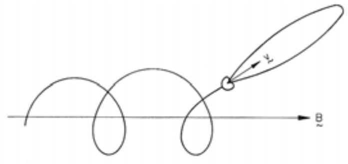
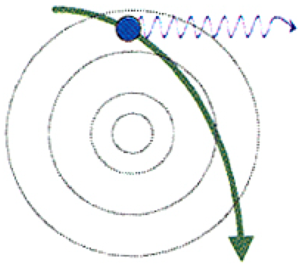
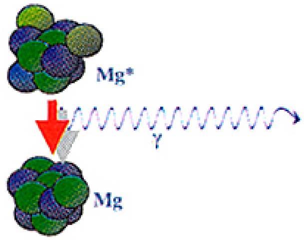
HE Astrophysics: Basic Radiation Mechanisms



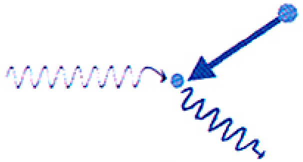
Synchrotron



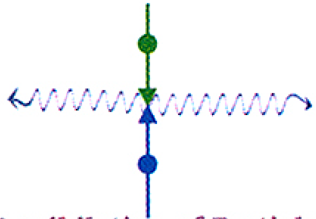
Accelerated Charged Particles



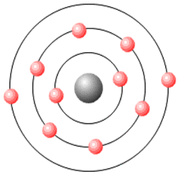
De-Excitation of Atomic Nuclei



Inverse Compton Scattering



Annihilation of Particle-Antiparticle Pairs



Characteristic X-rays

Blackbody Radiation

1. Planck functions (Brightness of a blackbody)

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\,\frac{1}{\left(\exp\frac{h\nu}{kT}-1\right)} \qquad \text{erg cm}^{-2}\text{sec}^{-1}\text{Hz}^{-1}\text{ster}^{-1}$$

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\,\frac{1}{\left(\exp\frac{hc}{\lambda kT}-1\right)} \qquad \text{erg cm}^{-2}\text{sec}^{-1}\text{cm}^{-1}\text{ster}^{-1}$$

$$B_{\tilde\nu}(T) = \frac{2hc^2\tilde\nu^3}{\left(\exp\frac{hc\tilde\nu}{kT}-1\right)} \qquad \text{erg cm}^{-2}\text{sec}^{-1}(\text{cm}^{-1})^{-1}\text{ster}^{-1}$$

$$B_\nu(T)\,d\nu = B_\lambda(T)\,d\lambda = B_{\tilde\nu}(T)\,d\tilde\nu$$

Rayleigh-Jean's law

$h\nu / kT \ll 1$

$$B_\nu(T) = 2\left(\frac{\nu}{c}\right)^2 kT$$

Wien's law

$h\nu / kT \gg 1$

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\exp\frac{-h\nu}{kT}$$

2. Stefan-Boltzmann law

$$\text{total emittance} = \pi\int_0^\infty B_\nu(T)\,d\nu = \sigma T^4 \text{ ergs cm}^{-2}\text{sec}^{-1}$$

$$\sigma = \frac{2\pi^5k^4}{15c^2h^3} = 5.67 \times 10^{-5} \text{ erg cm}^{-2}\text{ deg}^{-4}\text{ sec}^{-1}$$

3. Wien displacement law

Maximizing B_ν :

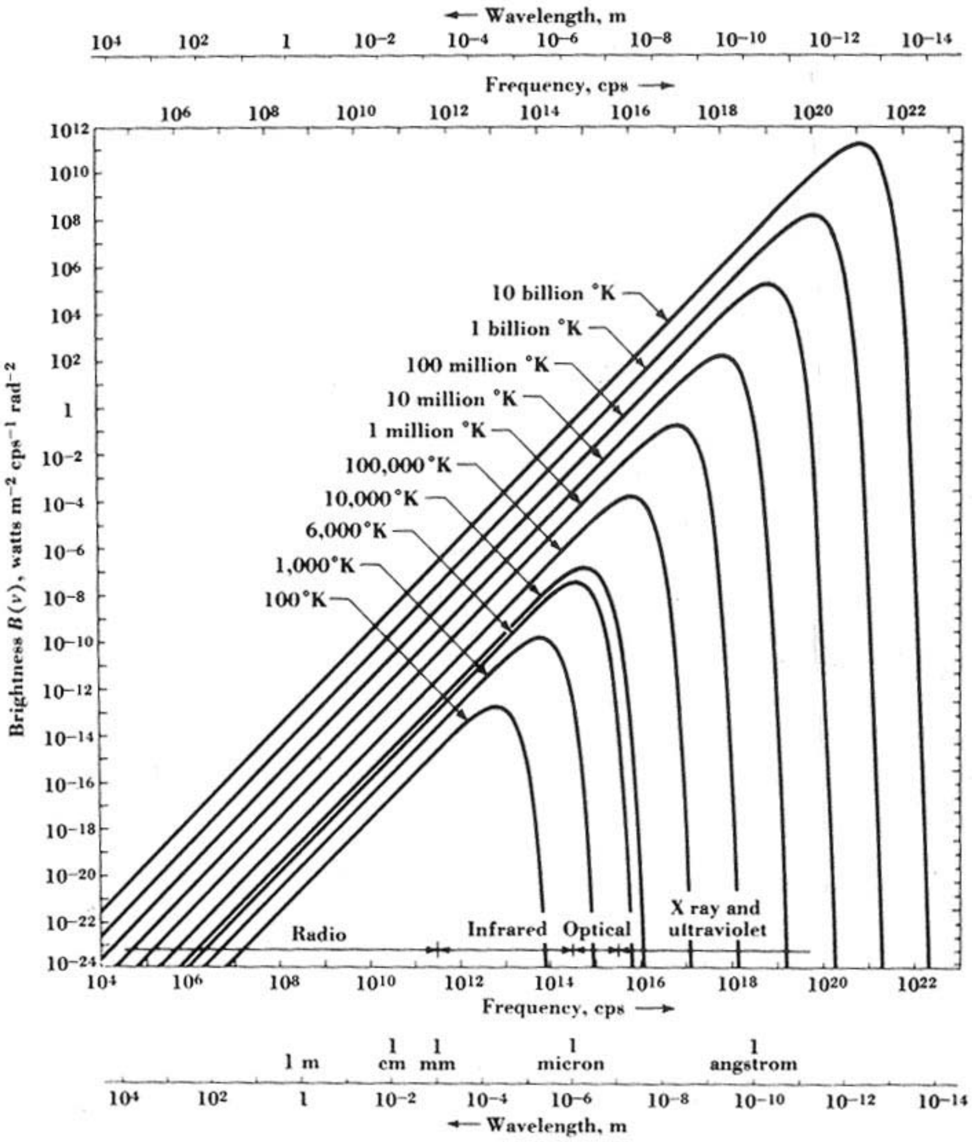
$\nu_m = 5.9 \times 10^{10}\ T$ Hz

$\lambda_m = 0.51\ T^{-1}$ cm

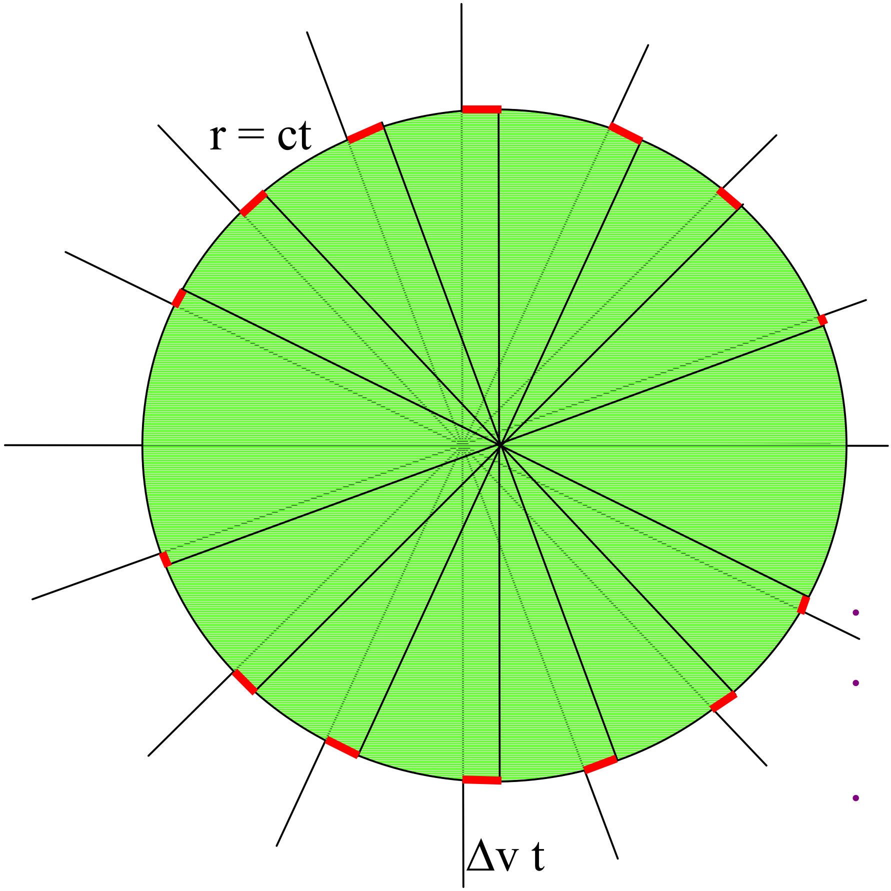
Maximizing B_λ :

$\nu_m = 10.3 \times 10^{10}\ T$ Hz

$\lambda_m = 0.29\ T^{-1}$ cm



Radiation from accelerated charged particle



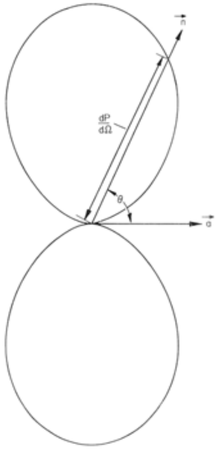


Fig. 3.1. The angular distribution of the power radiated by a non-relativistic charge undergoing an acceleration **a**. The power per unit solid angle radiated in the direction of the vector **n** is proportional to the radius vector as indicated in the figure.

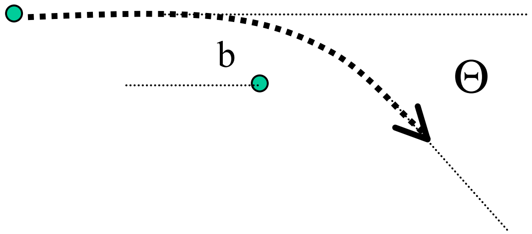
- The information about the charge acceleration is transmitted as a *pulse of electromagnetic radiation*
- The total radiation is approximated by the *Larmor formula*

$-(dE/dt) = |\ddot{p}|^2 / 6 o \varepsilon_0 c^3 = q^2 |\ddot{r}|^2 / 6 o \varepsilon_0 c^3$

where p is the dipole moment qr of the charge q

- The radiation pattern is of *dipolar form*, i.e. the power radiated varies as sin2Θ. There is no radiation along the acceleration direction
- The radiation is *polarized* with the electric field vector in the direction of the acceleration vector of the particle

Bremsstrahlung



- ☆ **Coulomb interaction of two charged particles**
  - ☞ "free-free" transitions
  - ☞ Bremsstrahlung emission
  - ☞ Free free emission
- ☆ **Non-relativistic and relativistic cases**
  - ☞ Lorentz transform between particle and observer frames
  - ☞ Decomposition of acceleration into parallel/perpendicular to v
  - ☞ Fourier analysis yields radiated spectral distribution
    - Flat up to limiting energy transfer, exponential cutoff
  - ☞ Gaunt factor describes target/environment-specific collision parameters
  - ☞ Thermal Bremsstrahlung from integration over Maxwellian velocities
  - ☞ Relativistic-particle Bremsstrahlung from QM treatment

Thermal Bremsstrahlung in Cosmic Sources

For a Maxwellian distribution of electron velocities, the spectral emission per unit volume is

$$dP_B(T)/dVd\nu = 6.8\mathrm{x}10^{-38}\ T^{-1/2}\ e^{-E/kT}\ N_e\ N_Z\ Z^2\ g_B(T,E) \quad [\mathrm{erg\ cm^{-3}\ s^{-1}\ Hz^{-1}}]$$

N_e = electron density
N_Z = ion density (charge z)
$E = h\ \nu$ = photon energy
$g_B(T,E)$ = Gaunt Factor $(E/kT)^{-0.4}$ for $E \ll kT$

The total bremsstrahlung emission is:

$$dP_B(T)/dV = 1.4 \times 10^{-27}\ T^{1/2}\ N_e\ N_Z\ Z^2\ g_B(T) \quad \text{where } g_B(T) \quad 1.2$$

For a plasma with cosmic abundances:

$$dP_B(T)/dV = 1.4 \times 10^{-27}\ T^{1/2}\ N_e^2\ ,$$

since $\Sigma\ N_e\ N_Z\ Z^2 \quad 1.4\ N_e^2$ for cosmic abundances

Bremsstrahlung occurs in optically thin thermal plasmas

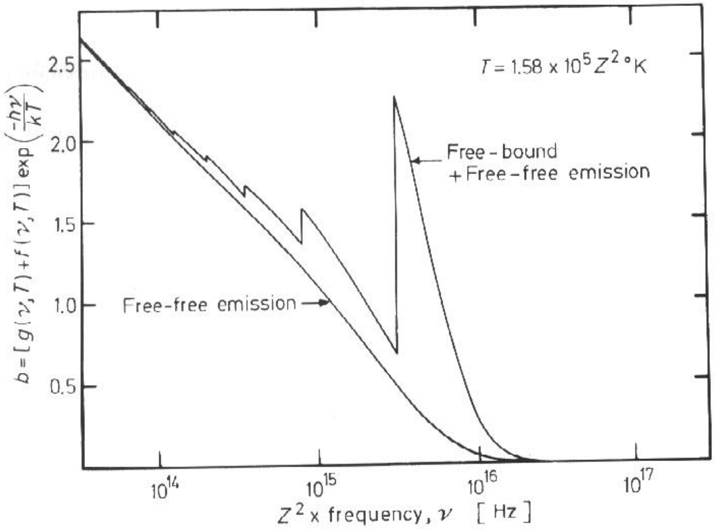
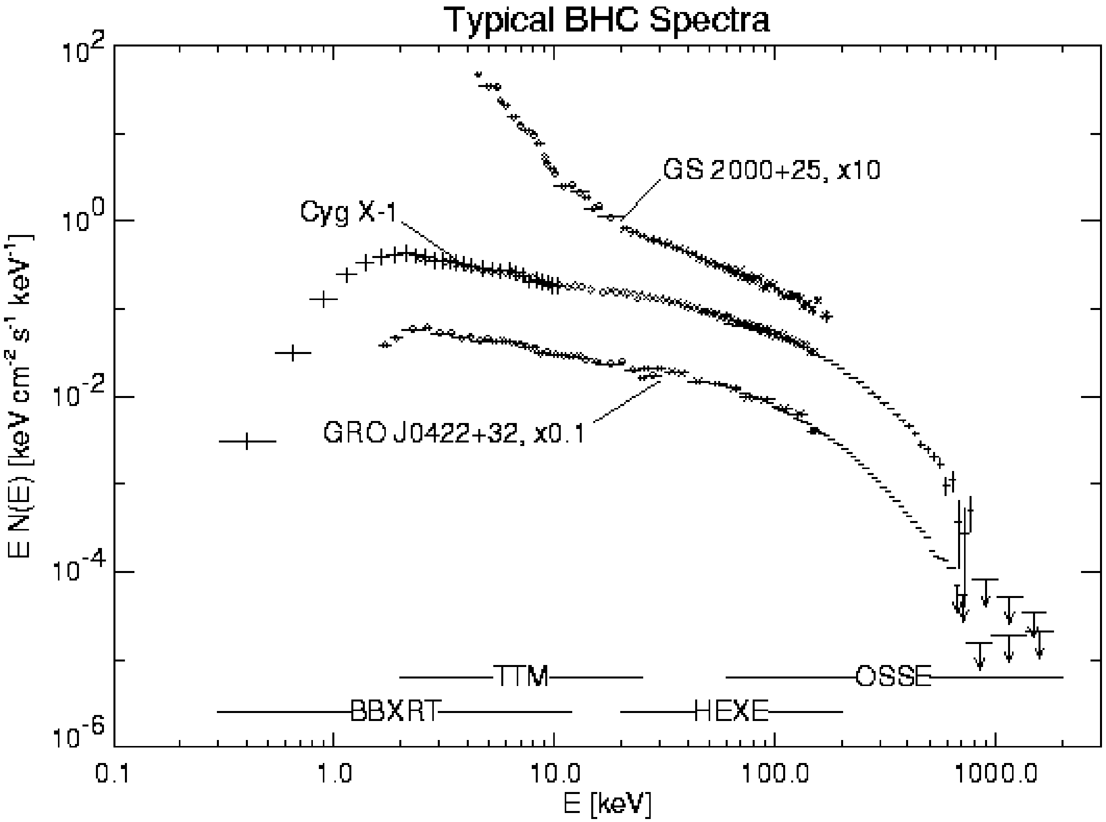


Fig. 1.5. The emission spectrum of a Maxwellian distribution of electrons at a temperature of $T = 2\pi^2 me^4 Z^2/(h^2k) = 1.58 \times 10^5 Z^2$ °K with exact Gaunt factors taken into account [after Brussard and van de Hulst, 1962]. The lower curve illustrates the spectrum of thermal bremsstrahlung emission, whereas the upper curve illustrates the combined spectrum of thermal bremsstrahlung and recombination radiation. The volume emissivity, ε_ν, is related to b by Eq. (1.237)

Typical Spectra of Compact Sources: Thermal & Non-Thermal Bremsstrahlung



(Cyg X-1: Wilms et al., 1996, ; GRO J0422+32, GS2000+25: Sunyaev et al., 1993, Kroeger [priv. comm.])

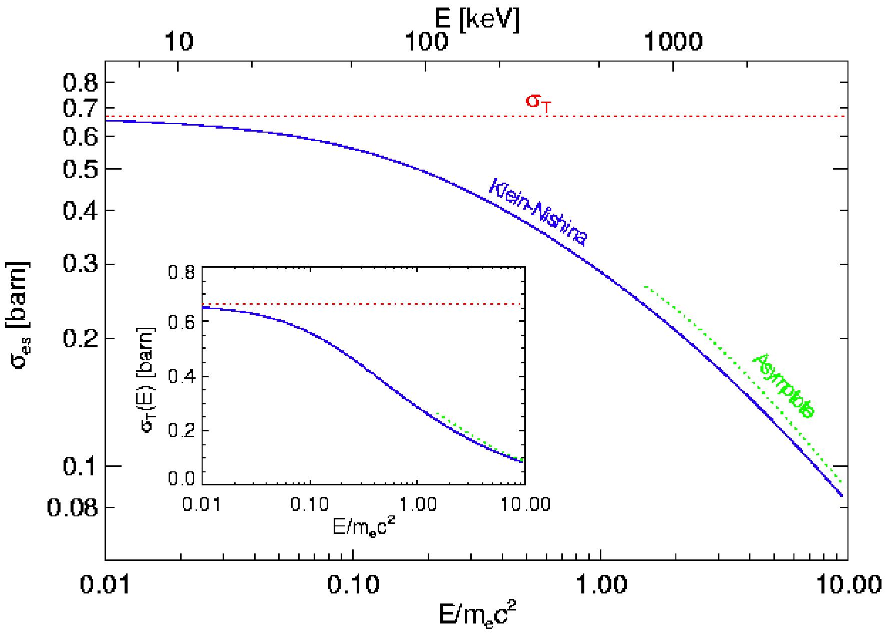
Klein-Nishina Cross Section

$$\sigma_{\mathrm{es}} = \frac{3}{4}\sigma_{\mathrm{T}}\left[\frac{1+x}{x^3}\left\{\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right\} + \frac{1}{2x}\ln(1+2x) - \frac{1+3x}{(1+2x)^2}\right]$$

where $x = E/m_e c^2$.

For $x \gg 1$,

$$\sigma \sim \frac{3}{8}\sigma_{\mathrm{T}}\frac{1}{x}\left(\ln 2x + \frac{1}{2}\right)$$



Cosmic Photons

- **Sources**
  - ☆ Thermal Emission (~10^9K Blackbody; Fireball)
  - ☆ Continuum Radiation from Accelerated Charged Particles

☞ Bremsstrahlung	(e- & nuclei)
☞ Synchrotron Radiation	(e- and magnetic field)
☞ Inverse Compton Radiation	(e- and low-energy photons)

  - ☆ Line Radiation from QM System Transitions

☞ Characteristic X-rays	(atomic shell)
☞ Cyclotron Radiation	(magnetic field)
☞ Annihilation of Positrons	(e.m. field)
☞ Decay of Pions	(nucleonic interactions)

- **Attenuation Processes**
  - ☆ Inelastic Scattering Processes
    - ☞ Compton Scattering
    - ☞ Photo-Ionization
  - ☆ Pair Production

Cross Sections

- Assume a slab of *thickness* **l** homogeneously filled with scatteres of some sort (electrons, atoms, molecules etc.).
- The *number density* of scatteres is **n** (in cm^{-3})
- The *cross section* **σ** is the area (in cm^2) that a scatterer presents to the photon for an interaction
- The *absorption coefficient* λ is the cross section multiplied by the number density: $\lambda = \sigma\, \mathbf{n}$ (in cm^{-1})
- The mean free path is $\mathbf{1}/\lambda$ (in cm)
- The *optical depth* τ is the absorption coefficient multiplied with the thickness: $\tau = \lambda\, \mathbf{l}$
- Finally, the transmitted light through the slab is

$$\mathbf{I} = \mathbf{I}_0\, \mathbf{e}^{-\tau} = \mathbf{I}_0\, \sigma\, \mathbf{n}\, \mathbf{l}$$

Thompson scattering

- Scattering of light from single electrons (Thomson scattering) has a total cross section

  $$\sigma_T = 8 \pi r_e^2 / 3 = 6.652 \times 10^{-25} \text{ cm},$$

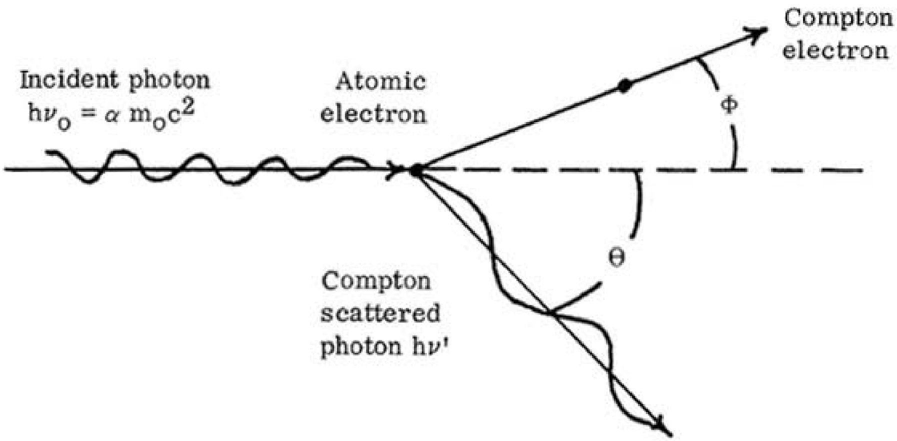
  where r_e is the classical radius of the electron:

  $$r_e = e^2 / m c^2 = 2.818 \times 10^{-13} \text{ cm}$$

- Scattering from atoms involves the cooperative effect of all atomic electrons and the cross section becomes correspondingly larger

- Thomson scattering applies only for low-energy photons, for higher energies the Klein-Nishina scattering cross section has to be applied (see below).

Compton Scattering



$$E' = \frac{E}{1+\frac{E}{m_ec^2}(1-\cos\theta)}$$

$$\sim E\left(1-\frac{E}{m_ec^2}(1-\cos\theta)\right)$$

$$\lambda'-\lambda = \frac{h}{m_ec}(1-\cos\theta)$$

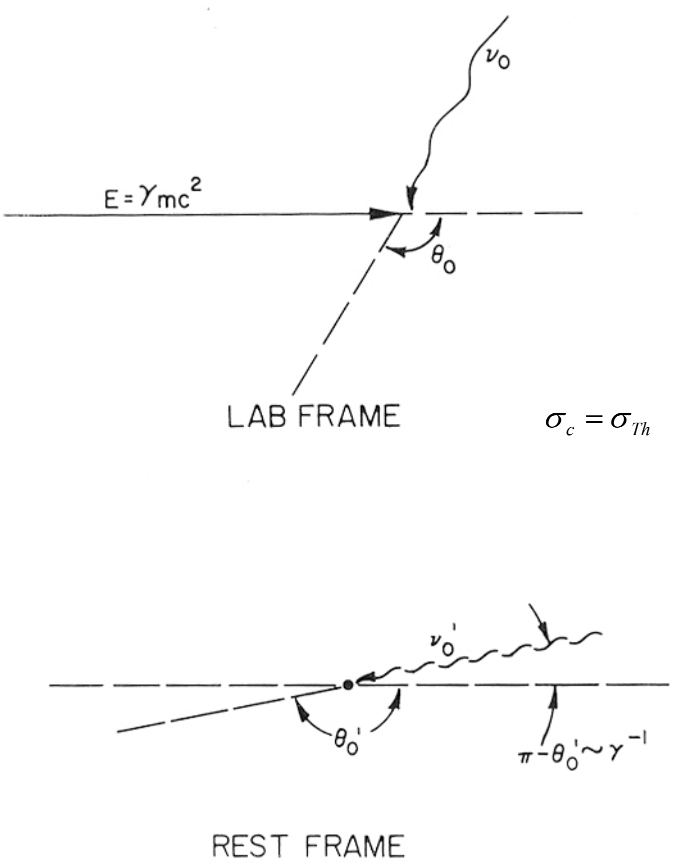
where $h/m_ec = 2.426\times10^{-10}$ cm (Compton wavelength).

Averaging over θ, for $E \ll m_ec^2$:

$$\frac{\Delta E}{E} \approx -\frac{E}{m_ec^2}$$

E.g., at 6.4 keV, $\Delta E \approx 0.2$ keV.

Inverse Compton Scattering



$\sigma_c = \sigma_{Th}$

Fig. 3.10. Compton collision between a relativistic electron and low frequency photon as viewed in both the laboratory and the rest frame of the electron.

☆ Energy Gain

- $E_\gamma = \Gamma^2 mc^2$ for high energies
- $E_\gamma = \Gamma^2 h\nu$ for lower energies

Total Power (= Energy Loss of e-)

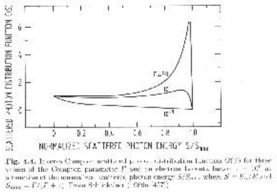
- $$-\frac{dE}{dx} = \int \sigma_c(E_\gamma, h\nu) N(h\nu) E_\gamma dE_\gamma d(h\nu)$$
- low-energy: Thompson scattering

  $$\sigma_c = \sigma_{Th}$$

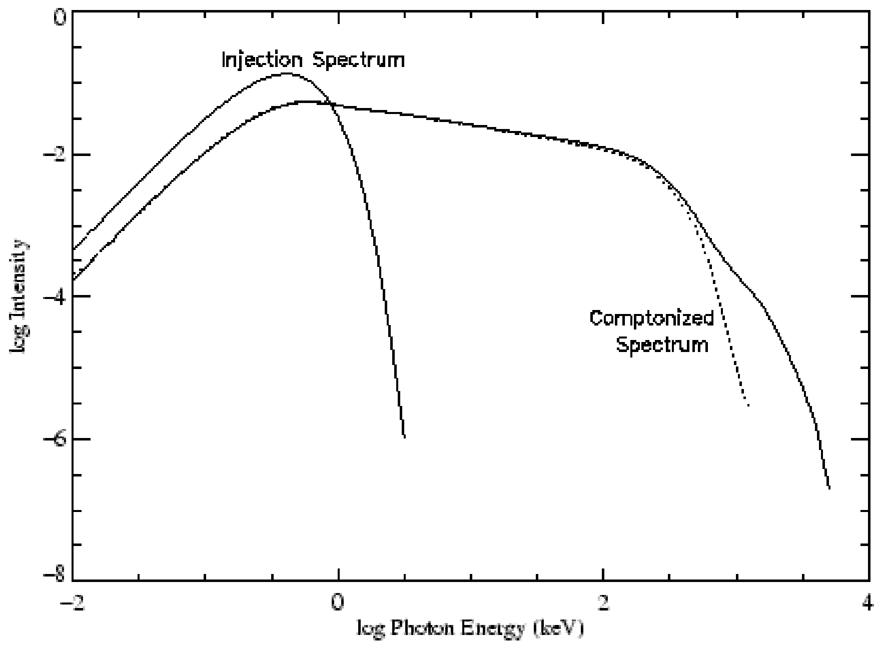
- High energy ($\Gamma h\nu >> mc^2$): Klein-Nishina, ~$1/h\nu$

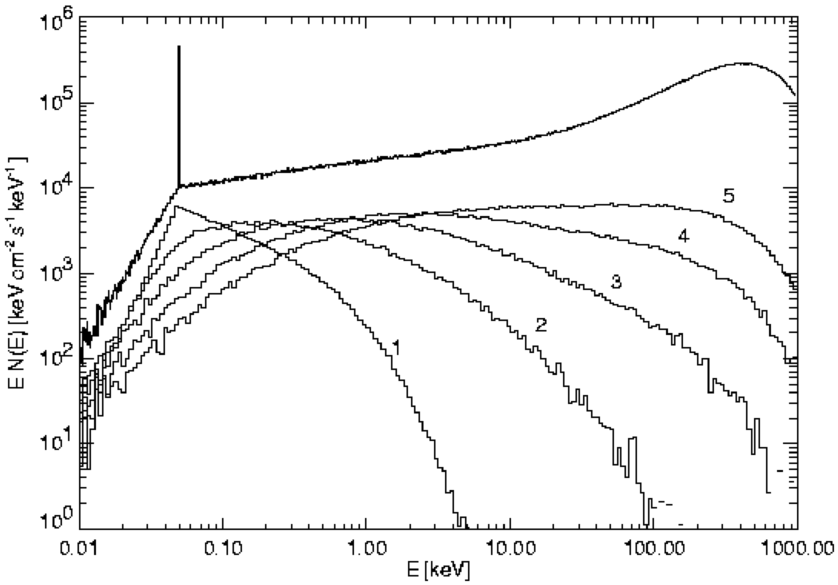
  $$\sigma_c \approx \frac{3}{8} \sigma_{Th} \left(\frac{mc^2}{\Gamma h\nu} \right) \left[\ln\left(\frac{2\Gamma h\nu}{mc^2} \right) + \frac{1}{2} \right]$$

- Energy spectrum ~ $h\nu$ N($h\nu$)
- Max Energy ~ 4 Γ^2 $E_{photons}$



Comptonization





Sphere with $\tau = 5$ and $kT_e = 0.4\, m_e c^2$ ($\sim 200\,\mathrm{keV}$), seed photons come from center of sphere.

- ☆ General energy exchange between e$^-$ and photons
  - Need rarefied gas (so no additional photons are produced)
  - Hot plasma (so energy gain of photons is significant)

$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{h\nu}{m_e c^2} + \frac{4kT_e}{m_e c^2}$$

  - For n scatterings, the energy gain is $\frac{\varepsilon'}{\varepsilon} = \left(1 + \frac{4kT_e}{m_e c^2}\right)^n \approx e^{\frac{4kT_e}{m_e c^2} n}$
  - For an optically-thick plasma, $\langle h\nu \rangle \sim 3kT_e$

Synchrotron radiation

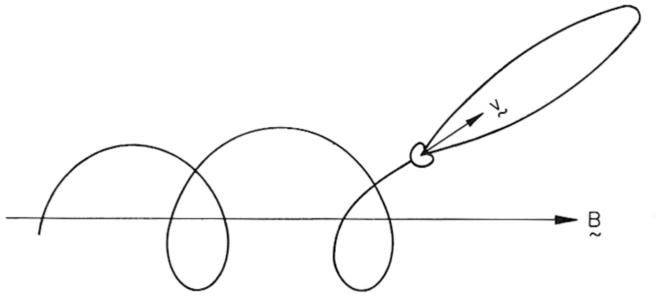


Fig. 3.2. A relativistic particle spiraling in a magnetic field emitting synchrotron radiation with the angular pattern as indicated.

Particle traversing magnetic field **B**

Gyration frequency:

$$\Omega = eB / \chi\ mc = 1.8 \text{ x } 10^{7}\ \chi^{-1}\ [\text{rad/s}]$$

Total Energy loss according to Larmor formula:

$$P = 2/3\ r_e^2\ c\ \chi^2\ B^2\ \beta^2 = 1.6 \text{ x } 10\text{-}15\ \chi^2\ B^2\ \beta^2 \sin^2\alpha\ \ [\text{erg/s}]$$

For an isotropic distribution of particle velocities, integrated over all angles:

$$P = 4/9\ r_e^2\ c\ \chi^2\ B^2\ \beta^{\,2} = 1.1 \text{ x } 10^{-15}\ \chi^2\ B^2\ \beta^2\ [\text{erg/s}]$$

Synchrotron Radiation Spectrum

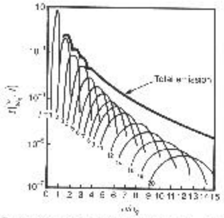


Figure 18.4. The spectrum of emission of the first 20 harmonics of mildly relativistic cyclotron radiation. The electron has $v = 0.4c$. (After G. Bekefi (1966). *Radiation processes in plasmas*, p. 205. New York: John Wiley and Sons, Inc.)

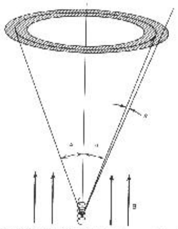
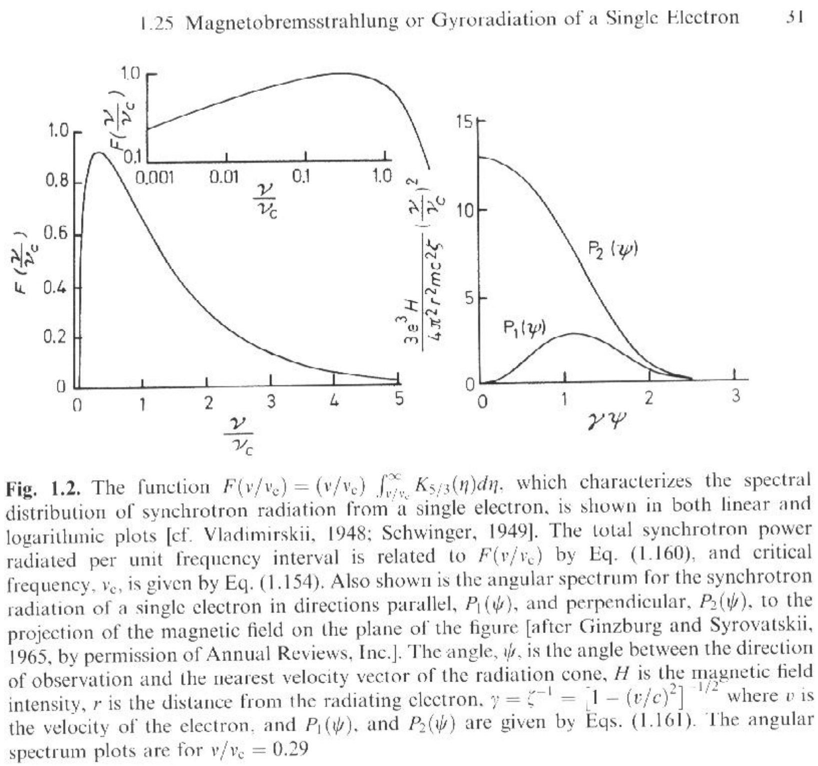


Figure 18.7. Synchrotron emission from a particle with pitch angle α. The radiation is confined to the shaded solid angle. (After G.B. Rybicki and A.P. Lightman (1979) *Radiative processes in astrophysics*, p. 178. New York: John Wiley and Sons.)

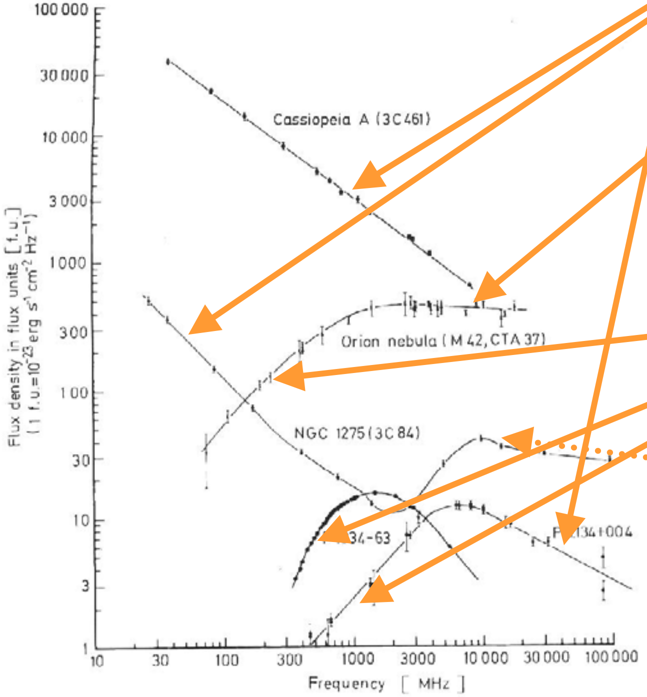
- **Low-E limit:**
  - ☞ Superposition of Lines at ν_g
- **High-E:**
  - ☆ Transformation into Observer's Frame
    - ☞ F.T. of e- Acceleration
    - ☞ Doppler Shift, Retarded Time
    - ☞ Forward Beaming
  - ☆ $j \sim \nu^{1/3}$ (low-ν) … $j \sim \sqrt{\nu} e^{-\frac{\nu}{\nu_c}}$



1.25 Magnetobremsstrahlung or Gyroradiation of a Single Electron 31

Fig. 1.2. The function $F(\nu/\nu_c) = (\nu/\nu_c)\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)d\eta$, which characterizes the spectral distribution of synchrotron radiation from a single electron, is shown in both linear and logarithmic plots [cf. Vladimirskii, 1948; Schwinger, 1949]. The total synchrotron power radiated per unit frequency interval is related to $F(\nu/\nu_c)$ by Eq. (1.160), and critical frequency, ν_c, is given by Eq. (1.154). Also shown is the angular spectrum for the synchrotron radiation of a single electron in directions parallel, $P_1(\psi)$, and perpendicular, $P_2(\psi)$, to the projection of the magnetic field on the plane of the figure [after Ginzburg and Syrovatskii, 1965, by permission of Annual Reviews, Inc.]. The angle, ψ, is the angle between the direction of observation and the nearest velocity vector of the radiation cone, H is the magnetic field intensity, r is the distance from the radiating electron, $\gamma = \zeta^{-1} = [1-(v/c)^2]^{-1/2}$ where v is the velocity of the electron, and $P_1(\psi)$, and $P_2(\psi)$ are given by Eqs. (1.161). The angular spectrum plots are for $\nu/\nu_c = 0.29$

Spectra from Cosmic Sources with Radiating e$^-$



Synchrotron radiation power law

Bremsstrahlung radiation (flat)

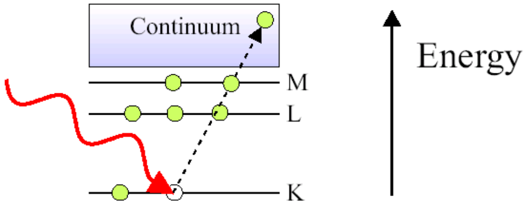
Self-absorption

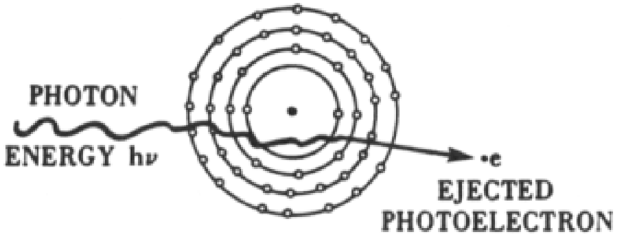
(Peculiar HE Emission)

Fig. 1.4. Radiofrequency spectra of sources exhibiting the power law spectrum of synchrotron radiation (Casseopeia A), the flat spectrum of thermal bremsstrahlung radiation with low frequency self absorption (Orion Nebula), unusual high frequency radiation (NGC 1275), and low frequency absorbtion processes (P1934 – 63 and P2134 + 004). The data for P2134 + 004 are from E. K. Conklin, and the other data are from Kellermann [1966], Hjellming, and Churchwell [1969], Terzian and Parrish [1970], and Kellermann, Pauliny-Toth, and Williams [1969].

Interactions of X-ray photons with matter

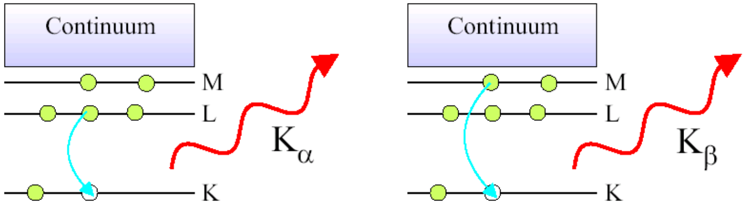
(a) Photoelectric absorption



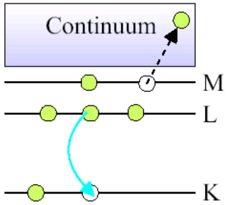


Photoelectric Absorption
(Courtesy of Dresser Atlas)

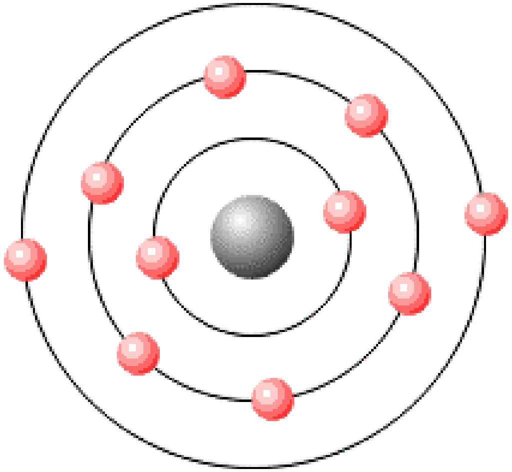
(b) Fluorescent X-ray emission



(c) Auger electron emission



Characteristic X-rays



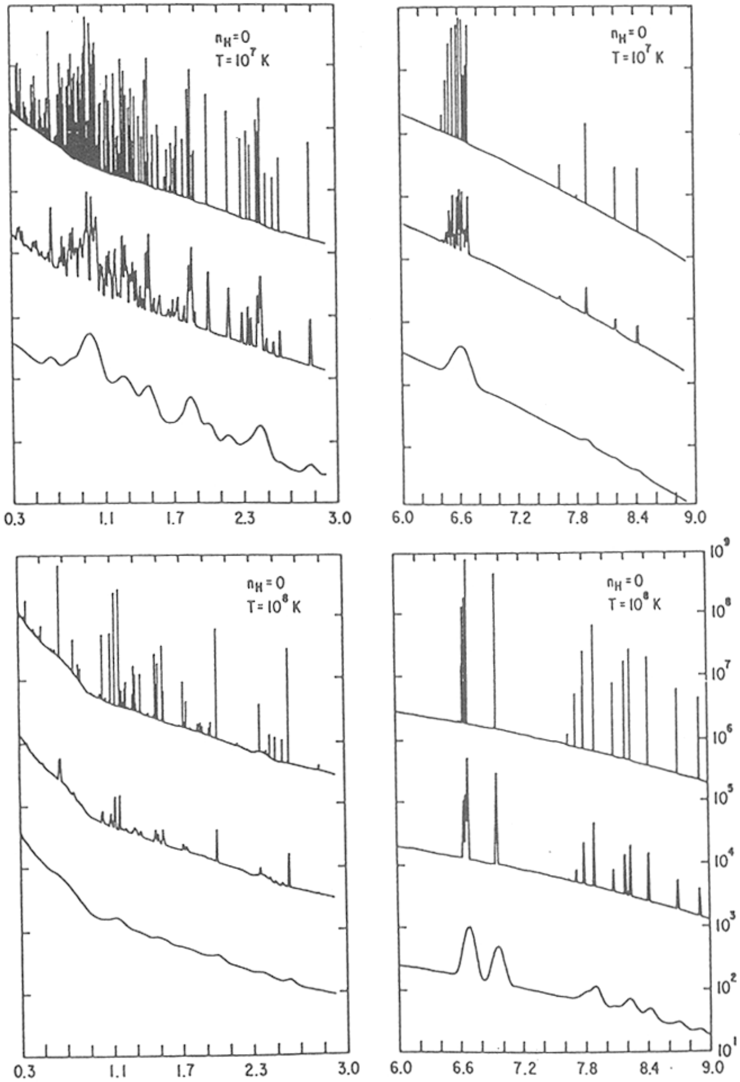
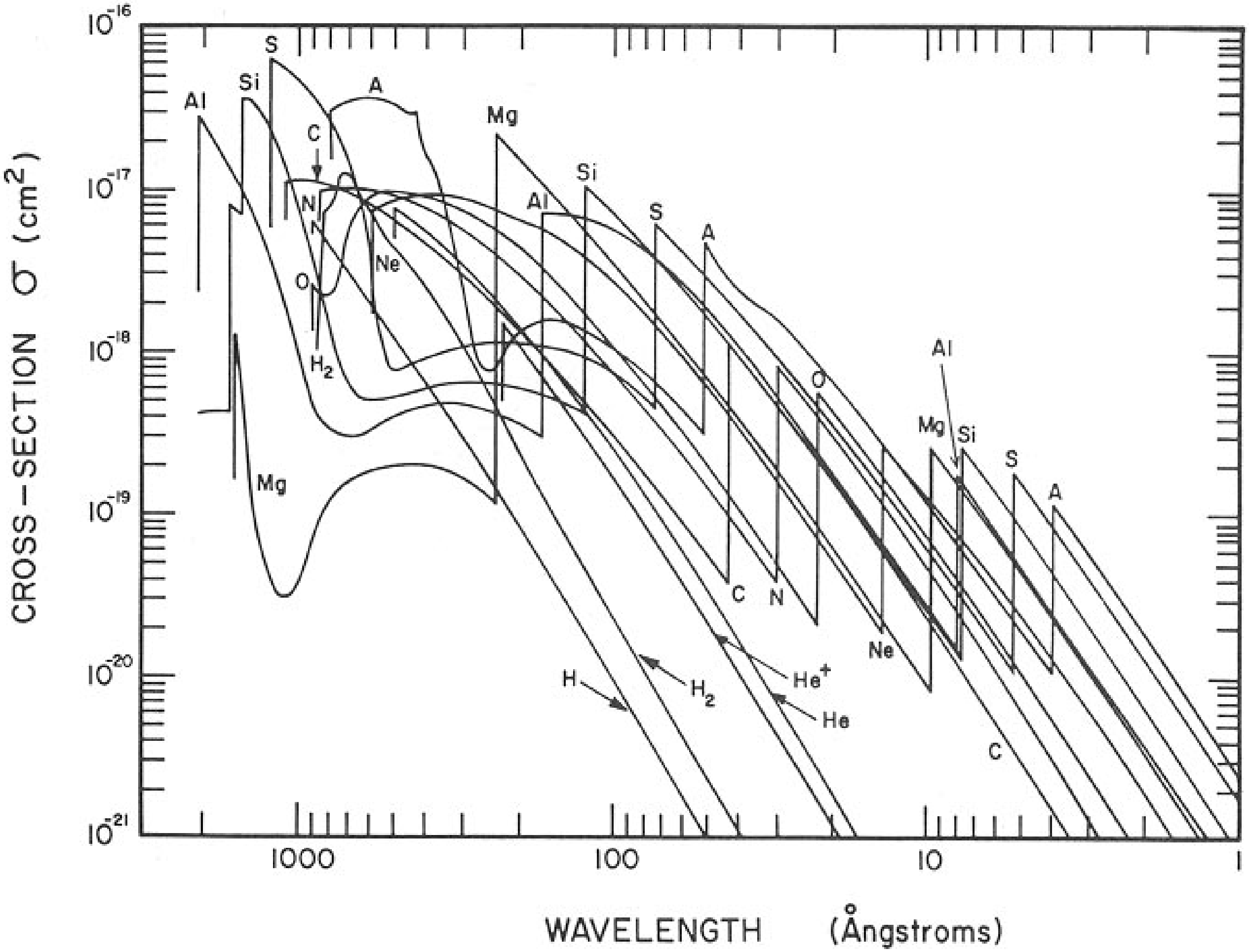


Figure 3. Coronal equilibrium spectra (arbitrary logarithmic intensity) of plasmas with solar abundances, in the energy bands (keV) containing Fe L- and Fe K-emission, as viewed with detectors having FWHM resolutions of 1, 10 and 100 eV (upper, middle and lower trace of each panel).

Photoelectric Crosssections of various elements



Outside Atomic Edges: $\sigma \sim Z^3 \lambda^3$

Annihilation of Positrons

- Antimatter e^+
  - ☆ From Pair Production
  - ☆ $m_e c^2 = 511$ keV
- Annihilation
  - ☆ 2-Photon Annihilation with e^-
  - ☆ Annihilation Paths
    - ☞ In flight
    - ☞ At rest / thermal
    - ☞ Via Positronium formation

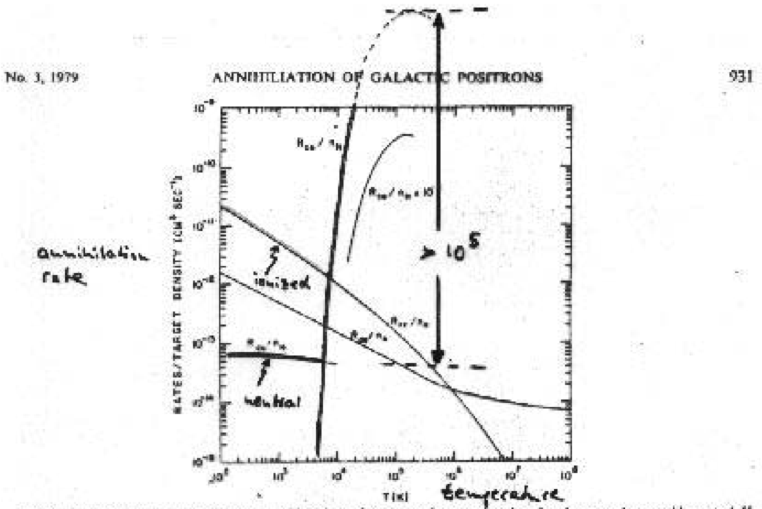


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FIG. 3.—Rates (per unit target density) at which thermal positrons form positronium by charge exchange with neutral H (R_{ce}/n_H) or by radiative recombination with free electrons (R_{rr}/n_e), and annihilate directly with free electrons (R_{df}/n_e) or with bound electrons (R_{db}/n_H), as functions of the gas temperature.

Pair Production

- **Photon-Photon Collisions**
    - ☞ Production of e^+e^- Pair

$$\sigma(E_1, E_2) = \frac{\pi r_0}{2}\left(1-\beta^2\right)\left[2\beta\left(\beta^2-2\right)+\left(3-\beta^4\right)\ln\left(\frac{1+\beta}{1-\beta}\right)\right]$$

$$\beta = \sqrt{1-\frac{\left(mc^2\right)^2}{E_1 E_2}}$$

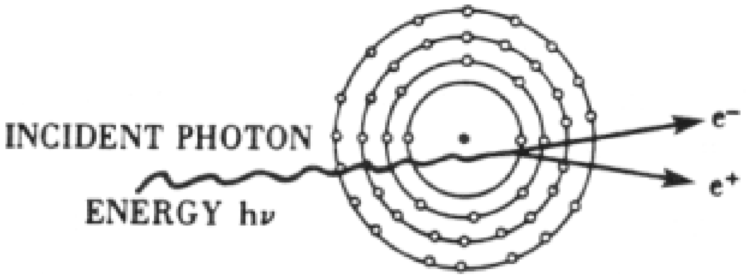
- **Thresholds**
    - ☆ Energy of Colliding Photons
        - ☞ hν > 511 keV (γγ)
        - ☞ hν > 1.022 MeV (with starlight/CMB)
    - ☆ Source Compactness
        - ☞ Compactness parameter l > 10

$$l = \frac{L\sigma_T}{Rm_e c^3} = 2\pi \frac{m_p}{m_e}\frac{L}{L_E}\frac{R_S}{R}$$

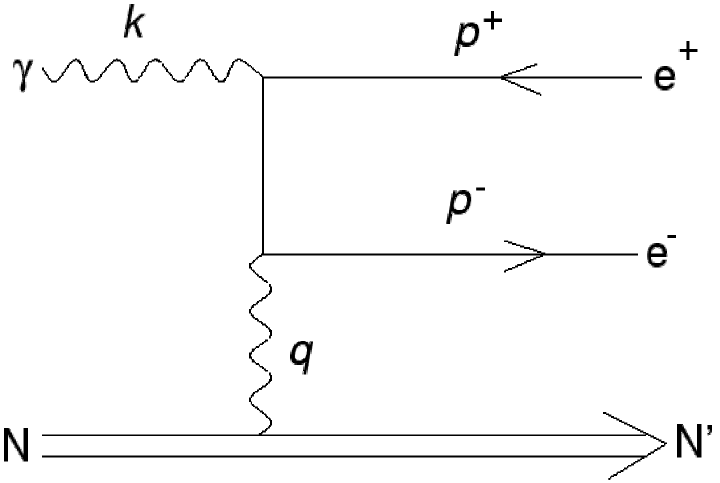
$$L_E = \frac{4\pi GMcm_e}{\sigma_T} \qquad R_S = \frac{2GM}{c^2}$$

Pair Production

- Pairs can be created as soon as $h\,\nu > 2\,m_e\,c^2$
- Pairs cannot be created in vacuum because energy and momentum need to be conserved; a third particle (e.g. nucleus) is required



Pair Production
(Courtesy of Dresser Atlas)



Nuclear De-Excitation

- Cosmic-Ray Collisions with ISM Gas Excite Nuclei
- De-Excitation Leads to Characteristic Gamma-Ray Emission

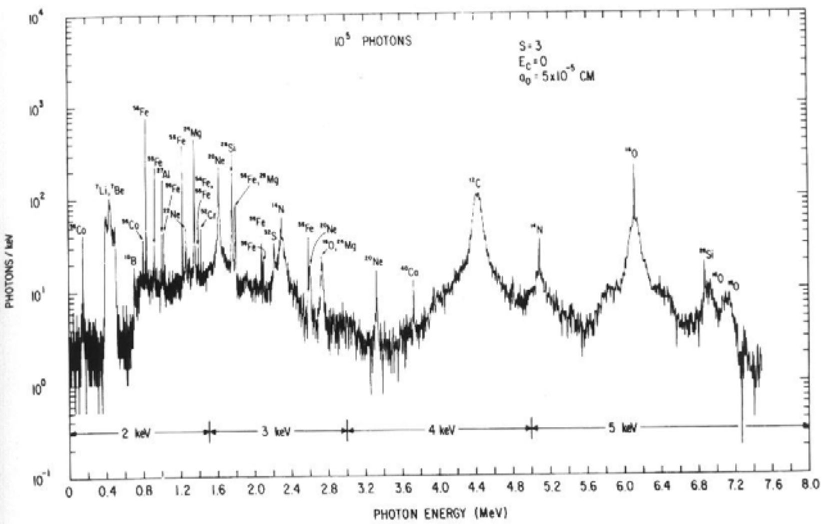
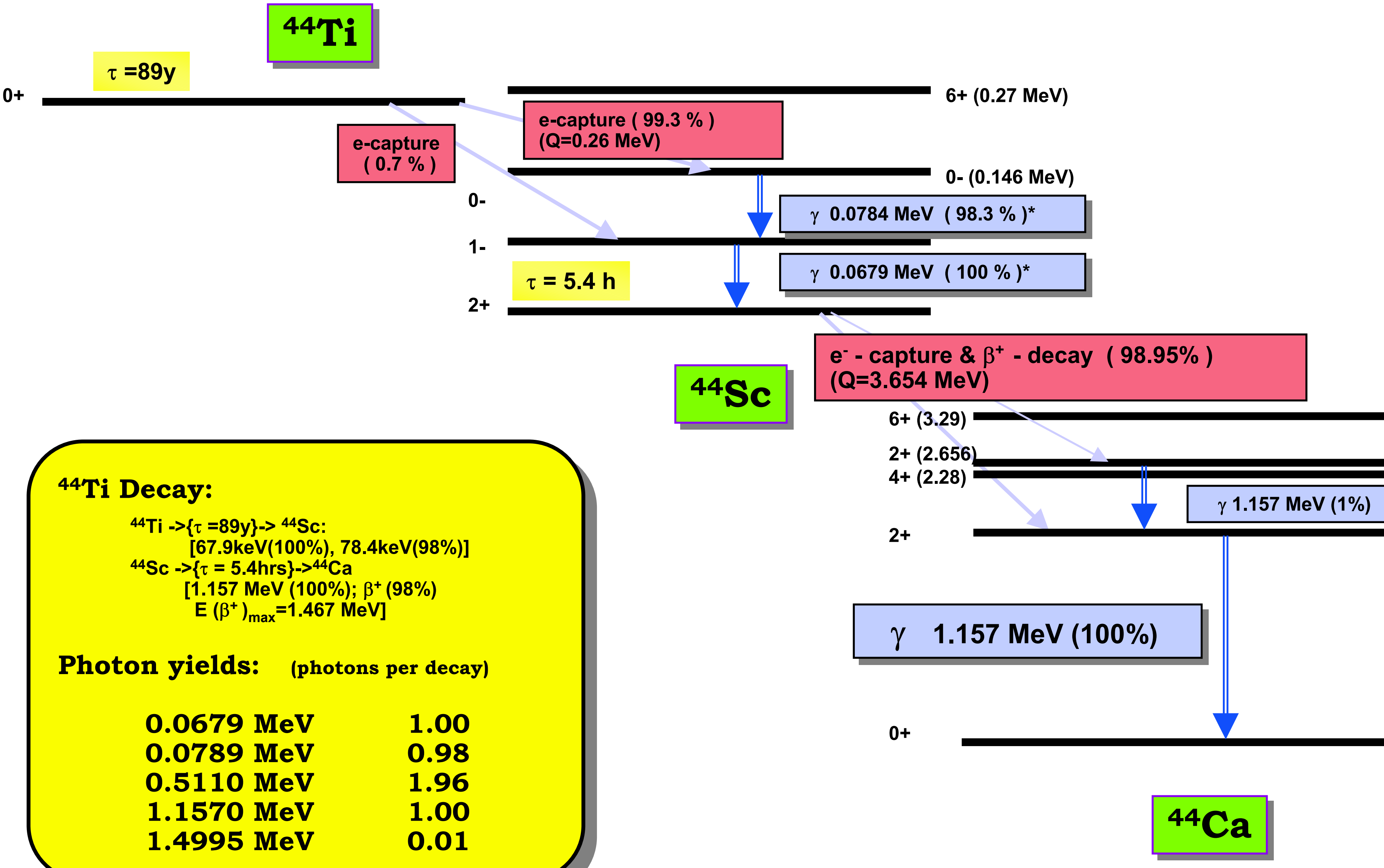


Fig. 5.12. Calculated γ-ray spectrum from energetic particles and ambient medium having solar composition. The energetic particles obey a power law spectrum in kinetic energy with spectral index s above a low-energy cutoff E_c, and the contribution from lines from interstellar grains of characteristic radius a_0 is included. From Ramaty et al. (1979 [419])

^{44}Ti Decay



^{44}Ti Decay:

^{44}Ti ->{τ =89y}-> ^{44}Sc:
[67.9keV(100%), 78.4keV(98%)]
^{44}Sc ->{τ = 5.4hrs}->^{44}Ca
[1.157 MeV (100%); β^+ (98%)
E $(\beta^+)_{max}$=1.467 MeV]

Photon yields: (photons per decay)

0.0679 MeV	1.00
0.0789 MeV	0.98
0.5110 MeV	1.96
1.1570 MeV	1.00
1.4995 MeV	0.01

Pion Production and - Decay Gamma-Rays

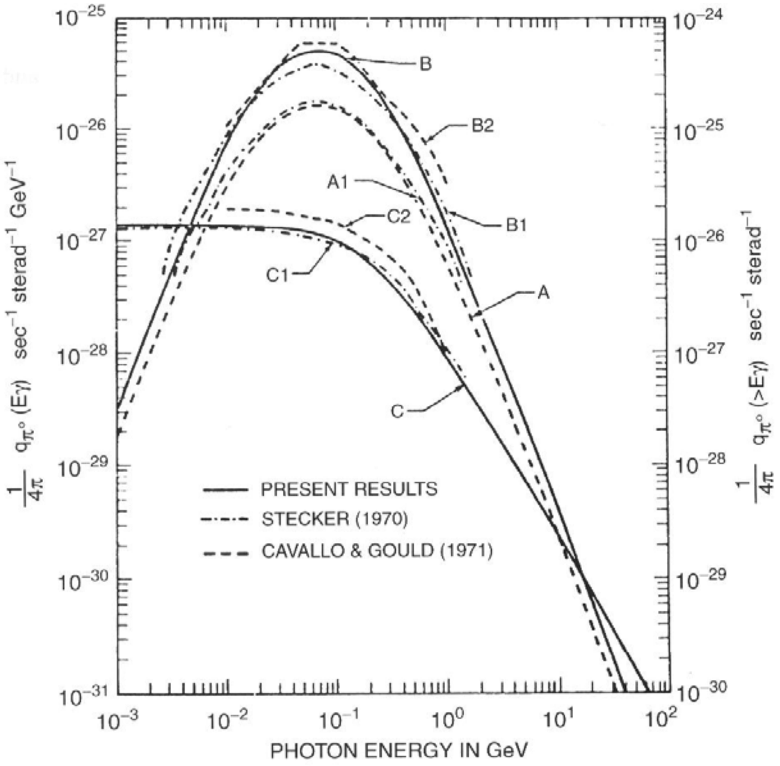


Fig. 5.7. Differential and integral source functions of γ-rays from the $\pi^0 \rightarrow 2\gamma$ decay calculated by various authors with different representations of the local interstellar cosmic ray proton spectrum. After Badhwar and Stephens (1977 [24])

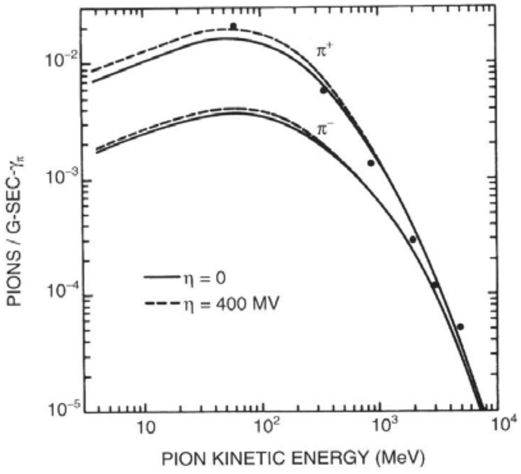


Fig. 5.6. Pion production spectrum in interstellar space giving the number of pions per s, gram and unit pion Lorentz factor. The **solid line** is calculated with the cosmic ray spectrum as observed near Earth whereas the **dashed line** is calculated with the demodulated cosmic ray spectrum. From Ramaty (1974 [415])

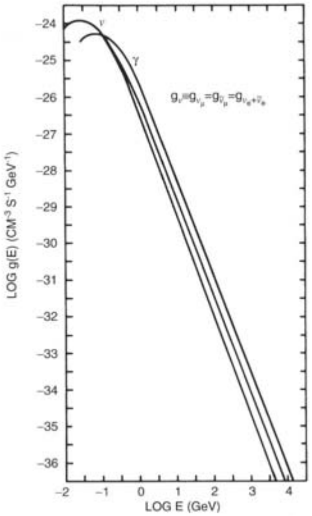


Fig. 5.10. Differential source function of neutrinos and γ-rays from the decay of pions produced by interactions of cosmic rays in our galactic neighborhood per hydrogen atom. The **γ-ray curve** and the **upper neutrino curve** are calculated for cosmic rays having a spectral index of 2.67 between 10 and 3×10^6 GeV; the **lower neutrino curve** is for a cosmic ray spectrum with index 2.75. The spread in the curve is indicative of the uncertainty in such calculations. From Stecker (1979 [520])

- Nuclear Interactions in CR/Gas Collisions Lead to π^o
- π^o -> γγ
- Source Function is Unclear (deconvolved from γ-ray emission)

Cosmic Ray Interactions and Photon Sources

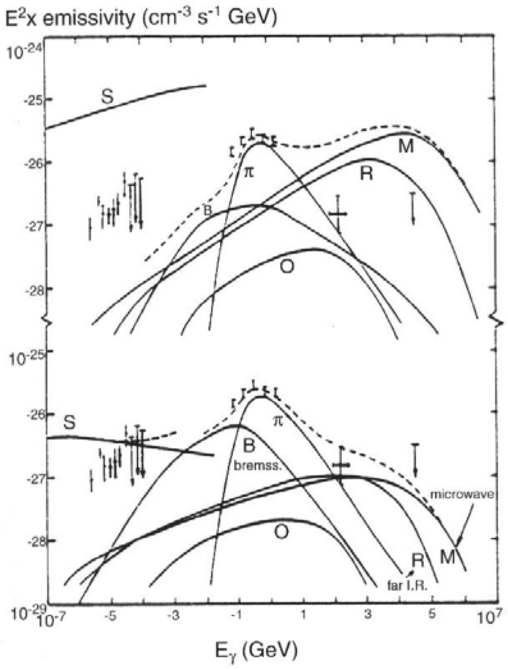
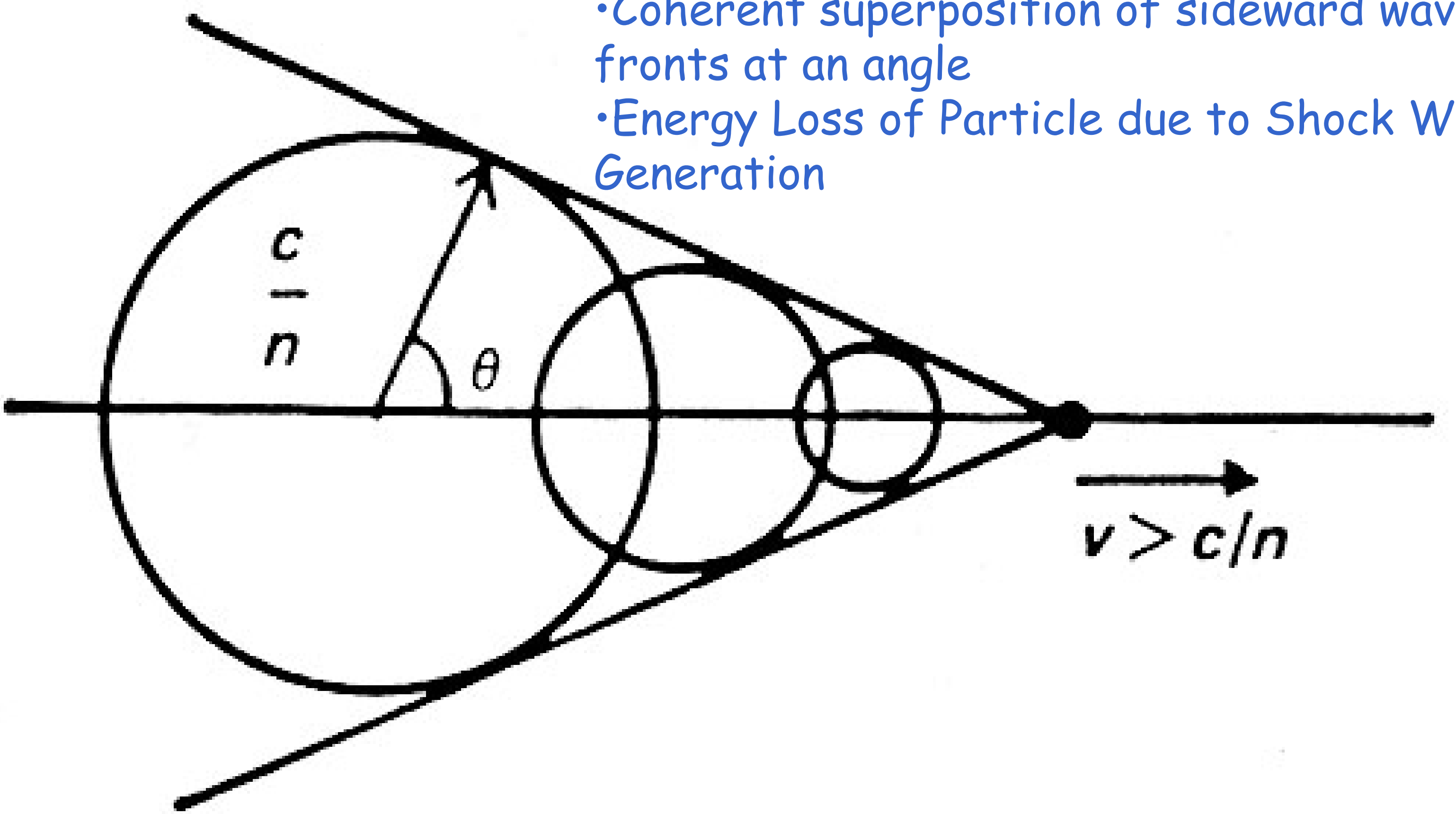


Fig. 6.13. The predicted X-ray and γ-ray emissivity from various production processes: inverse Compton-interactions with the microwave background (**M**), far infrared radiation (**R**), starlight (**O**); synchrotron radiation (**S**); nonthermal bremsstrahlung (**B**); the decay of neutral pions produced in nuclear interactions of cosmic rays (π). The total emissivity is shown as the **dashed curve**. The predictions are compared with the derived emissivity from observations in different energy bands. From Protheroe and Wolfendale (1980 [409])

- Diffuse High-Energy Radiation is Created from Variety of Processes
  - ☆ Charged-particle radiation in plasmas and magnetic fields
  - ☆ Inverse-Compton scattering on starlight and microwave background radiation

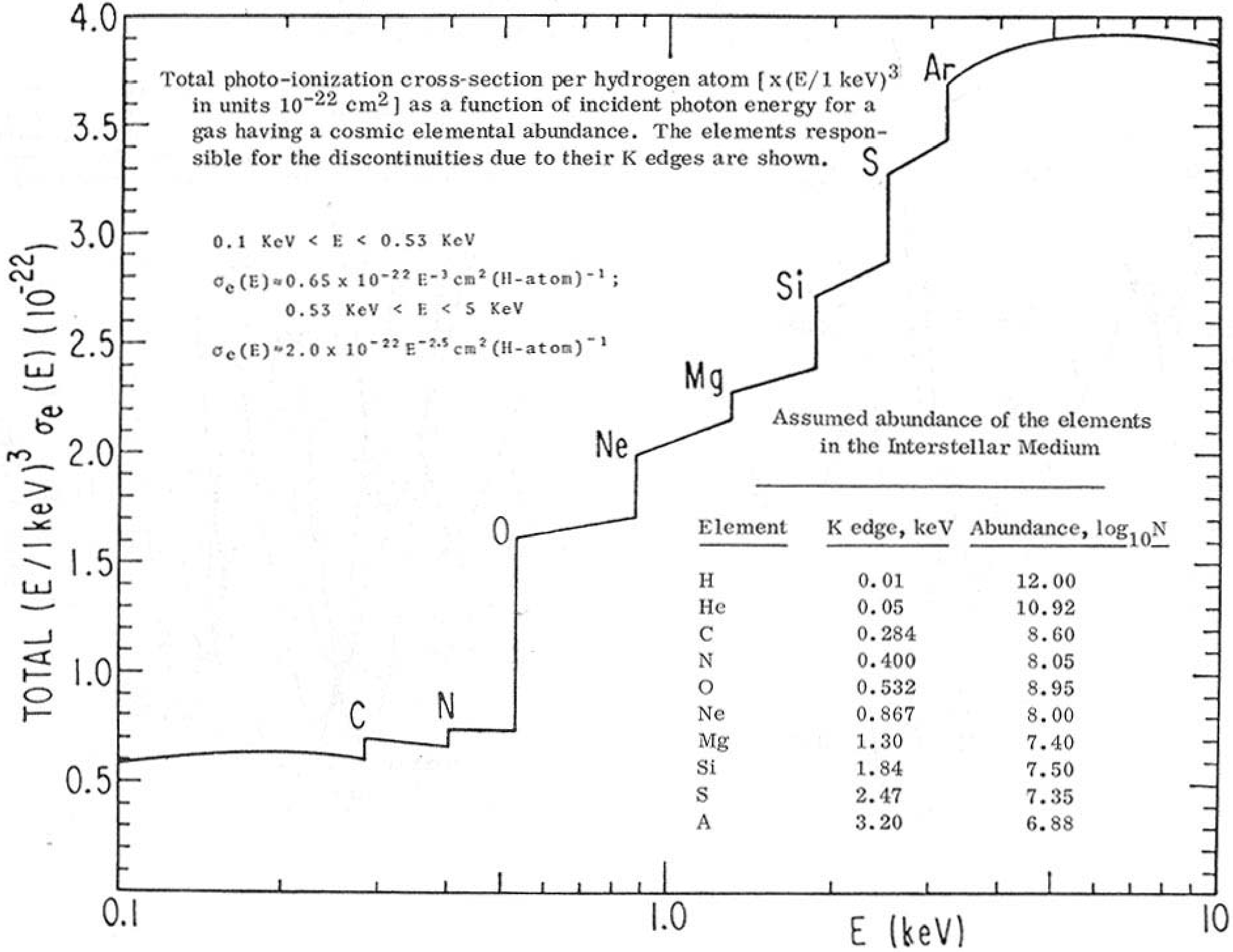
Cerenkov Radiation

- Particle Propagation through Medium at v=local light velocity
- Coherent superposition of sideward wave fronts at an angle
- Energy Loss of Particle due to Shock Wave Generation



$$\cos\theta = \frac{c}{nv}$$

Interstellar Medium Photo Crossection (Cosmic Abundances)



Total photo-ionization cross-section per hydrogen atom [$\times (E/1\,keV)^3$ in units 10^{-22} cm^2] as a function of incident photon energy for a gas having a cosmic elemental abundance. The elements responsible for the discontinuities due to their K edges are shown.

0.1 KeV $< E < 0.53$ KeV

$\sigma_e(E) \approx 0.65 \times 10^{-22} E^{-3}\,cm^2\,(H\text{-}atom)^{-1}$;

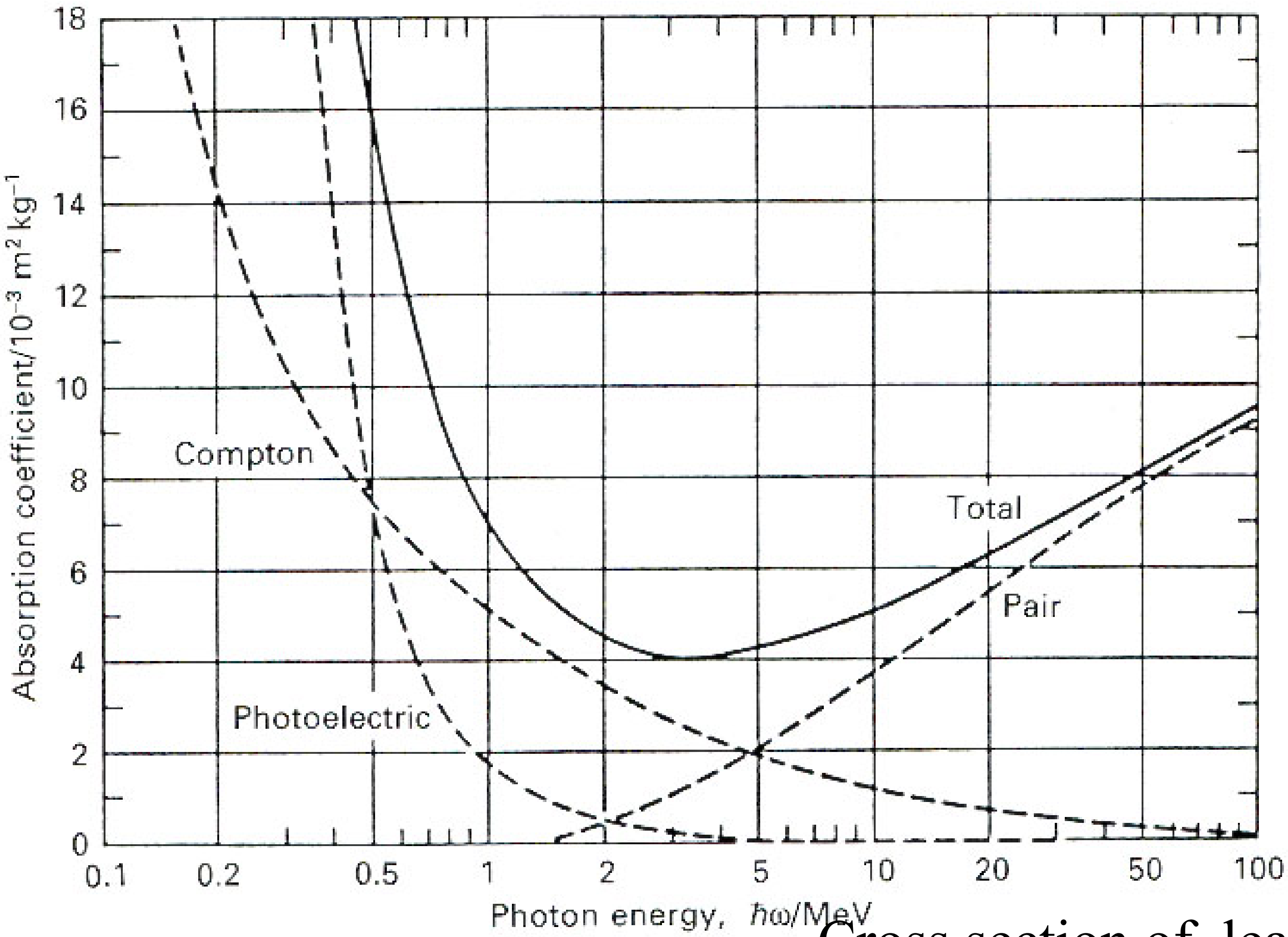
0.53 KeV $< E < 5$ KeV

$\sigma_e(E) \approx 2.0 \times 10^{-22} E^{-2.5}\,cm^2\,(H\text{-}atom)^{-1}$

Assumed abundance of the elements in the Interstellar Medium

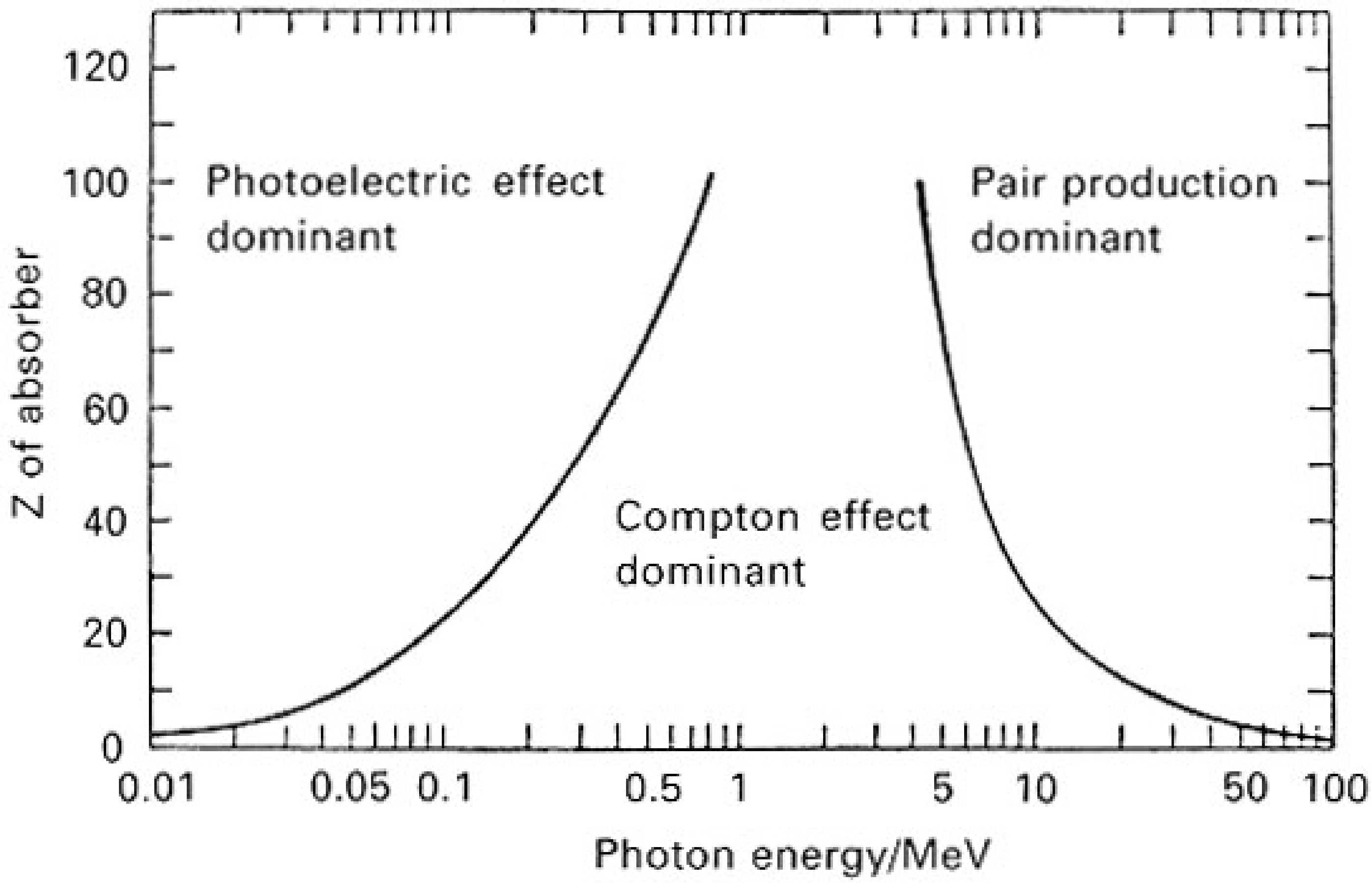
Element	K edge, keV	Abundance, $\log_{10} N$
H	0.01	12.00
He	0.05	10.92
C	0.284	8.60
N	0.400	8.05
O	0.532	8.95
Ne	0.867	8.00
Mg	1.30	7.40
Si	1.84	7.50
S	2.47	7.35
A	3.20	6.88

Interaction of HE photons with matter

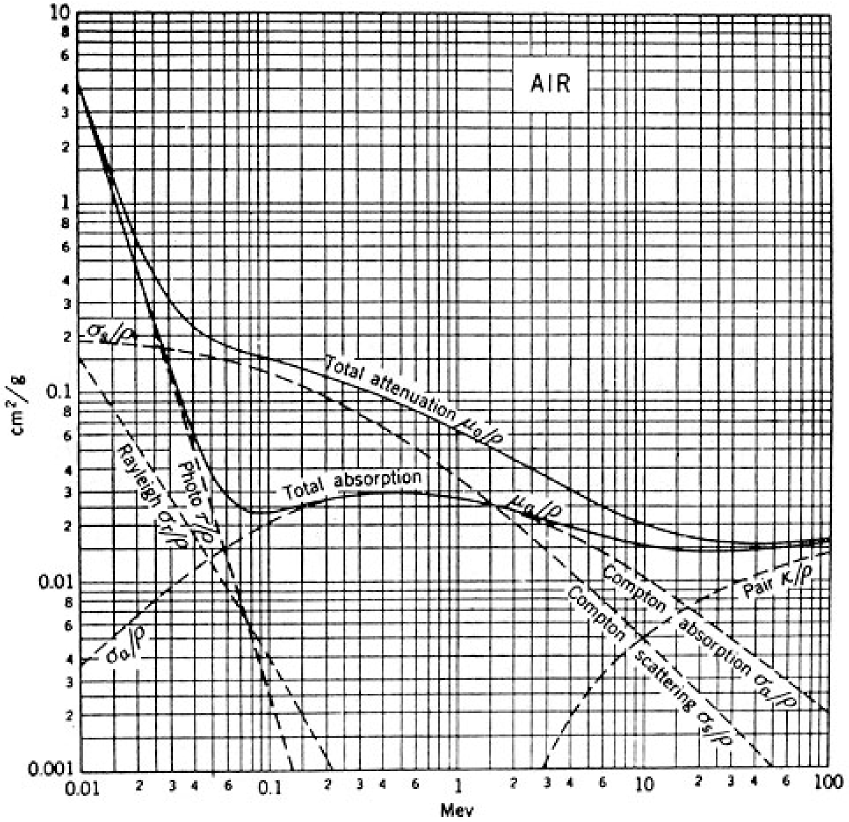


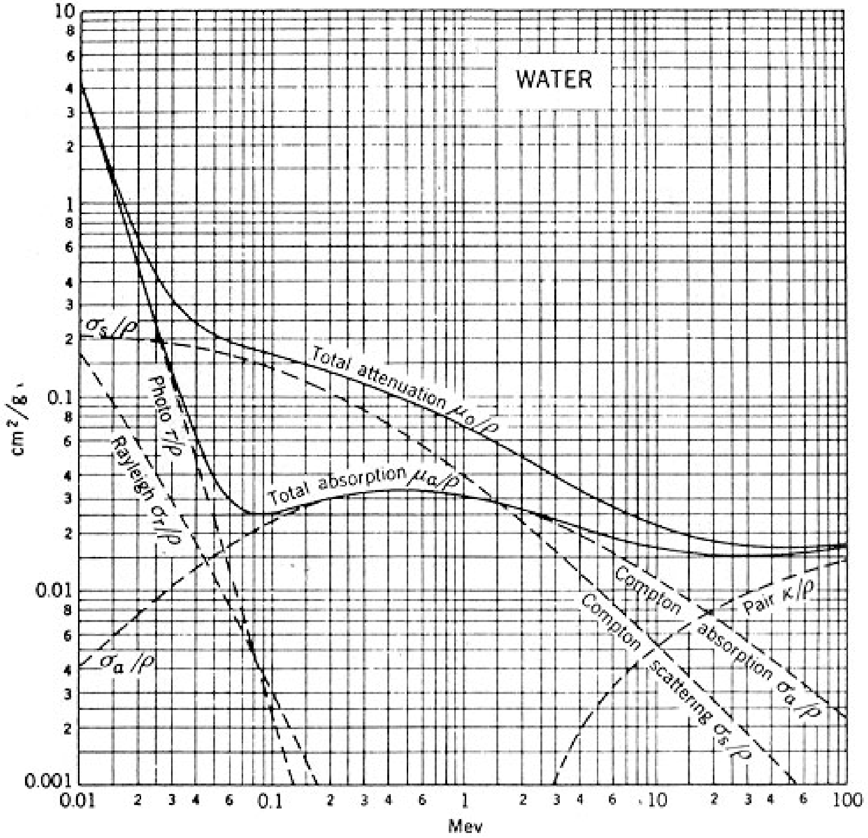
Cross section of lead

Interaction of HE photons with matter



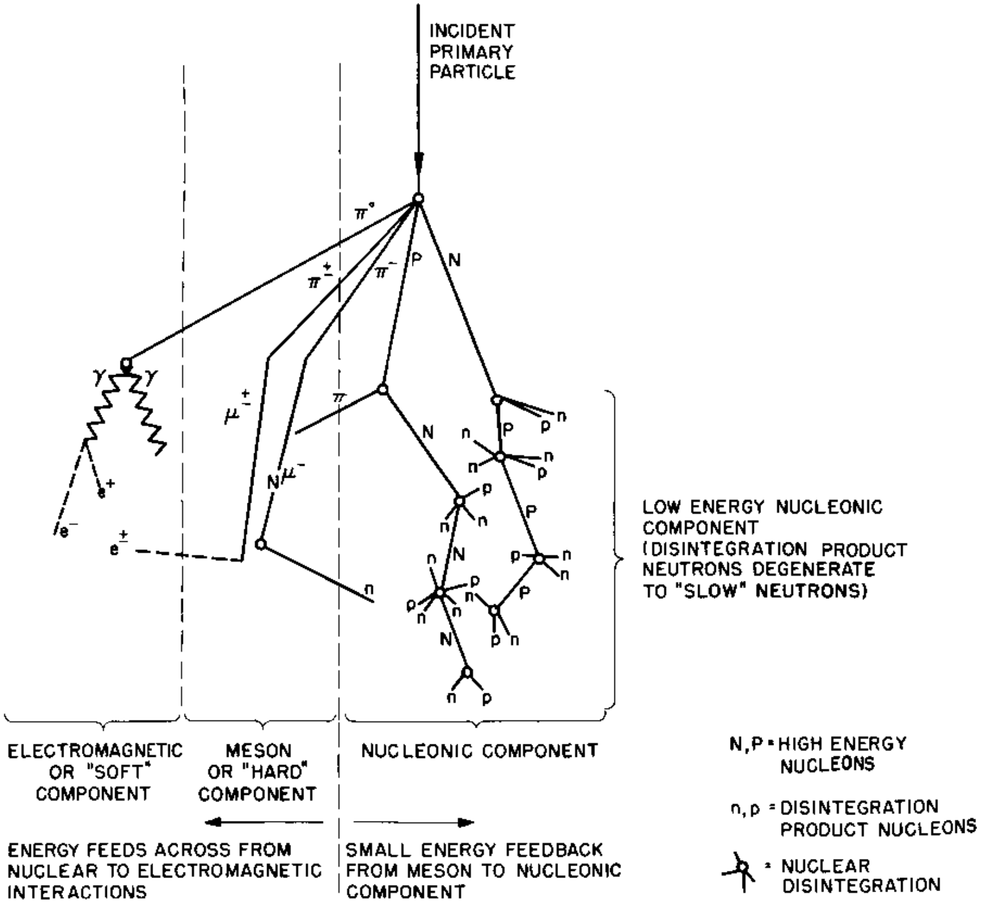
Attenuation in Matter



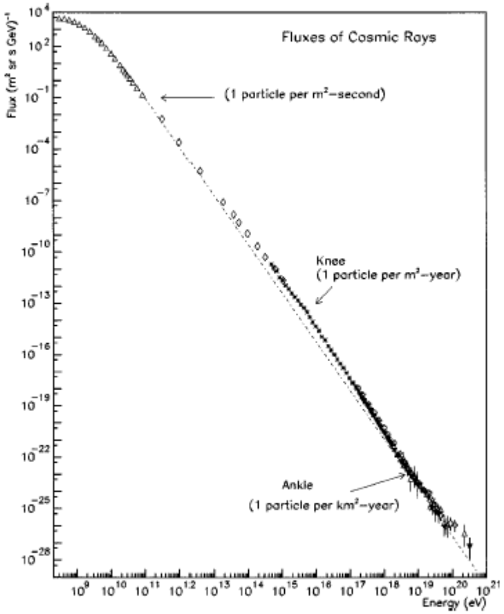


HE-Photons from Cascade Interactions

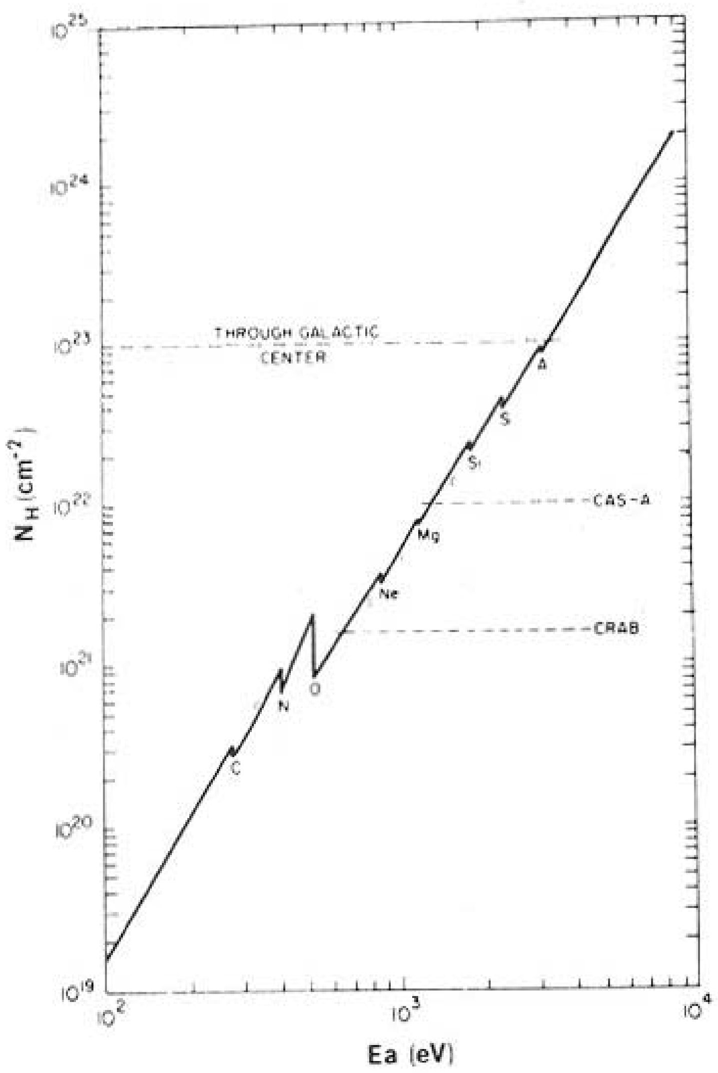
- **Two Aspects**
  - ☞ Electromagnetic Cascade
  - ☞ Nucleonic Cascade
- **In Cosmic Sources: CR Gamma-Rays**
- **In Instruments: Background**



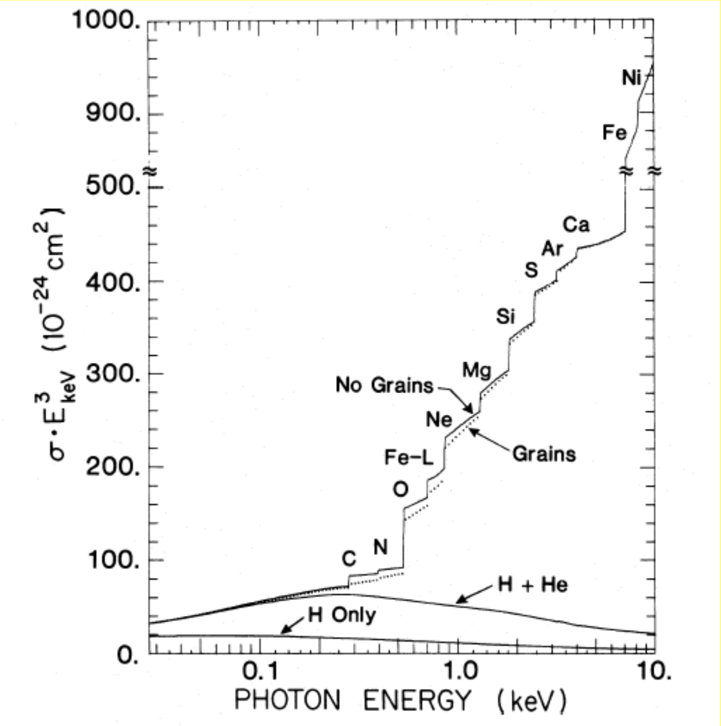
Schematic Diagram of Cosmic Ray Shower



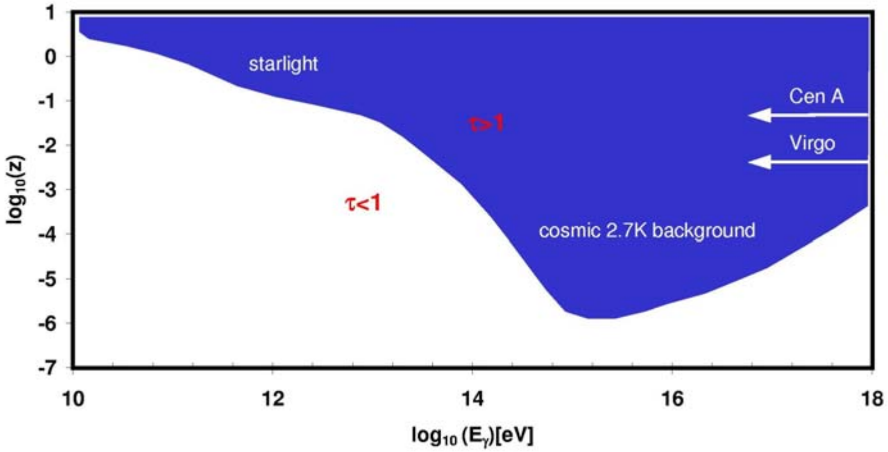
ISM Absorption Cross Sections / X-ray Horizon



Transparency of the Universe



Morrison & MacCammon ApJ 1983



$$\tau_x = 2\cdot 10^{-26}\left(\frac{h\nu}{1keV}\right)^{-8/3}\int N_H dl$$

- Interaction of Photons
  - ☆ Absorption (Dust, Gas)
    - ☞ Structure from
      - – Dust Particles, Molecules
      - – Atomic-Shell Electrons
  - ☆ Scattering
    - ☞ Inelastic:
      - – Comptonization
      - – Inverse-Compton Boosting