

Inverse Compton: sun, stars and interstellar

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1 Inverse Compton from Sun and Stars

This document generated on November 12, 2012.

Documentation of software, with examples and derivation of formulae. More information can be found in README and in the driver programme provided. This will also be included here in future.

2 Examples

These examples were generated using linear and logarithmic intervals of the angle θ from the sun. The graphics were generated with the *idl* commands output at run time.

The profiles show the decrease at the solar limb due to occultation of gamma rays from beyond the sun, and also the effect of the anisotropy of the solar radiation field, which reduces to almost zero the emission between the earth and the sun, due to IC scattering at very small angles. In contrast, the emission at small angles outside the disk includes the head-on scattering from beyond 1 AU, which is maximum. Hence the emission here is dominated by the distant regions.

3 Anisotropic IC

Define

ϵ_1 = target photon energy in lab

ϵ_2 = gamma-ray energy in lab

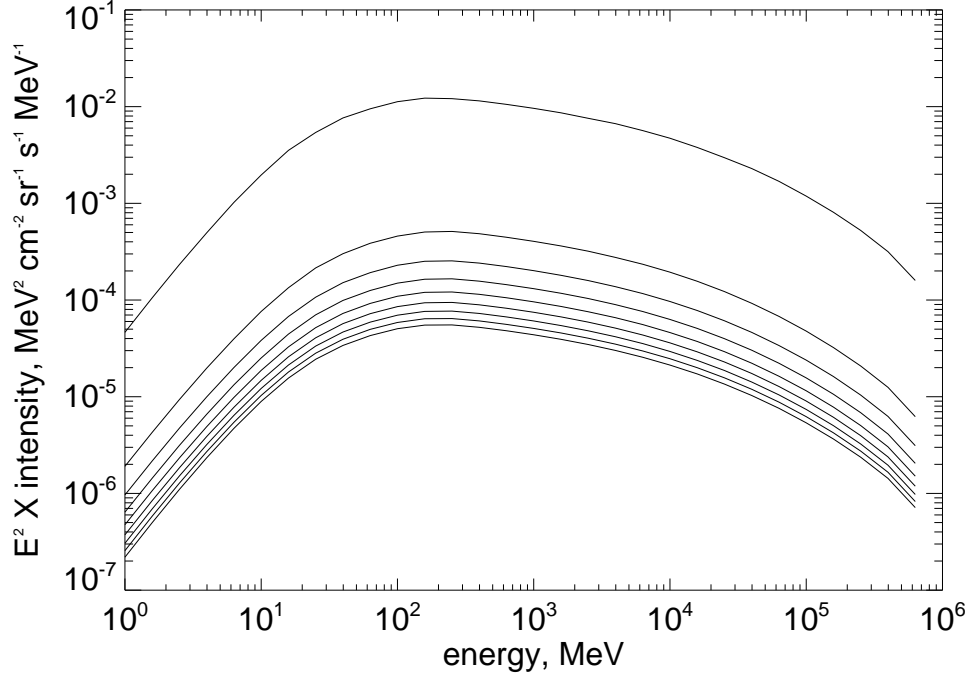


Figure 1: Spectra for various angles from sun:
 $0.3^\circ, 5^\circ, 10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ, 35^\circ, 40^\circ$

ϵ'_1 = target photon energy in electron system

ϵ'_2 = gamma-ray energy in electron system

E_e = electron energy

η = scatter angle of photon in lab

η' = scatter angle of photon in electron system

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{r_e^2}{2} \left(\frac{\epsilon'_2}{\epsilon'_1} \right)^2 \left[\frac{\epsilon'_2}{\epsilon'_1} + \frac{\epsilon'_1}{\epsilon'_2} - \sin^2 \eta' \right]$$

$$\epsilon'_2 = \frac{\epsilon'_1}{1 + \frac{\epsilon'_1}{m_e} (1 - \cos \eta')}$$

$$d\Omega = 2\pi \sin \eta' d\eta' = 2\pi d \cos \eta'$$

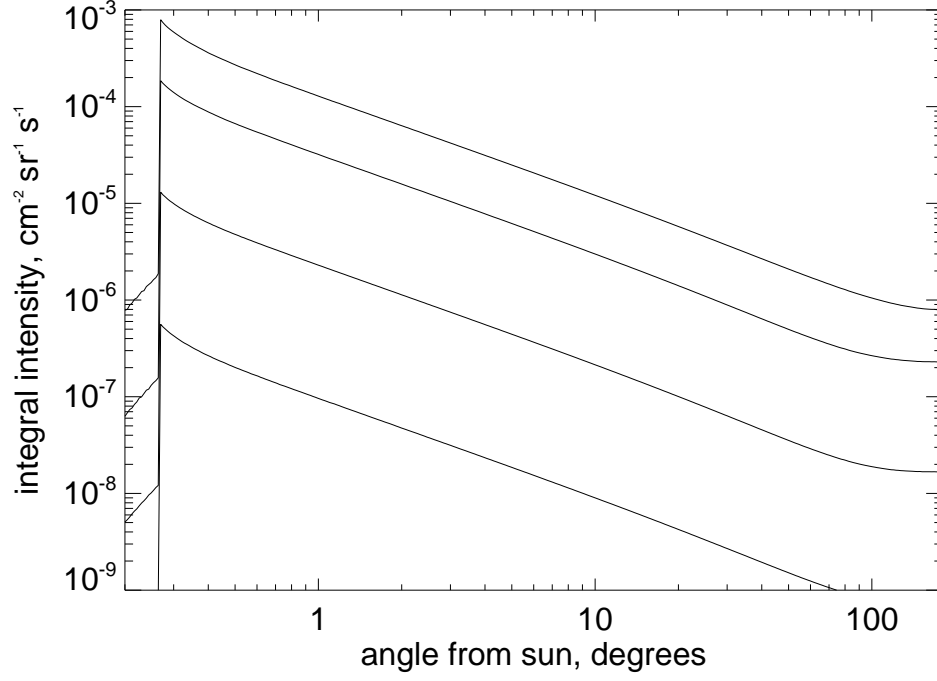


Figure 2: Angular profiles for various integral energies

$$d\epsilon'_2 = \frac{\epsilon'_1}{m_e} \frac{\epsilon'_1}{[1 + \frac{\epsilon'_1}{m_e}(1 - \cos \eta')]} d \cos \eta' = \frac{\epsilon'_1}{m_e} \frac{(\epsilon'_2)^2}{\epsilon'_1} d \cos \eta'$$

$$\frac{d \cos \eta'}{d\epsilon'_2} = \frac{m_e}{(\epsilon'_2)^2}$$

$$\frac{d\sigma_{KN}}{d\epsilon'_2} = \frac{d\sigma_{KN}}{d\Omega} \frac{d\Omega}{d \cos \eta'} \frac{d \cos \eta'}{d\epsilon'_2} = 2\pi \frac{m_e}{(\epsilon'_2)^2} \frac{d\sigma_{KN}}{d\Omega}$$

$$\frac{d\sigma_{KN}}{d\epsilon'_2} = \pi r_e^2 \frac{m_e}{(\epsilon'_1)^2} \left[\frac{\epsilon'_2}{\epsilon'_1} + \frac{\epsilon'_1}{\epsilon'_2} - \sin^2 \eta' \right]$$

$$\epsilon_2 = \gamma \epsilon'_2 (1 - \cos \eta')$$

$$(1 - \cos \eta') = \frac{m_e}{\epsilon'_1} \left(\frac{\epsilon'_1}{\epsilon'_2} - 1 \right)$$

$$\frac{\epsilon_2}{E_e} = \gamma \epsilon'_2 (1 - \cos \eta') \frac{1}{\gamma m_e} = \epsilon'_2 \frac{m_e}{\epsilon'_1} \left(\frac{\epsilon'_1}{\epsilon'_2} - 1 \right) \frac{1}{m_e} = 1 - \frac{\epsilon'_2}{\epsilon'_1}$$

$$\frac{d\sigma_{KN}}{d\epsilon_2} = \frac{d\sigma_{KN}}{d\epsilon'_2} \frac{d\epsilon'_2}{d\epsilon_2} = \frac{d\sigma_{KN}}{d\epsilon'_2} \frac{\epsilon'_1}{E_e}$$

$$\frac{d\sigma_{KN}}{d\epsilon_2} = \frac{\epsilon'_1}{E_e} \pi r_e^2 \frac{m_e}{(\epsilon'_1)^2} \left[\frac{\epsilon'_2}{\epsilon'_1} + \frac{\epsilon'_1}{\epsilon'_2} - \sin^2 \eta' \right] = \pi r_e^2 \frac{m_e}{\epsilon'_1 E_e} \left[\frac{\epsilon'_2}{\epsilon'_1} + \frac{\epsilon'_1}{\epsilon'_2} - \sin^2 \eta' \right]$$

$$\epsilon'_1 = \gamma \epsilon_1 (1 - \cos \eta)$$

$$\frac{dR}{d\epsilon_2} = n(\epsilon_1) c (1 - \cos \eta) \frac{d\sigma_{KN}}{d\epsilon_2} = n(\epsilon_1) c \frac{\epsilon'_1}{\gamma \epsilon_1} \frac{d\sigma_{KN}}{d\epsilon_2}$$

$$\frac{dR}{d\epsilon_2} = n(\epsilon_1) c (1 - \cos \eta) \frac{d\sigma_{KN}}{d\epsilon_2} = n(\epsilon_1) c \frac{\epsilon'_1}{\gamma \epsilon_1} \pi r_e^2 \frac{m_e}{\epsilon'_1 E_e} \left[\frac{\epsilon'_2}{\epsilon'_1} + \frac{\epsilon'_1}{\epsilon'_2} - \sin^2 \eta' \right]$$

$$\frac{dR}{d\epsilon_2} = n(\epsilon_1) c \pi r_e^2 \frac{m_e}{\gamma \epsilon_1 E_e} \left[\frac{\epsilon'_2}{\epsilon'_1} + \frac{\epsilon'_1}{\epsilon'_2} - \sin^2 \eta' \right]$$

$$\frac{dR}{d\epsilon_2} = n(\epsilon_1) c \pi r_e^2 \frac{m_e^2}{\epsilon_1 E_e^2} \left[\frac{\epsilon'_2}{\epsilon'_1} + \frac{\epsilon'_1}{\epsilon'_2} - \sin^2 \eta' \right]$$

Define

$$v = \frac{\epsilon_2}{E_e}$$

$$\frac{\epsilon'_2}{\epsilon'_1} = 1 - \frac{\epsilon_2}{E_e} = 1 - v$$

$$1 - \cos \eta' = \frac{m_e}{\epsilon'_1} \left[\frac{1}{1 - v} - 1 \right] = \frac{m_e}{\epsilon'_1} \frac{v}{1 - v}$$

$$-\sin^2 \eta' = \cos^2 \eta' - 1 = \left[1 - \frac{m_e}{\epsilon'_1} \frac{v}{1 - v} \right]^2 - 1 = \left(\frac{m_e}{\epsilon'_1} \right)^2 \left(\frac{v}{1 - v} \right)^2 - 2 \frac{m_e}{\epsilon'_1} \frac{v}{1 - v}$$

$$\frac{d\sigma_{KN}}{d\epsilon_2} = \pi r_e^2 \frac{m_e}{\epsilon'_1 E_e} \left[\left(\frac{m_e}{\epsilon'_1} \right)^2 \left(\frac{v}{1 - v} \right)^2 - 2 \frac{m_e}{\epsilon'_1} \frac{v}{1 - v} + (1 - v) + \frac{1}{1 - v} \right]$$

$$\frac{dR}{d\epsilon_2} = n(\epsilon_1) c \pi r_e^2 \frac{m_e^2}{\epsilon_1 E_e^2} \left[\left(\frac{m_e}{\epsilon'_1} \right)^2 \left(\frac{v}{1-v} \right)^2 - 2 \frac{m_e}{\epsilon'_1} \frac{v}{1-v} + (1-v) + \frac{1}{1-v} \right]$$

To make it look like MS2000 equation 8:

$$\frac{dR}{d\epsilon_2} = n(\epsilon_1) c \pi r_e^2 \frac{m_e^2}{\epsilon_1 E_e^2 (1-v)^2} \left[\left(\frac{m_e}{\epsilon'_1} \right)^2 v^2 - 2 \frac{m_e}{\epsilon'_1} v(1-v) + (1-v)^3 + 1-v \right]$$

$$\frac{dR}{d\epsilon_2} = n(\epsilon_1) c \pi r_e^2 \frac{m_e^2}{\epsilon_1 (E_e - \epsilon_2)^2} \left[\left(\frac{m_e}{\epsilon'_1} \right)^2 v^2 - 2 \frac{m_e}{\epsilon'_1} v(1-v) + (1-v)^3 + 1-v \right]$$

$$\frac{dR}{d\epsilon_2} = n(\epsilon_1) c \pi r_e^2 \frac{m_e^2}{\epsilon_1 (E_e - \epsilon_2)^2} \left[\left(\frac{m_e}{\epsilon'_1} \right)^2 v^2 - 2 \frac{m_e}{\epsilon'_1} v(1-v) + 2 - 4v + 3v^2 - v^3 \right]$$

$$\frac{dR}{d\epsilon_2} = n(\epsilon_1) c \pi r_e^2 \frac{m_e^2}{\epsilon_1 (E_e - \epsilon_2)^2} \left[2 - 2v \left(\frac{m_e}{\epsilon'_1} + 2 \right) + v^2 \left(\left(\frac{m_e}{\epsilon'_1} \right)^2 + \frac{2m_e}{\epsilon'_1} + 3 \right) - v^3 \right]$$

This is MS2000 form with

$$v = \frac{\epsilon_2}{\gamma m_e}$$

and

$$\zeta = -\eta$$

4 Gamma-ray emissivity - δ -function approximation to IC cross-section

Simple isotropic low-energy power-laws case:

$$I(\epsilon_2) = \frac{1}{4\pi} \int q(\epsilon_2) ds$$

emissivity

$$q(\epsilon_2) d\epsilon_2 = N(E_e) dE_e n_{ph} \sigma_T c$$

$$q(\epsilon_2) = N(E_e) \frac{dE_e}{d\epsilon_2} n_{ph} \sigma_{TC}$$

$$\epsilon_2 = \frac{4}{3} \gamma^2 \epsilon_1$$

$$\epsilon_2 = \frac{4}{3} \left(\frac{E_e}{m_e} \right)^2 \epsilon_1$$

$$d\epsilon_2 = \frac{4}{3} \cdot 2 \frac{E_e}{m_e^2} dE_e \epsilon_1$$

$$\frac{dE_e}{d\epsilon_2} = \frac{3}{8} \cdot \frac{m_e^2}{E_e} \frac{1}{\epsilon_1}$$

$$q(\epsilon_2) = N(E_e) \frac{3}{8} \cdot \frac{m_e^2}{E_e} \frac{1}{\epsilon_1} n_{ph} \sigma_{TC}$$

$$N(E_e) = A E_e^{-p}$$

$$q(\epsilon_2) = A E_e^{-p} \frac{3}{8} \cdot \frac{m_e^2}{E_e} \frac{1}{\epsilon_1} n_{ph} \sigma_{TC}$$

$$q(\epsilon_2) = A E_e^{-(p+1)} \frac{3}{8} \cdot \frac{m_e^2}{\epsilon_1} n_{ph} \sigma_{TC}$$

$$E_e = \left(\frac{3}{4} \frac{\epsilon_2 m_e^2}{\epsilon_1} \right)^{1/2}$$

$$q(\epsilon_2) = A \left(\frac{3}{4} \frac{\epsilon_2 m_e^2}{\epsilon_1} \right)^{-(p+1)/2} \frac{3}{8} \frac{m_e^2}{\epsilon_1} n_{ph} \sigma_{TC}$$

$$q(\epsilon_2) = A \left(\frac{3}{4} \frac{m_e^2}{\epsilon_1} \right)^{-(p+1)/2} \frac{3}{8} \frac{m_e^2}{\epsilon_1} n_{ph} \sigma_{TC} \epsilon_2^{-(p+1)/2}$$

$$q(\epsilon_2) = \frac{A}{2} \left(\frac{3}{4} \frac{m_e^2}{\epsilon_1} \right)^{-(p-1)/2} n_{ph} \sigma_{TC} \epsilon_2^{-(p+1)/2}$$

$$q(\epsilon_2) = \frac{A}{2} \left(\frac{3}{4} \frac{m_e^2}{\epsilon_1} \right)^{-(p-1)/2} n_{ph} \sigma_{TC} \epsilon_2^{-(p+1)/2}$$

5 Planck spectrum - δ -function approximation to IC cross-section

$$n_{ph}(\epsilon_1) = \frac{1}{\pi^2 \hbar^3 c^3} \frac{\epsilon_1^2}{e^{\epsilon_1/kT} - 1}$$

$$q(\epsilon_2) = \int \frac{A}{2} \left(\frac{3}{4} \frac{m_e^2}{\epsilon_1} \right)^{-(p-1)/2} n_{ph}(\epsilon_1) \sigma_{TC} \epsilon_2^{-(p+1)/2} d\epsilon_1$$

$$q(\epsilon_2) = \sigma_{TC} \epsilon_2^{-(p+1)/2} \frac{A}{2} \left(\frac{3}{4} m_e^2 \right)^{-(p-1)/2} \int \epsilon_1^{(p-1)/2} n_{ph}(\epsilon_1) d\epsilon_1$$

$$q(\epsilon_2) = \sigma_{TC} \epsilon_2^{-(p+1)/2} \frac{A}{2} \left(\frac{3}{4} m_e^2 \right)^{-(p-1)/2} \frac{1}{\pi^2 \hbar^3 c^3} \int \frac{\epsilon_1^{2+(p-1)/2}}{e^{\epsilon_1/kT} - 1} d\epsilon_1$$

For $p = 3$

$$q(\epsilon_2) = \sigma_{TC} \epsilon_2^{-(p+1)/2} \frac{A}{2} \left(\frac{3}{4} m_e^2 \right)^{-(p-1)/2} \frac{1}{\pi^2 \hbar^3 c^3} w_{ph}$$

where $w_{ph} = \int \epsilon_1 n_{ph}(\epsilon_1) d\epsilon_1$ = energy density

Define

$$y = \epsilon_1/kT$$

$$\epsilon_1 = kTy$$

$$q(\epsilon_2) = \sigma_{TC} \epsilon_2^{-(p+1)/2} \frac{A}{2} \left(\frac{3}{4} m_e^2 \right)^{-(p-1)/2} \frac{1}{\pi^2 \hbar^3 c^3} \int \frac{(kTy)^{2+(p-1)/2}}{e^y - 1} kT dy$$

$$q(\epsilon_2) = \sigma_{TC} \epsilon_2^{-(p+1)/2} \frac{A}{2} \left(\frac{3}{4} m_e^2 \right)^{-(p-1)/2} \frac{1}{\pi^2 \hbar^3 c^3} (kT)^{3+(p-1)/2} \int \frac{y^{2+(p-1)/2}}{e^y - 1} dy$$

For $p = 3$

$$\int \frac{y^3}{e^y - 1} dy = \frac{\pi^4}{15}$$

$$q(\epsilon_2) = \sigma_{TC} \epsilon_2^{-2} \frac{A}{2} \left(\frac{3}{4} m_e^2 \right)^{-1} \frac{1}{\pi^2 \hbar^3 c^3} (kT)^4 \frac{\pi^4}{15}$$

6 Planck spectrum - low-energy approximation to IC cross-section

Blumenthal and Gould (1970) eq 2.43, 2.44, 2.45

$$\frac{dR}{d\epsilon_2} = 8\pi r_o^2 c \frac{1}{4\epsilon_1 \gamma^2} n_{ph}(\epsilon_1) (2x \ln x + x + 1 - 2x^2)$$

$$x = \frac{\epsilon_2}{4\gamma^2 \epsilon_1}, \quad 0 < x < 1$$

$$\frac{dR}{d\epsilon_2} = \frac{8\pi r_o^2 c}{\epsilon_2} n_{ph}(\epsilon_1) (2x \ln x + x + 1 - 2x^2) x$$

$$q(\epsilon_2) = \int_{\epsilon_1} \int_{x=0}^{x=1} \frac{1}{\pi^2 \hbar^3 c^3} \frac{\epsilon_1^2}{e^{\epsilon_1/kT} - 1} \frac{8\pi r_o^2 c}{\epsilon_2} (2x \ln x + x + 1 - 2x^2) x d\epsilon_1 N(E_e) dE_e$$

$$q(\epsilon_2) = \frac{8\pi r_o^2 c}{\epsilon_2} \int_{\epsilon_1} \frac{1}{\pi^2 \hbar^3 c^3} \frac{\epsilon_1^2}{e^{\epsilon_1/kT} - 1} d\epsilon_1 \int_{x=0}^{x=1} (2x \ln x + x + 1 - 2x^2) x N(E_e) dE_e$$

$$N(E_e) = A E_e^{-p}$$

$$E_e = \gamma m = \left(\frac{\epsilon_2}{4\epsilon_1 x} \right)^{\frac{1}{2}} m$$

$$dE_e = \frac{1}{2} \left(\frac{\epsilon_2}{4\epsilon_1} \right)^{\frac{1}{2}} x^{-3/2} m dx$$

$$\begin{aligned} & \int_{x=0}^{x=1} (2x \ln x + x + 1 - 2x^2) x N(E_e) dE_e \\ &= \int_{x=0}^{x=1} (2x \ln x + x + 1 - 2x^2) x A \left(\frac{\epsilon_2}{4\epsilon_1 x} \right)^{-\frac{p}{2}} m^{-p} \frac{1}{2} \left(\frac{\epsilon_2}{4\epsilon_1} \right)^{\frac{1}{2}} x^{-3/2} m dx \\ &= \frac{A}{2} m^{-p+1} \int_{x=0}^{x=1} (2x \ln x + x + 1 - 2x^2) x \left(\frac{\epsilon_2}{4\epsilon_1 x} \right)^{-\frac{p}{2}} \left(\frac{\epsilon_2}{4\epsilon_1} \right)^{\frac{1}{2}} x^{-3/2} dx \\ &= \frac{A}{2} m^{-p+1} \left(\frac{\epsilon_2}{4\epsilon_1} \right)^{-\frac{p-1}{2}} \int_{x=0}^{x=1} (2x \ln x + x + 1 - 2x^2) x^{\frac{p-1}{2}} dx \end{aligned}$$

$$q(\epsilon_2) = \frac{8\pi r_o^2 c}{\epsilon_2} \int_{\epsilon_1} \frac{1}{\pi^2 \hbar^3 c^3} \frac{\epsilon_1^2}{e^{\epsilon_1/kT} - 1} d\epsilon_1 \frac{A}{2} m^{-p+1} \left(\frac{\epsilon_2}{4\epsilon_1}\right)^{-\frac{p-1}{2}} \int_{x=0}^{x=1} (2x \ln x + x + 1 - 2x^2) x^{\frac{p-1}{2}} dx$$

$$q(\epsilon_2) = 8\pi r_o^2 c \frac{A}{2} m^{-p+1} \epsilon_2^{-\frac{p+1}{2}} \int_{\epsilon_1} \frac{1}{\pi^2 \hbar^3 c^3} \frac{\epsilon_1^2}{e^{\epsilon_1/kT} - 1} d\epsilon_1 \left(\frac{1}{4\epsilon_1}\right)^{-\frac{p-1}{2}} \int_{x=0}^{x=1} (2x \ln x + x + 1 - 2x^2) x^{\frac{p-1}{2}} dx$$

$$q(\epsilon_2) = \epsilon_2^{-\frac{p+1}{2}} 8\pi r_o^2 c \frac{A}{2} m^{-p+1} \int_{x=0}^{x=1} (2x \ln x + x + 1 - 2x^2) x^{\frac{p-1}{2}} dx \int_{\epsilon_1} \frac{1}{\pi^2 \hbar^3 c^3} \frac{\epsilon_1^2}{e^{\epsilon_1/kT} - 1} d\epsilon_1 (4\epsilon_1)^{\frac{p-1}{2}}$$

$$q(\epsilon_2) = \epsilon_2^{-\frac{p+1}{2}} 8\pi r_o^2 c \frac{A}{2} m^{-p+1} 4^{\frac{p-1}{2}} \int_{x=0}^{x=1} (2x \ln x + x + 1 - 2x^2) x^{\frac{p-1}{2}} dx \int_{\epsilon_1} \frac{1}{\pi^2 \hbar^3 c^3} \frac{\epsilon_1^{2+\frac{p-1}{2}}}{e^{\epsilon_1/kT} - 1} d\epsilon_1$$