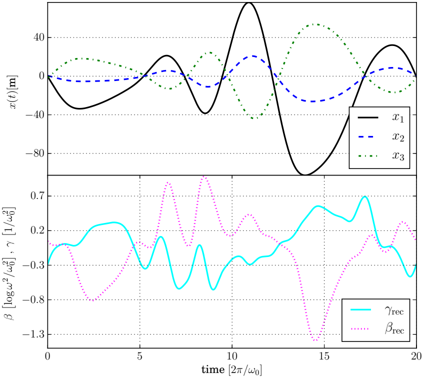
$$\mathcal{P}(s_i|d) = \frac{\mathcal{P}(d|s_i)\mathcal{P}(s_i)}{\mathcal{P}(d)}$$

THE BAYESIAN NETWORK OF DSC



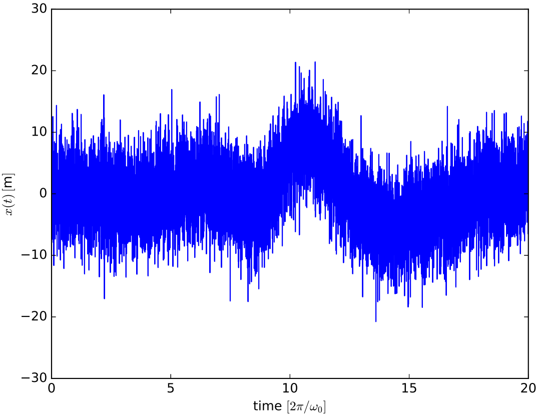
MODEL SELECTION- THE SYSTEM CLASSES





TEST CASE- SNR= 10

$$\Delta_{i,j} = \log \mathcal{P}(d|s_i) - \log \mathcal{P}(d|s_j)$$



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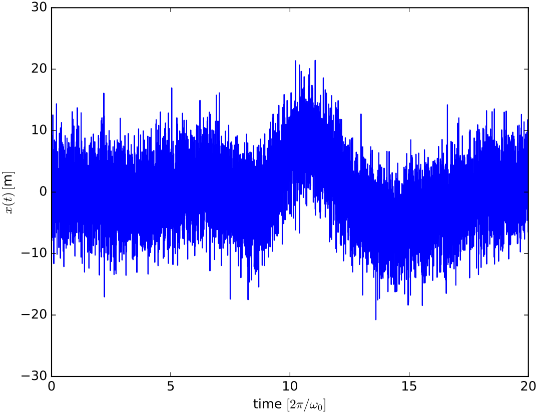
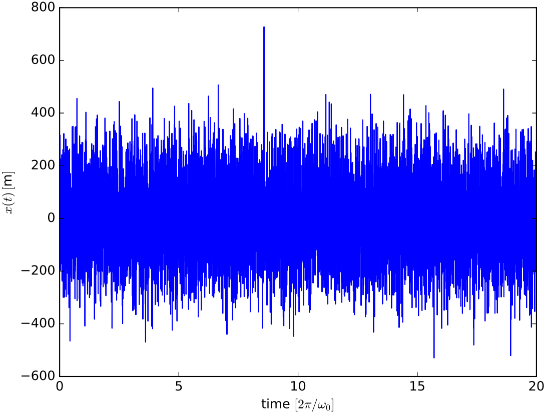


SNR=10	$\Delta_{i,j=1}$	$\Delta_{i,j=2}$	$\Delta_{i,j=3}$
d_{s_1}	0	-4352	-1757

TEST CASE- SNR= 0.01

$$\Delta_{i,j} = \log \mathcal{P}(d|s_i) - \log \mathcal{P}(d|s_j)$$



SNR=0.01	$\Delta_{i,j=1}$	$\Delta_{i,j=2}$	$\Delta_{i,j=3}$
d_{s_1}	0	-6	-5

PERFORMANCE OF DSC

$$\Delta_{i,j} = \log \mathcal{P}(d|s_i) - \log \mathcal{P}(d|s_j)$$

SNR=0.01	$\Delta_{i,j=1}$	$\Delta_{i,j=2}$	$\Delta_{i,j=3}$
d_{s_1}	0	-6	-5
d_{s_2}	-8	0	-10
d_{s_3}	0	0	0
SNR=0.1			
d_{s_1}	0	-144	-55
d_{s_2}	-151	0	-139
d_{s_3}	-1	-12	-0
SNR=10			
d_{s_1}	0	-4352	-1757
d_{s_2}	-6355	0	-5724
d_{s_3}	-60	-136	0

CONCLUSION

- DSC algorithm is established:
  1. Analyzes training data from system classes to construct abstract classifying information
  2. Confronts data with the system classes, to state the probability which system class explains observations best
- The classification ability of the DSC-algorithm has successfully been demonstrated in realistic numerical tests
- The DSC-algorithm should be applicable to a wide range of model selection problems

Thanks for your attention!

CLASSIFICATION- THE LIKELIHOOD

$$\boldsymbol{d} = \boldsymbol{R}_{\text{OBS}} x + n = \boldsymbol{R}_{\text{OBS}}\, \boldsymbol{R}^{(\text{s})}\, \xi + n\,.$$

$$\begin{aligned}\mathcal{P}(d|s_i) &= \int \mathcal{D}x\, \mathcal{P}(d|x)\mathcal{P}(x|s_i) \\ &= \int \mathcal{D}x\; \mathscr{G}(d - \boldsymbol{R}_{\text{OBS}} x, N) \\ &\qquad \times \mathscr{G}(x, {\boldsymbol{R}^{(\text{s})}}^{\dagger}\, \Xi\, \boldsymbol{R}^{(\text{s})}) \\ &\propto \frac{1}{\sqrt{|D|}} \exp\left(\frac{1}{2} j^{\dagger} D j\right)\end{aligned}$$

with

$$j = {\boldsymbol{R}^{(\text{s})}}^{\dagger}\, \boldsymbol{R}_{\text{OBS}}^{\dagger} N^{-1} d$$

and

$$D^{-1} = {\boldsymbol{R}^{(\text{s})}}^{\dagger}\, \boldsymbol{R}_{\text{OBS}}^{\dagger} N^{-1} \boldsymbol{R}_{\text{OBS}}\, \boldsymbol{R}^{(\text{s})} + \boldsymbol{\Xi}^{-1}.$$