

ELLIPTICAL GALAXIES WITH DARK MATTER. II. OPTIMAL LUMINOUS-DARK MATTER DECOMPOSITION FOR A SAMPLE OF BRIGHT OBJECTS

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ABSTRACT

A procedure for the analysis of photometric and kinematical profiles of elliptical galaxies based on a family of spherical, pressure supported, self-consistent two-component models described in a previous paper is developed as a tool to study the presence of dark halos in such systems. Several tests on simulated objects are performed in order to show that this procedure is able to recognize the presence of dark matter, when sufficiently accurate and extended profiles are available; some of these tests have been carried out on flattened objects (E4 and E6), in order to establish how geometry can affect our conclusions. Then we have fitted the most accurate and extended data available for a set of 10 bright nearly round ellipticals. Very good photometric and kinematical fits are obtained; in some of these cases, the preferred models have a massive and diffuse dark component. In general, the amount of dark matter found inside R_e is not too large, of the order of the luminous mass. Typically, the mass-to-light ratio that we derive inside R_e is $M_L/L_B \sim 7 M_\odot/L_\odot$ for the stellar component, and $(M/L_B)_{R_e} \sim 12 M_\odot/L_\odot$ for the sum of the two components.

For NGC 4472 the velocity dispersion profile of the best-fit model is found to be well in agreement with the observed kinematics of the globular cluster system; in addition, an analysis devised to assess the statistical significance of the results obtained indicates that the amount of dark matter present in this galaxy exceeds that of the luminous component at a 3 sigma confidence level.

Subject headings: dark matter — galaxies: elliptical and lenticular, cD — galaxies: fundamental parameters — galaxies: kinematics and dynamics

1. INTRODUCTION

In this second paper, devoted to modeling elliptical galaxies with dark matter, we present detailed fits to the photometry and to the velocity dispersion profiles for a set of bright objects. The dynamical basis for the models used in the fitting procedure is given in the first paper (Bertin, Saglia, & Stiavelli 1992, hereafter Paper I), where we have constructed a large data base of fully self-consistent two-component models, characterized by a wide range in the ratios of scale lengths and masses for the luminous and for the dark components. These models are consistent with the most widely accepted physical pictures of galaxy formation, i.e., dissipationless collapse and merging. As a result of the modeling process, for the galaxies under investigation we have determined the gradient of the mass-to-light ratio as a function of radius, i.e., the “optimal” decomposition of the relevant mass distribution into stars and dark matter. For each galaxy, we provide the values for the total luminous mass M_L and for the dark mass M_D as identified by the best-fit models, together with an estimated range of uncertainty on these values.

Some obvious limitations are noted in the approach adopted here. In the first place, the fitting procedure described in § 2, being based on a rather well-defined physical picture of galaxy formation, excludes classes of pressure tensor profiles that would otherwise be admissible for an arbitrary collisionless stellar dynamical equilibrium system. Secondly, the models

used, being spherical, fully pressure supported, and based on one simple analytical form for the relevant distribution functions, are expected to provide only an approximate description of the real galaxies.

As to the former limitations, we recognize that other modeling methods that do not make assumptions on the processes of galaxy formation (see Tonry 1983; Richstone & Tremaine 1984), can give important indications on the pressure anisotropy profiles in elliptical galaxies. It should be noted that while a larger intrinsic radial anisotropy would require larger amounts of dark matter, tangential anisotropy could give a flat profile without the need for dark matter. On the other hand, we are not aware of any study of galaxy formation mechanisms able to justify systems with tangential anisotropy and to give quantitative predictions on its profile. However, rather than enter a full discussion of the relative merits of various possible modeling methods, here we would like to show what are the quantitative conclusions that can be drawn from one fitting procedure based on a physically consistent picture, ready to abandon it if it fails to provide satisfactory fits to the observations. In this context, it is crucial to make sure that, *within the assumptions made*, the use of simple models leads to an approximately correct determination of the relevant mass distributions. Thus we have performed several tests in order to set quantitative limits to the application of the adopted fitting procedure. The tests have been performed on simulated observations, by applying our “dynamical diagnostics” to theoretical objects (constructed either analytically or by means of N -body simulations) for which all the intrinsic properties are known in advance (in contrast to real galaxies). As will be

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shown in § 3, these tests on simulated observations have been particularly encouraging.

The galaxies studied in this paper (see § 4) have been chosen among the bright objects characterized by fairly round isophotes and by insignificant amounts of rotation. In order to proceed a step beyond a first modeling (see Bertin, Saglia, & Stiavelli 1988, hereafter BSS88; see also van der Marel, Binney, & Davies 1990) where no dark matter has to be invoked, we should refer to a global fit based on cases with accurate and radially extended kinematical profiles (see also Introduction to Paper I). Clearly, accuracy and radial extension cannot be easily improved at the same time, and it is not trivial to assess the proper requirements in order to be able to discriminate between models with and without dark matter with a given confidence level (see discussion in § 3.3). In view of some recently obtained excellent kinematical data (e.g., Davies & Birkinshaw 1988) we have decided to give priority to the accuracy of the data by sacrificing in part the available radial extension. The fits (see § 5) turn out indeed to discriminate between cases where dark matter is required and those where dark matter is not needed. Of course, some issues will be left unresolved until both accurate and radially extended kinematical data will become available (see also Bertin et al. 1989).

The astrophysically interesting results of this study (see §§ 5 and 6) are that in some cases (particularly NGC 4472, 4374, 4636, 7626) we do find, by the adopted fitting procedures, convincing evidence for the presence of dark matter already within the half-light radius R_e ; these cases happen to be galaxies that are known X-ray emitters. The case of NGC 4472 is particularly clear, because for this object the identified model with dark matter can naturally account for the observed velocity dispersion in the globular cluster system; in addition, for this object all the models identified at a 3 sigma confidence level (see discussion in § 5.2) have more mass in the dark component than in the luminous component.

2. FITTING PROCEDURE

The set of 2939 accepted two-component models of the main survey described in Paper I is used to fit photometric and kinematic data of elliptical galaxies in a procedure that extends the method described in a previous paper and restricted to models without dark matter (BSS88). For each of the 2939 models of the main survey we produce a fit to a given galaxy by assuming that the projected surface density Σ_L of the luminous component of the model be proportional to the surface luminosity \mathcal{L} of the galaxy, at each location R . This allows us to determine the scale S of the model, i.e., the equivalent in arcsec of the unit length of the model, by minimizing the quantity χ_P^2 :

$$\chi_P^2 = \sum_{R_{\min}}^{R_{\max}} \{[\mu(R) - \mu_L(R/S) + P]/\delta\mu(R)\}^2. \quad (1)$$

We set $P = -2.5 \log(\hat{M}_L/L)$, where the mass of the luminous component \hat{M}_L is expressed in model units. The quantity $\mu(R)$ is the surface brightness (mag arcsec⁻²) at distance R from the center of the galaxy, $\delta\mu$ is the error on the value of μ , and μ_L is the projected mass density of the luminous component of the model, expressed in magnitudes. The sum extends from R_{\min}^P to R_{\max}^P (see BSS88).

Once the value of S has been determined, the code proceeds to fit the kinematic data (i.e., the velocity dispersion profile of the galaxy). This fit determines the central projected velocity dispersion of the model considered here. The velocity scale S_K

is then calculated by minimizing the quantity

$$\chi_K^2 = \sum_{R_{\max}^K} \{[\sigma_{\text{obs}}(R) - S_K \sigma_{\text{mod}}(R/S)]/\delta\sigma(R)\}^2, \quad (2)$$

where σ_{obs} is the measured velocity dispersion, $\delta\sigma$ the related error, σ_{mod} the projected velocity dispersion of the luminous component predicted by the model, R_{\max}^K the maximum radius where kinematical data are available and S is the scale selected by the photometric fit. Therefore we find

$$S_K = \frac{\Sigma \sigma_{\text{mod}}(R/S) \sigma_{\text{obs}}(R) / \delta\sigma(R)^2}{\Sigma \sigma_{\text{mod}}^2(R/S) / \delta\sigma(R)^2}. \quad (3)$$

Finally, the program computes the total luminous mass of the galaxy and the amount of dark matter predicted by the model under study (see BSS88).

We note that in principle the fitting procedure may give different results on the mass content in the galaxy depending on the radial extent of the performed fits, since the scales S and S_K depend on the values of R_{\max}^P and R_{\max}^K considered. Therefore, whenever the data that are fitted cover only a fraction of the half-luminosity radius, as often is the case for accurate CCD data for large nearby objects, one should show that the final conclusions are not too sensitive to the adopted values of R_{\max}^P and R_{\max}^K . This point will be considered further in § 3 and in § 5.

We now address the nontrivial issue of defining the best-fit models to a given galaxy and of selecting models within a given confidence level. We may start out by introducing a *global* $\bar{\chi}^2$:

$$\bar{\chi}^2 = \frac{\chi_P^2 + \chi_K^2}{N_P + N_K - 6}. \quad (4)$$

The $\bar{\chi}^2$ parameter is normalized to the total number of data points of the photometric (N_P) and kinematical (N_K) fits, minus the number of parameters (these are the scale S , the velocity scale S_K and the four dimensionless parameters that specify a given two-component model). If N_P and N_K correctly represent the number of independent data points and if $\delta\mu$ and $\delta\sigma$ were Gaussian errors, then $\bar{\chi}^2$ would be a random variable distributed according to the χ^2 distribution of mean 1 and variance $2/(N_P + N_K - 6)$ (see Abramowitz & Stegun 1970). In this case the confidence level of a given fit would be one minus the probability that $\bar{\chi}^2$ exceeds the actual value of $\bar{\chi}^2$. The selection of models with $\bar{\chi}^2 < 1 + 3\sqrt{2/(N_P + N_K - 6)}$ corresponds to the so-called "3 sigma limit" with confidence level 99.7%. In principle the derived value of $\bar{\chi}^2$ (see eq. [4]) overestimates the real minimum value of $\bar{\chi}^2$, since $\partial\chi_K^2/\partial S \neq 0$, but we have verified that this dependence is very weak. In practice, since the values of the photometric errors are not well under control, it is better to derive S by minimizing only χ_P^2 . The resulting values of $\bar{\chi}_P^2$ turn out to be smaller than unity. This prompted us to reassess the significance of the results obtained in terms of *fiducial* photometric errors (see discussion in § 5).

3. TEST OF THE FITTING PROCEDURE ON SIMULATED OBJECTS

In this section we present the results of several tests of the fitting procedure on simulated objects. Our aim is to explore the capabilities of our method to reconstruct the amount and the distribution of dark matter present in a given galaxy (§ 3.1), and to decide whether the method can be applied to systems that are intrinsically nonspherical (§ 3.2). In the last subsection

(§ 3.3) the role played by the accuracy in a given set of data is discussed.

3.1. Spherical Objects

We have considered as “test-objects” seven different spherical, self-consistent models with variable amounts and distributions of dark matter and we have applied the fitting program to the projected densities and velocity dispersion profiles of their luminous components. Their physical properties are described in detail by Saglia (1990).

In order to simulate the analysis of excellent but realistic data, the “photometric” fit has been applied to the projected densities (of the luminous component) of the test objects at distances $R_e/10 < R < 4R_e$, while in the kinematical fit only the velocity dispersions (of the luminous component) at distances $R_e/10 < R < R_e$ have been considered. In this analysis no “noise” has been added to the test models: a constant (fictitious) value for the errors $\delta\mu$ and $\delta\sigma$ has been adopted in equations (1) and (2) (see also following § 3.3).

Since for the bright objects studied in the present paper the profiles are radially more restricted, in order to make sure that the results are not too sensitive to the choice of R_{\max}^P and R_{\max}^K , we have performed additional tests based on $R_{\max}^P = R_e$ and $R_{\max}^K = R_e/3$, which represents a more common situation. Indeed the results of the fits turn out to be substantially unchanged; but we note that in some of these cases accepted photometric fits can be obtained with differences in the estimated scale length of the test object as large as 50% (see discussion of NGC 4636 in § 5).

From the results of these tests we can draw the following conclusions (see Paper I for some of the terminology used here):

1. The fitting program is able to recognize the presence of dark matter in a given test object, as shown by an examination of the χ_P^2 - χ_K^2 correlation plots for normal (i.e., with extended and usually more massive halos) and reverse (i.e., with compact and often lighter halos) models.
2. The algorithm is able to derive (within 20%) the scale lengths and the masses of the two components and with a better precision the masses enclosed within R_e only when the dark component is distributed differently from the luminous component, i.e., it is not dominant in the central parts and it is diffuse.
3. The fitting procedure is flexible enough to recognize the presence of dark mass even for systems populated by stellar orbits that are *not* associated with the f_∞ -distribution function.
4. If the system under study has a pressure anisotropy profile structure that is drastically different from what is argued on the basis of our physical assumptions, as in the case of a test object heavily populated with radial orbits, then the fitting procedure is still able to get some quantitative answers; however, the quality of the fit is degraded. This means that when dealing with *observed* galaxies our models can be disproved if certain kinematical features turn out to be not well fitted. In contrast, very good kinematical fits are obtained in our sample of bright objects (see § 5).
5. In line with the expectations of some analytical estimates (see Appendix B of Paper I), modest rotation ($V_*/\sigma(o) \simeq 0.13$) does not affect the fits significantly.

To be sure, these conclusions have been drawn only on the basis of the seven cases considered.

3.2. Flattened Objects

With the aim of testing the dependence of our diagnostics on the geometry of the system under investigation, we have

applied our fitting program to two highly flattened test objects F1 and F2, the first without dark matter, prolate, and characterized by axial ratios 1:3, the second oblate, with dark matter ($M_L/M_D = 0.6$, $r_L/r_D = 0.79$), and with axial ratios 1:2.5. Stellar dynamical models so flattened and characterized by a realistic luminosity profile are presently *not* available in analytical form. Therefore, the test objects have been constructed by means of an N -body code derived from that of van Albada (1982) as described by Bertin & Stiavelli (1989). Model F1 has been produced by simulating galaxy formation via collisionless collapse starting from cold initial conditions. Model F2 instead, has been obtained by a procedure analogous to the one described by Bertin & Stiavelli (1989), by relaxing an approximate flattened configuration. Its evolution has been monitored for about $12t_{\text{cr}}$ (defined in terms of the total mass M and the total energy \mathcal{E} as $t_{\text{cr}} = GM^{5/2}/|2\mathcal{E}|^{3/2}$) to check that the system actually reached equilibrium.

For each test object, two fits have been performed, one based on simulated observations from a viewing angle that makes the object most flattened (F1E6, F2E4, see Fig. 1) and the other from a viewing angle that makes the object appear roundest (F1E0, F2E0). Of course, it is the second case that applies better to the work presented in this paper, which is devoted to galaxies of the E0–E1 type. For each case the “photometry” of the test object has been derived using the programs described by Bender & Möllenhoff (1987), after smoothing of the projected density of the luminous component. The projected density profile is given as a function of the circularized distance from the center $\sqrt{1 - \epsilon}a$, where $1 - \epsilon$ is the average axial ratio and a is the distance along the major axis. The quantity ϵ has been set to 0 for test objects F1E0 and F2E0, and to 0.55 and 0.4 for F1E6 and F2E4, respectively. In order to improve the signal to noise ratio of the profiles of test objects F2, four noncorrelated flashes of the N -body configuration have been averaged. A mean velocity dispersion profile has been obtained by averaging the projected velocity dispersion profiles of the luminous component along the major and minor apparent axes of the model and folding the data with respect to the center. The properties of the best-fit models are shown in Figure 2 and the results of the fits are summarized in Table 1. Note that even in these nonspherical cases r_L and r_D refer to *spheres* containing half of the relevant masses. The fits have been performed using constant values of $\delta\mu$ and $\delta\sigma$. From this analysis, the performance of the fits is found to be satisfactory both for the intrinsically prolate F1 and the intrinsically oblate F2 models.

It turns out that better estimates of the physical parameters of the flattened test objects are obtained when these are seen flattened also in projection. Even better results are obtained when the test objects are intrinsically spherical. On the other hand, for intrinsically nonspherical objects seen round in projection, *our method seems to underestimate the amount of dark matter present*. Based on these considerations, we have preferred to focus our attention on E0–E2 galaxies (see §§ 5 and 6).

It has sometimes been argued (see, e.g., Bertola et al. 1991) that the use of spherical models to fit nonspherical objects leads to unreliable results. This is certainly true if kinematical tracers (such as gas) are used that rely on the structure of *closed* orbits. In fact, for these cases one may argue that the structure of these orbits is exceedingly sensitive not only to geometry and viewing angles of the system, but also to the detailed mass distributions (which tends to be badly represented by Stäckel potentials) and especially to figure rotation. The use of the *velocity dispersion profile of the stars*, on intuitive grounds, is expected to be less sensitive to the above mentioned factors.

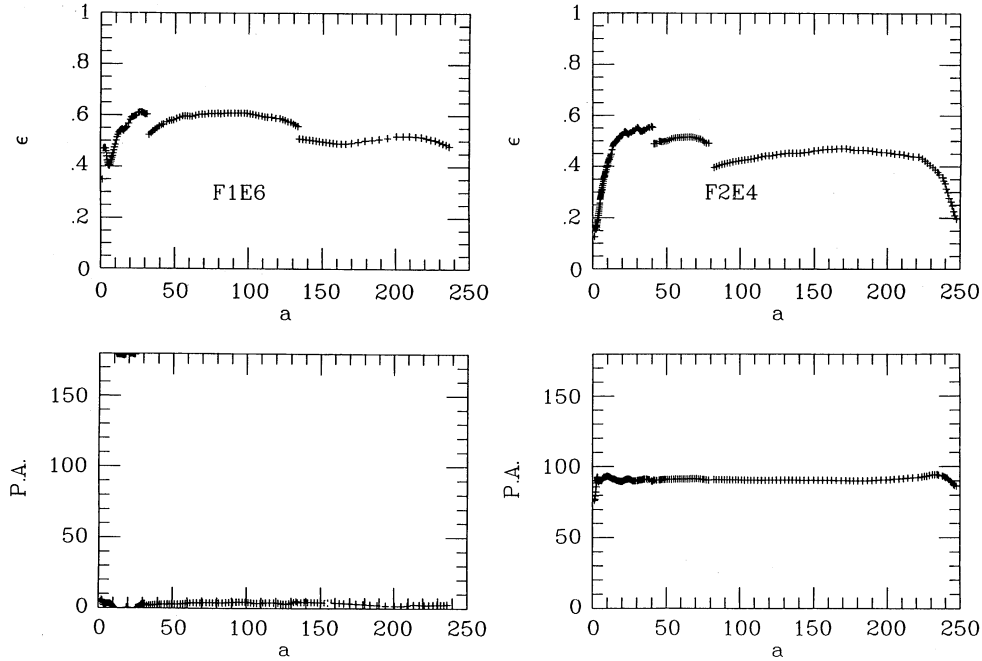


FIG. 1.—Photometric properties of flattened test-objects. The ellipticity ϵ (top) and the position angle P.A. (bottom) are shown as a function of the radial distance along the major axis a for the models F1 (left) and F2 (right) seen edge-on. The various profiles extend to $4R_e$. Discontinuities are not intrinsic, but correspond to the use of different smoothings of the data.

This point of view is indeed supported by the simulated observations presented in this subsection.

3.3. Role of the Accuracy of the Analyzed Data

All the tests performed so far assume that the photometric and the kinematical profiles are known (at least out to R_{\max}^P and R_{\max}^K) with infinite precision. Kinematical profiles are much more noisy than photometric profiles, and only recently stellar kinematics with error bars smaller than 10 km s^{-1} have been derived for some galaxies (see Davies & Birkinshaw 1988), although only out to a fraction of the half-luminosity radius of the galaxies. It is therefore important to examine the role of the accuracy of the analyzed data on the results of the fitting procedure.

The following simple argument can give an estimate of the required precision on the kinematical data for a reliable determination of the amount of dark matter inside R_e . Referring to equation (15) of Paper I, in general we expect to measure a ratio $\mathcal{R}_0 = \sigma_L(R_e)/\sigma_L(0) = 0.53$ if $M_D(R_e) = 0$, and a ratio

$\mathcal{R}_1 = 0.76$ if $M_D(R_e)/M_L(R_e) = 1$. So, in order to discriminate between the two cases with a “3 sigma” level of confidence, the error on \mathcal{R} should be smaller than $\delta\mathcal{R} \approx (\mathcal{R}_1 - \mathcal{R}_0)/(3 \times 2) = 0.04$. If we have only two measurements of the stellar velocity dispersion profile, $\sigma_0 \pm \delta\sigma_0$, $\sigma_1 \pm \delta\sigma_1$, where $\sigma_i = \sigma(R_e)$, we have

$$\frac{\delta\mathcal{R}}{\mathcal{R}} = \sqrt{\left(\frac{\delta\sigma_0}{\sigma_0}\right)^2 + \left(\frac{\delta\sigma_1}{\sigma_1}\right)^2}, \quad (5)$$

and therefore

$$\frac{\delta\sigma_1}{\sigma_1} = \sqrt{\left(\frac{\delta\mathcal{R}}{\mathcal{R}}\right)^2 - \left(\frac{\delta\sigma_0}{\sigma_0}\right)^2}, \quad (6)$$

$$\delta\sigma_1 = \mathcal{R}\sigma_0 \sqrt{\left(\frac{\delta\mathcal{R}}{\mathcal{R}}\right)^2 - \left(\frac{\delta\sigma_0}{\sigma_0}\right)^2}. \quad (7)$$

Thus, if $\sigma_0 = (300 \pm 5) \text{ km s}^{-1}$, we derive from equation (7) $\delta\sigma_1 = 8 \text{ km s}^{-1}$, while for $\sigma_0 = (200 \pm 7) \text{ km s}^{-1}$, $\delta\sigma_1 = 3 \text{ km s}^{-1}$.

TABLE 1
SIMULATION OF FLATTENED NONROTATING OBJECTS

Model	r_L	r_D	$M_L(R_e)$	$M_D(R_e)$	$M(R_e)$	M_L	M_D	M
F1E0	77				522			1257
Best fit	60				597			1662
Range	57–62				557–618			1521–2055
F1E6	77				522			1257
Best fit	108				505			1406
Range	84–109				409–520			1400–1985
F2E0	72	91	146	338	484	471	785	1257
Best fit	67	96	125	129	254	290	407	697
Range	67–78	70–149	20–180	82–216	236–284	46–416	207–927	623–973
F2 E4	72	91	146	338	484	471	785	1257
Best fit	46	146	194	107	301	459	892	1350
Range	43–46	104–220	62–225	83–217	279–311	145–532	541–1804	1057–1949

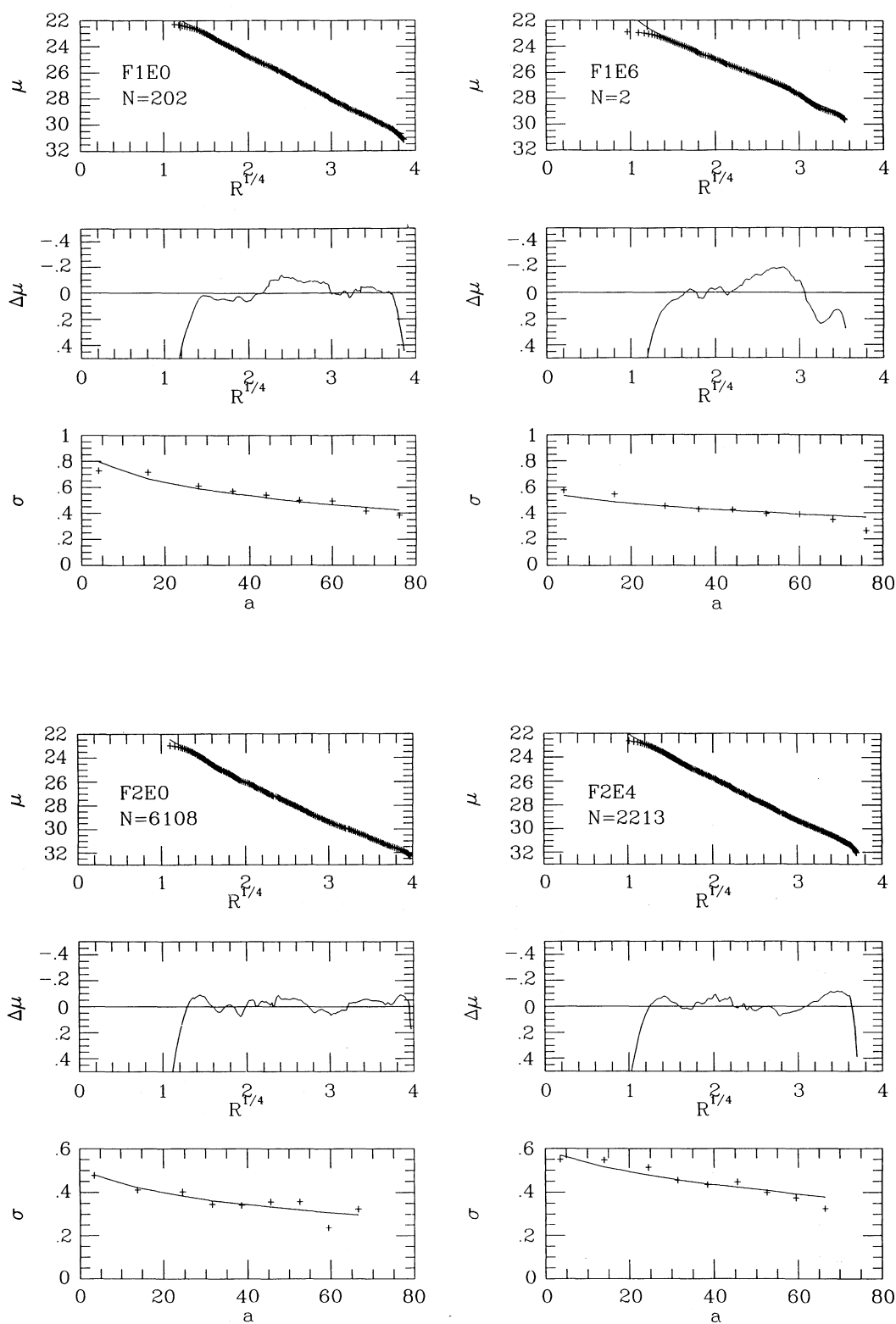


FIG. 2.—Best-fits to the flattened test-objects. The luminosity profiles (top frames) of the test objects (symbols) are compared to those predicted by the best-fit models (solid lines), as a function of the $\frac{1}{4}$ power of the circularized distance from the center (see text); the residuals $\Delta\mu = \mu_{\text{obj}} - \mu_{\text{mod}}$ are shown in the middle frames. The corresponding projected velocity dispersion profiles (symbols: test objects; solid lines: best-fit model) are given in the bottom frames as a function of the radial distance taken along the major axis a .

TABLE 2
GLOBAL PARAMETERS OF THE ANALYZED GALAXIES AND CODING OF THE FITS

Object	Type ^a	<i>D</i> (Mpc)	<i>B_T</i>	Band	FWHM	<i>N_p</i>	<i>R_{min}^p</i>	<i>R_{max}^p</i>	Source	<i>N_k</i>	<i>R_{max}^k</i>	Source	Code
NGC 1399.....	E1P	28	−21.7 ^b	R	1"	32	3".45	66".26	b	10	37".3	b	F
				R	1	32	3.45	66.26	b	29	87.9	c	B
				B	1	55	3.1	562	c	29	87.9	c	BSKY
				B	1	82	3.1	3162	c	29	87.9	c	BCLEA
NGC 1404.....	E1	28	−21.36 ^b	R	0.9	33	3.1	66.3	b	12	39.4	b	F
NGC 1549.....	E0	23	−21.25 ^b	R	1.4	33	3.1	66.3	b	10	50.1	b	F
NGC 3379.....	E1	16	−20.65 ^d	B		97	2.7	234	e	10	64.8	f	D
				B		97	2.7	234	e	8	38.6	g	DB
NGC 4278.....	E1	16	−20.01 ^d	R	2	32	6.2	118.9	h	8	34.3	g	P
NGC 4374.....	E1	20	−21.43 ^h	R	1.6	38	5.5	163.3	h	8	36.3	g	P
				B	2.4	23	5.0	446.6	i	8	36.3	g	MI
				B	2.4	20	5.0	316	i	9	63.3	f, g	MIDBD
NGC 4472.....	E2	20	−22.24 ^h	R	2	37	5.4	170.8	h	8	42.8	g	P
				B	3.3	24	5.0	562.3	i	9	63.8	f, g	MI
				B	3.3	41	4.2	316	h, i	9	63.8	f, g	PMI
NGC 4486.....	E0p	20	−22.04 ^h	B		185	6.1	1186	j	14	69.7	k	S
				B	1	62	5.47	1186	j, l	8	71.8	g, k	YDBS
NGC 4636.....	E0	20	−21.36 ^h	R	2.1	36	5.9	165.3	h	8	37.2	g	P
				B	1.4	9	5.6	316.2	m	9	65.8	f, g	KI
NGC 7626.....	E1p	77	−22.44 ^b	r	2	30	3.65	85.05	n	15	34.46	o	K

^a de Vaucouleurs, de Vaucouleurs, & Corwin 1976.

^b Franx, Illingworth, & Heckman 1989a, 1989b.

^c Bicknell et al. 1989.

^d van der Marel, Binney, & Davies 1990.

^e De Vaucouleurs & Capaccioli 1979.

^f Davies 1981.

^g Davies & Birkinshaw 1988.

^h Peletier et al. 1990.

ⁱ Michard 1985.

^j de Vaucouleurs & Nieto 1978, 1979.

^k Sargent et al. 1978.

^l Young et al. 1978.

^m King 1978.

ⁿ Kent 1984.

^o Jedrzejewski & Schechter 1988.

s^{-1} . If N data points have been measured, in the best case we can require the single data points to have error bars lower than $\sqrt{N} \delta\sigma_1$. Therefore, better results are obtained if bright galaxies with high central velocity dispersion are considered, since from equation (7) we see that in these cases $\delta\sigma_1$ can be larger.

By performing more realistic (with respect to § 3.1) simulated observations with given Gaussian noise, we find that, since the present accuracy of CCD photometries is of the order of 0.03–0.05 mag arcsec^{−2}, it is possible to recognize unambiguously the presence of an amount of dark matter inside R_e equal to the luminous mass only if the velocity dispersion profile is measured with a precision better than 10% out to such a radius.

4. A SET OF BRIGHT ELLIPTICAL GALAXIES

We now turn our attention to the set of 10 round in projection, slowly rotating ellipticals that have been fitted. Table 2 summarizes the properties of the selected objects, together with the sources for the relevant profiles. Six of these objects have been modeled by us before (BSS88), without dark matter, on the basis of different photometric and kinematical data.

The surface brightness profiles considered in Table 2 have been obtained for the majority of the objects using CCD detectors, with typical accuracy better than 0.1 mag. In a few cases, other (photographic photometry) sources have also been considered in order to cover a larger radial extent. The seeing (FWHM in arcsec) of the various observations is given in

Table 2 (except for NGC 3379 and NGC 4486, since the photometries of these objects have been obtained by considering various observations with different seeing conditions), together with the minimum and the maximum radii R_{min}^p and R_{max}^p considered in the photometric fit (see § 2, eq. [1]). The total number N_p of photometric data used in the fit is also given in the table.

As to the kinematics, we have referred to the recent and accurate data derived by Franx, Illingworth, & Heckman (1989b), Jedrzejewski & Schechter (1988), Bicknell et al. (1989), and Davies & Birkinshaw (1988). For NGC 4374, NGC 4472, and NGC 4636 we have performed fits to kinematical profiles obtained by adding to the profiles derived by Davies & Birkinshaw (1988) the last data points measured by Davies (1981). In addition, for NGC 3379 we have considered also the velocity dispersion profile measured by Davies (1981), and for NGC 4486 the extended kinematical data of Sargent et al. (1978). In Table 2 we give for each galaxy the distance from the center of the last available kinematical point R_{max}^k , together with the total number of data N_k considered in the fit and the corresponding source. The velocity dispersion profiles taken from Franx, Illingworth, & Heckman (1989b) refer to the major axis of the galaxies and have been folded by considering the data from both sides of the center. The velocity dispersion profile of NGC 1399 has been obtained from Bicknell et al. (1989) by folding and averaging the data points from the two sides of the

center (along the major axis):

$$r = \frac{r_+ + r_-}{2} \quad (8)$$

$$\sigma(r) = \left(\frac{\sigma_+}{\delta\sigma_+} + \frac{\sigma_-}{\delta\sigma_-} \right) / \left(\frac{1}{\delta\sigma_+} + \frac{1}{\delta\sigma_-} \right) \quad (9)$$

$$\delta\sigma(r) = 1 / \sqrt{\frac{1}{\delta\sigma_+^2} + \frac{1}{\delta\sigma_-^2}} \quad (10)$$

Here r_+ and r_- are the distances of the points on the two sides of the galaxy ($r_+ \approx r_-$), σ_+ and σ_- the relative values of the velocity dispersions, and $\delta\sigma_+$ and $\delta\sigma_-$ the error bars. The data in the inner 5" have been excluded (see discussion in the following § 5). The last two points have been measured on the western side of the galaxy only. The velocity dispersions given by Davies & Birkinshaw (1988) are averaged over all slit position angles. Finally, the velocity dispersion profile for NGC 7626 (Jedrzejewski & Schechter 1988) has been obtained by folding and averaging the velocity dispersion data measured along the two sides of the major axis of the galaxy as for the data of Bicknell et al. (1989) (see eqs. [8]–[10]). The sets of data used for the fits (photometry and kinematics) are listed in Table 2 and there identified by a code in the last column.

5. THE FIT

To each set of data of Table 2 we have applied the fitting procedure described in § 2 using a constant value for the photometric errors ($\delta\mu = 0.1 \text{ mag arcsec}^{-2}$). For each case we have selected all the normal and reverse models with χ_P^2 and χ_K^2 parameters lower than some prescribed values. These upper limits have been chosen (somewhat arbitrarily, see following § 5.2) to be of the order of or lower than the values of χ_P^2 and χ_K^2 derived from the one-component (i.e., $M_D = 0$) f_∞ fits (in Table 3 we report the results of such fits), which have been carried out for the purpose of comparison. Among the selected models we have identified a "best-fit" solution, by trying to minimize both the χ_P^2 and the χ_K^2 parameters (further discussion is given in § 5.2).

In Tables 4–7, the first column of the tables identifies the objects, while in the second column the code specifies the data used in the fit (see Table 2), with an additional N to indicate fits by normal models. For each code two rows are reported: the first gives the parameters of the selected "best-fit," the second the ranges for the acceptable fits. In Table 4, the "range" row for χ_P^2 and χ_K^2 gives the upper limits adopted in the selection. In

Table 5, $M(R_e)$ is the value of the spherical mass enclosed inside R_e . Only the properties of the best-fit cases are summarized in the tables; complete data on all the fits performed are listed by Saglia (1990). The global values of M/L_B are not actually reliable, given the limited range of the kinematical profiles available. For the same reason, it would have been fairer to record the values of $(M/L_B)_{R_{\text{max}}}$, i.e., referred to the sphere sampled by the kinematical data, but we have preferred to list the values of $(M/L_B)_{R_e}$ which have an easier physical interpretation and are more easily compared to one another.

For all the 10 galaxies considered here we have obtained good fits of their photometric and kinematical data, by considering models with a dark component. The selected models illustrated in the figures have always χ_P^2 smaller than the number of data points. In general the quality of the kinematical fit is also very good. Figure 3 shows the photometric and kinematical best fits for eight of the ten galaxies. The luminosity excesses in the center would have been removed if we had performed the appropriate seeing convolution.

We may summarize the conclusions that can be drawn from the analysis of these fits by saying that we find no evidence for dark matter in NGC 1549, marginal evidence in NGC 1399, NGC 1404, NGC 3379, NGC 4278, NGC 4486, and relatively good evidence in NGC 4374, NGC 4472, NGC 4636, and NGC 7626, for which the two components fits improves significantly upon one component fits. In our view the most convincing case for dark matter is that of NGC 4472.

In many cases (NGC 4472, NGC 4636, NGC 7626) the obtained evidence is for massive ($M_L/M_D < 0.5$) and diffuse ($r_L/r_D < 0.5$) halos (see Table 6), with modest amounts of dark matter inside R_e [$(M_L/M_D)_{R_e} > 1$]. In three cases where X-ray data are available, we observe a correlation between the extent of X-ray emission and the derived halo scalelength. For NGC 4374 we have found best-fit models that have significant amounts of dark matter even inside R_e .

For the cases of NGC 4374 and NGC 4472, which in our view provide the strongest evidence for the presence of dark matter, the results of the fitting procedure are illustrated separately in more detail (similar plots have been produced for all the other galaxies of Table 2; see Saglia 1990). For NGC 4472, in Figure 4 we show the correlation plots χ_P^2 – χ_K^2 obtained for normal and reverse models separately (fits PMIN and PMIR). Only normal models minimize the parameters χ_P^2 and χ_K^2 and these have a diffuse dark halo. The set of identified models at a 3 sigma level (see following § 5.2) is represented in Figure 5 in the $(r_L/r_D, M_L/M_D)$ physical plane (see Paper I, Fig. 5). In Figure 6 we show for the best-fit solution the fit to the photometric profile, the residuals $\Delta\mu \doteq \mu - \mu_{\text{mod}}$, and the fit to

TABLE 3
FIT BY ONE-COMPONENT f_∞ MODELS^a

Object	Code	χ_P^2	χ_K^2	Ψ	R_e	r_M	M	(M/L_B)
NGC 1399.....	B	17.7	40.5	30	51"	67"	5.2	7.0
NGC 1404.....	F	9	53.9	6	91.2	120	4.8	8.8
NGC 1549.....	F	6.5	44.1	25	76	101	2.7	5.5
NGC 3379.....	DB	55.3	9.9	30	65	84	1.5	5.2
NGC 4278.....	P	9	8.1	6	110	144.9	2.7	17.2
NGC 4374.....	P	13.7	53.2	25	69	92.6	5	8.6
NGC 4472.....	P	4.2	65.8	12	131	172.3	8.8	7.2
NGC 4486.....	YDBS	81.9	5.7	12	132	173	8.9	8.7
NGC 4636.....	P	6.8	38.5	7.7	229	300.7	5.4	9.9
NGC 7626.....	K	3.4	119	9.6	45	59.6	7.8	5.3

^a Masses are in $10^{11} M_\odot$, and M/L ratios are in solar units.

TABLE 4
RESULTS OF THE TWO-COMPONENT FITS TO THE GALAXIES OF TABLE 2

Object	Code	χ^2_P	χ^2_K	R_e (Kpc)	R_e	r_L	r_D
NGC 1399.....	BN	3.53	6.91	13.6	100%0	135%6	838%5
	BN	7.50	9.00	9.8–13.6	72.1–100.0	97.8–136.0	654.0–838.5
NGC 1404.....	FN	6.04	60.50	8.3	61.5	89.6	91.1
	FN	7.75	69.30	4.6–9.5	34.1–69.8	46.2–108.0	85.3–395.9
NGC 1549.....	FN	2.43	37.91	9.5	85.1	114.6	420.3
	FN	3.10	56.70	9.5–11.2	85.1–100.0	114.6–134.7	396.8–671.3
NGC 3379.....	DBN	59.76	5.12	4.8	61.5	82.9	167.9
	DBN	59.85	7.00	4.4–5.0	56.4–64.1	75.8–86.6	128.1–450.1
NGC 4278.....	PN	3.36	7.31	4.4	57.1	78.1	480.2
	PN	6.00	11.20	4.4–7.8	57.1–100.0	78.1–148.8	95.6–492.1
NGC 4374.....	PN	1.99	4.51	12.6	130.0	179.2	184.3
	PN	2.88	7.00	7.1–18.1	73.6–186.7	99.9–264.5	163.1–366.7
NGC 4472.....	PMIN	5.89	5.04	12.0	123.9	166.2	466.1
	PMIN	9.75	12.64	10.3–14.6	106.5–151.0	142.3–204.4	385.9–1266.1
NGC 4486.....	YDBSN	66.14	2.72	13.1	134.8	181.3	379.5
	YDBSN	82.60	2.80	12.6–13.1	129.6–134.8	175.1–181.3	178.8–448.3
NGC 4636.....	PN	6.70	4.06	15.6	160.8	214.9	1277.2
	PN	6.80	5.60	15.6–19.4	160.8–200.0	214.9–267.7	861.9–1517.7
NGC 7626.....	KN	1.70	29.51	15.3	40.9	54.6	353.2
	KN	2.24	35.00	14.9–15.3	39.9–41.1	53.3–54.9	320.5–382.3

the velocity dispersion profile. Note that we have incorporated in the kinematical profile the velocity dispersion data points of the globular clusters obtained by Mould et al. (1990). The predictions of the best-fit model (identified on the basis of the stellar data alone) are found to be well consistent with the data out to $R = 300'' \approx 2.5R_e$. In Figure 7 we show more in detail the results of the optimal luminous-dark decomposition for the mass distribution of NGC 4472, in terms of volume densities, cumulative masses, and force fields as measured by the circular velocity profile V ; in addition, in this composite figure we show the gradient of the cumulative mass-to-light ratio. In Figures 8

and 9 we show the analogous plots obtained for NGC 4374 (data PN).

5.1. Detailed Discussion of Individual Objects

5.1.1. NGC 4374

This galaxy of the Virgo cluster is known as a radio and an X-ray source with X-ray emission measured out to $R_X \approx R_e$ (e.g., Sarazin 1990). We have fitted both the photometric profiles of Peletier et al. (1990) and of Michard (1985). A proper matching of the two photometries is impossible, since they have different slopes. The best-fit models selected using the two

TABLE 5
RESULTS OF THE TWO-COMPONENT FITS TO THE GALAXIES OF TABLE 2^a

Object	Data	$M_L(R_e)$	$M_D(R_e)$	$M(R_e)$	M_L	M_D	M
NGC 1399.....	BN	2.5	1.5	4.0	5.9	37.3	43.2
	BN	2.0–2.5	1.0–1.9	3.1–4.2	4.8–5.9	30.2–37.5	35.7–43.2
NGC 1404.....	FN	1.1	1.5	2.6	2.5	4.0	6.5
	FN	0.8–1.6	0.5–1.9	1.6–2.9	1.8–3.7	3.3–6.8	5.5–10.2
NGC 1549.....	FN	0.7	1.0	1.7	1.7	6.3	8.0
	FN	0.7–1.0	0.9–1.3	1.7–2.2	1.7–2.4	6.3–12.8	8.0–15.1
NGC 3379.....	DBN	0.6	0.4	1.0	1.4	1.9	3.3
	DBN	0.5–0.7	0.3–0.5	1.0–1.1	1.3–1.7	1.1–5.0	2.8–6.3
NGC 4278.....	PN	0.8	0.2	1.1	2.0	3.7	5.7
	PN	0.2–0.9	0.2–1.6	1.1–1.8	0.5–2.2	1.7–4.5	2.8–6.0
NGC 4374.....	PN	1.8	2.7	4.6	4.2	6.9	11.1
	PN	0.3–2.1	1.7–4.4	3.0–6.5	0.7–4.9	4.9–15.6	9.7–17.4
NGC 4472.....	PMIN	3.3	1.7	5.1	7.9	13.4	21.3
	PMIN	2.6–3.6	1.3–3.2	4.4–6.5	6.0–8.4	8.4–49.6	16.7–56.8
NGC 4486.....	YDBSN	3.3	1.7	5.0	7.8	9.0	16.8
	YDBSN	2.8–3.4	1.3–2.0	4.5–5.1	6.6–8.1	4.1–9.8	10.7–17.1
NGC 4636.....	PN	2.4	0.9	3.3	5.9	22.5	28.4
	PN	2.0–2.8	0.9–2.5	3.3–4.5	4.8–6.6	11.0–38.5	17.4–43.4
NGC 7626.....	KN	3.3	1.2	4.6	8.1	38.0	46.1
	KN	3.3–3.4	1.2–1.5	4.5–4.8	7.9–8.2	30.3–47.2	38.3–55.3

^a Masses are in $10^{11} M_\odot$.

TABLE 6
RESULTS OF THE TWO-COMPONENT FITS TO THE GALAXIES
OF TABLE 2

Object	Code	$\frac{r_L}{r_D}$	$\left(\frac{M_L}{M_D}\right)_{R_e}$	$\frac{M_L}{M_D}$
NGC 1399.....	BN	0.2	1.6	0.2
	BN	0.1–0.2	1.3–2.4	0.1–0.2
NGC 1404.....	FN	1.0	0.7	0.6
	FN	0.2–1.0	0.5–2.9	0.3–0.8
NGC 1549.....	FN	0.3	0.8	0.3
	FN	0.2–0.3	0.7–1.1	0.2–0.3
NGC 3379.....	DBN	0.5	1.5	0.7
	DBN	0.2–0.6	1.2–2.7	0.3–1.5
NGC 4278.....	PN	0.2	4.0	0.5
	PN	0.2–1.0	0.1–4.0	0.1–0.8
NGC 4374.....	PN	1.0	0.7	0.6
	PN	0.3–1.1	0.1–1.2	0.0–1.0
NGC 4472.....	PMIN	0.4	2.0	0.6
	PMIN	0.1–0.5	0.9–2.7	0.1–1.0
NGC 4486.....	YDBSN	0.5	2.0	0.9
	YDBSN	0.4–1.0	1.6–2.7	0.7–1.6
NGC 4636.....	PN	0.2	2.7	0.3
	PN	0.2–0.3	0.8–2.7	0.1–0.6
NGC 7626.....	KN	0.2	2.7	0.2
	KN	0.1–0.2	2.2–2.7	0.2–0.3

photometries are different. Larger values of r_L are generally found using the photometry of Peletier et al. (1990; fits P). However, the selected *normal* and *reverse* models have all substantial amounts of dark matter for both photometries. Dark matter is present in significant amounts even inside R_e .

5.1.2. NGC 4472

NGC 4472 is the brightest galaxy in the Virgo cluster. It is a radio and an X-ray source with very extended X-ray emission (out to $R_x \simeq 4R_e$, see, e.g., Sarazin 1990). As for the previous object, NGC 4374, we have considered both photometries of

Peletier et al. (1990) and Michard (1985). In this case it is possible to match the two sets of data, so we have extended the photometry of Peletier et al. (1990) with the last 5 data points of Michard (1985). *Reverse* models are not able to minimize both the χ_P^2 and the χ_K^2 parameters (see Fig. 4). The selected (*normal*) models have a large fraction of dark matter, but this is in the form of a diffuse halo. In the central regions of the galaxy the amount of dark matter present is low. The scale length of the dark halo is much larger than the scale length of the luminous component. Note how the selected best-fit model (see Fig. 6) has a velocity dispersion profile well consistent with the data derived from the globular cluster system of the galaxy (Mould et al. 1990) out to $R \approx 2.5R_e$.

5.1.3. NGC 4636

This galaxy of the Virgo cluster is known to be a radio and an X-ray source with X-ray emission extending to about $R_x \simeq 2R_e$ (e.g., Sarazin 1990). It would be desirable to supplement the photometric profile of Peletier et al. (1990) with more extended data. Unfortunately, a proper matching with the King (1978) photometric profile is not easy to perform. Large differences (50%) are found in the derived half-mass radii using the two photometries. In any case, it is not possible to minimize both χ_P^2 and χ_K^2 parameters for *reverse* models. The selected *normal* models have all a massive and diffuse dark halo. The invisible halo has a scale length many times larger than the scale length of the stellar (luminous) component of the galaxy. We conclude that we find evidence for a diffuse massive dark halo embedding NGC 4636.

5.1.4. NGC 7626

NGC 7626 is the brightest galaxy of Pegasus I. It has a shell structure and peculiar central stellar kinematics. It is known as a radio and an X-ray source. The velocity dispersion profile of this galaxy is peculiar, so that the quality of the kinematical fits

TABLE 7
RESULTS OF THE TWO-COMPONENT FITS TO THE GALAXIES OF TABLE 2 AND COMPARISON WITH
ONE-COMPONENT $[(M/L_B)']$ FITS AND OTHER AUTHORS^a

Object	Code	$\frac{M_L}{L_B}$	$\left(\frac{M}{L_B}\right)_{R_e}$	$\frac{M}{L_B}$	$\left(\frac{M}{L_B}\right)'$	Other
NGC 1399.....	BN	8	12.8	58.0	7	5.1–8.2 ^b
	BN	6.5–8.0	9.7–13.2	48.0–58.0		
NGC 1404.....	FN	4.6	10.8	12	8.8	
	FN	3.3–6.7	6.8–12.4	10.1–18.8		
NGC 1549.....	FN	3.5	8.1	16.2	5.5	
	FN	3.5–4.9	8.1–10.7	16.2–30.6		
NGC 3379.....	DBN	5	8.4	11.7	5.2	6.3 ^c
	DBN	4.6–6.0	8.0–9.4	9.7–22.2		
NGC 4278.....	PN	12.5	15.7	36.1	17.2	11 ^c
	PN	3.0–13.8	15.7–25.5	18.0–38.3		
NGC 4374.....	PN	7.2	17.9	19.2	8.6	11–19 ^d
	PN	1.1–8.4	12.2–25.3	16.8–30.0		
NGC 4472.....	PMIN	6.5	9.8	17.4	7.2	8.9 ^c , 6.5–86 ^d
	PMIN	4.9–6.9	8.6–12.4	13.6–46.4		
NGC 4486.....	YDBSN	7.7	11.6	16.5	8.7	
	YDBSN	6.4–7.9	10.2–11.7	10.5–16.7		
NGC 4636.....	PN	10.8	14.8	52.1	9.9	9.2–120 ^d
	PN	8.9–12.1	14.8–19.8	31.9–79.7		
NGC 7626.....	KN	5.5	7.5	31.3	5.3	
	KN	5.4–5.6	7.4–7.9	26.0–37.5		

^a Mass-to-light ratios are given in solar units.

^b Bicknell et al. 1989.

^c van der Marel, Binney, & Davies 1990.

^d Katz & Richstone 1985.

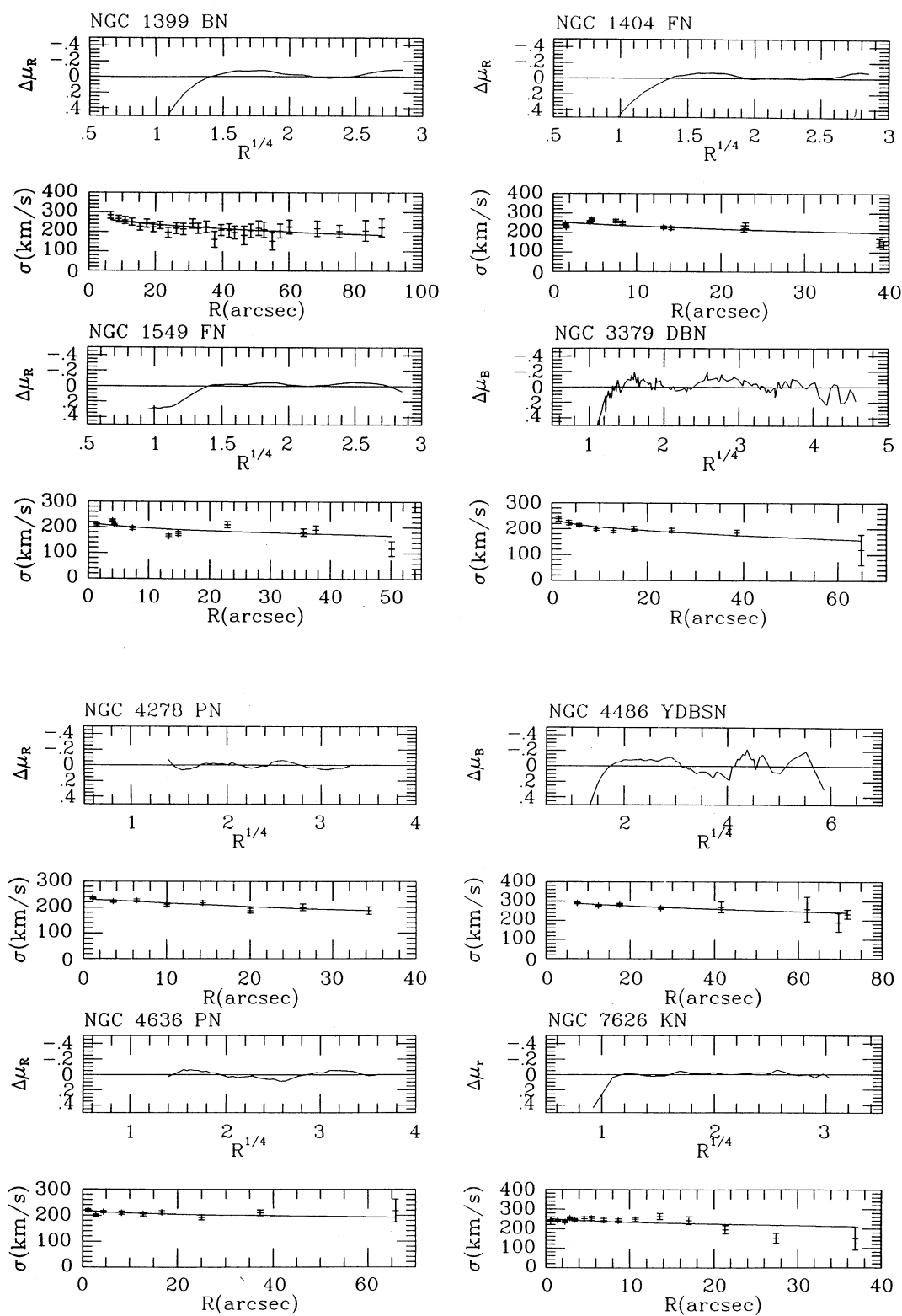


FIG. 3.—Best-fits to eight elliptical galaxies (all the galaxies of Table 2, with the exception of NGC 4374 and NGC 4472). For each galaxy two frames are shown based on the data and on the best-fit model identified by the relevant codes: the residuals $\Delta\mu = \mu - \mu_{\text{mod}}$ (top: $\Delta\mu > 0$ implies that the model is more luminous than the galaxy), and the velocity dispersion profiles (bottom: data points carry error bars). For NGC 3379 and NGC 4636 an additional outer data point taken from Davies (1981) has been added to the data of Davies & Birkinshaw (1988).

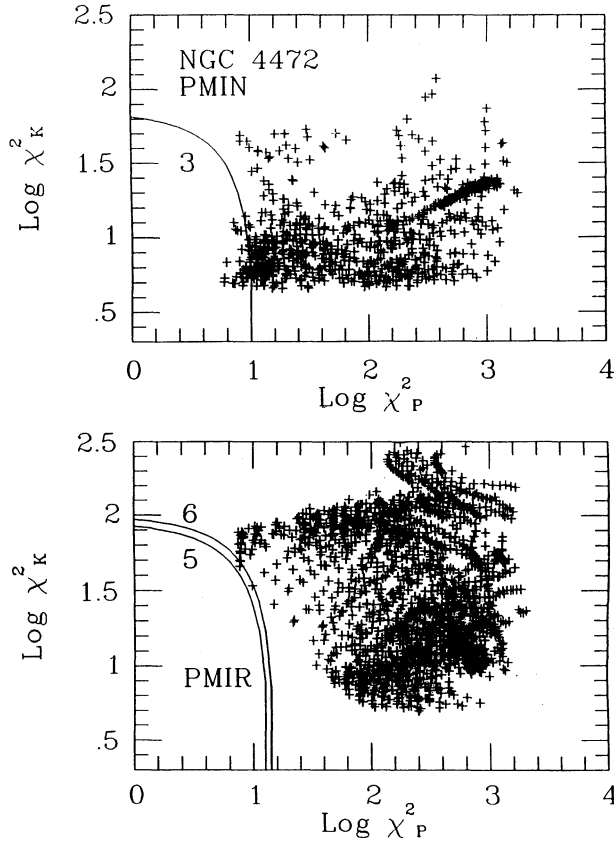


FIG. 4.—Correlation between χ_p^2 and χ_k^2 for NGC 4472 (*top*: normal models; *bottom*: reverse models). On the basis of these diagrams reverse models are excluded at 5 sigma level and the models accepted with 99.7% confidence level (3 sigma) have all substantial amounts of dark matter ($M_D > M_L$, see Fig. 5). Contours labeled by n identify the n sigma limit.

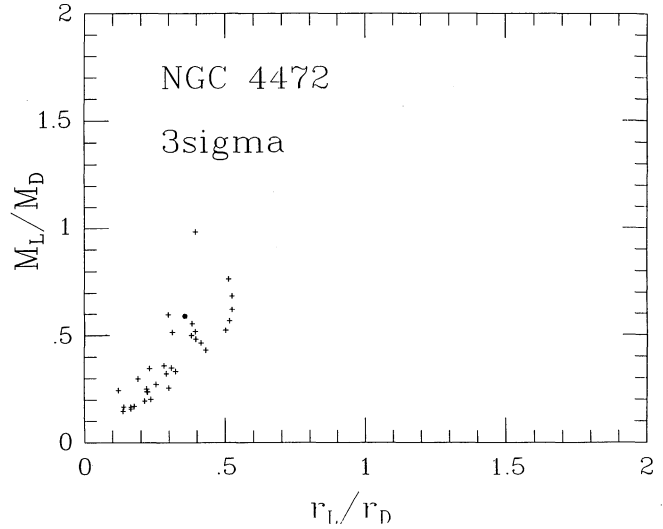


FIG. 5.—Set of identified models at a 3 sigma level (see Fig. 4) is represented in the physical plane (r_L/r_D , M_L/M_D) for NGC 4472. The point marked by a special symbol identifies the best-fit model $N = 418$.

is always low, with high values of χ_k^2 . The best photometric and kinematical fits are obtained with *normal* models. *Reverse* models have in general higher values of χ_k^2 . These selected (*normal*) models have a large fraction of dark matter, quite diffuse: the scale length of the dark halo is 4–8 times larger than that of the luminous component. We conclude that also for NGC 7626 there is evidence for a massive and diffuse dark halo, but caution is required by the unusual kinematics displayed by this object.

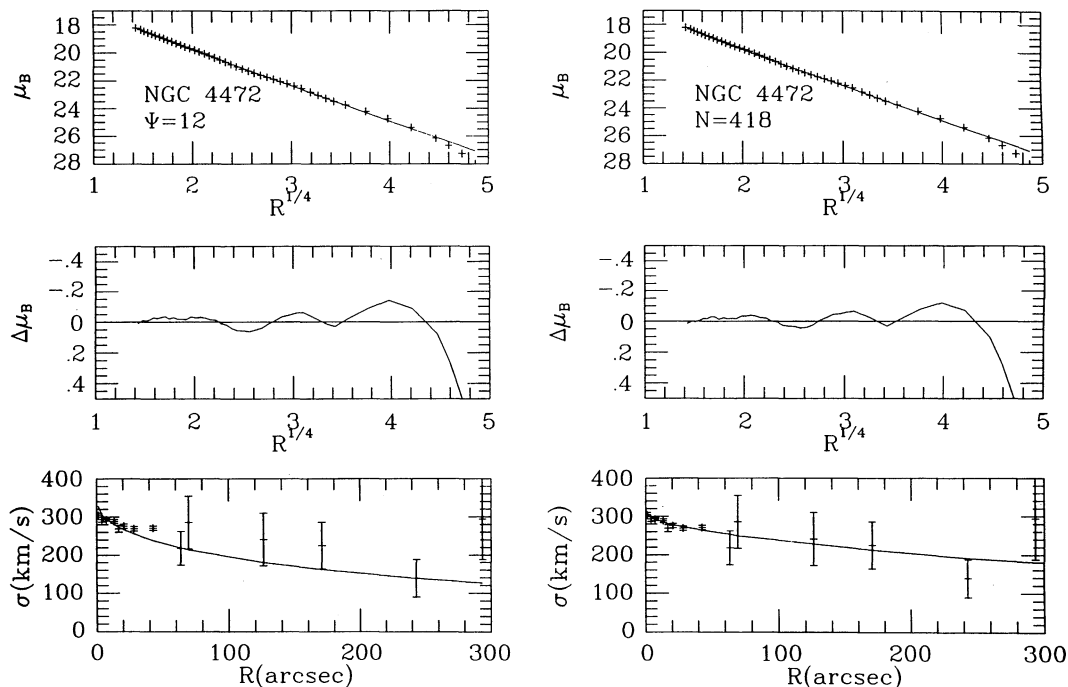


FIG. 6.—Best-fits to NGC 4472 with (*right*: model $N = 418$) and without (*left*: model $\Psi = 12$) dark matter, for the data PMI. Luminosity profiles are at the top, photometric residuals in the middle, and velocity dispersion profiles at the bottom. The five outermost kinematic data points refer to the globular cluster system (see text).

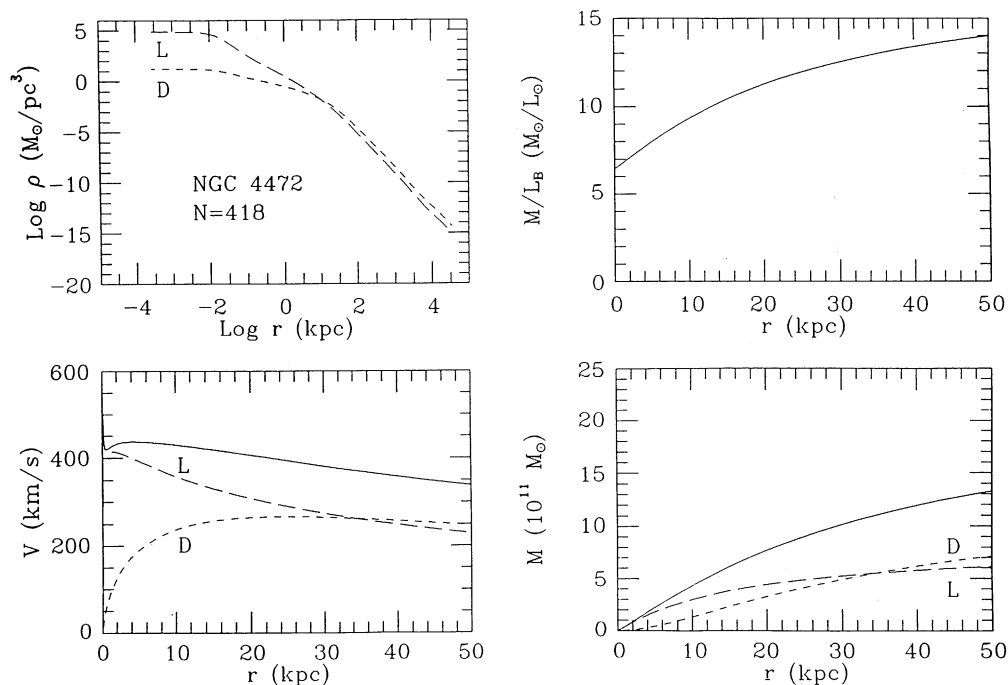


FIG. 7.—Intrinsic properties of the best-fit model ($N = 418$) for NGC 4472 (data PMI). Volume densities (*top left*), circular velocity profile decomposition (*bottom left*), mass-to-light ratio (*top right*), and integrated mass (*bottom right*). Solid lines identify global quantities, long-dashed lines refer to the luminous component, and short-dashed lines to the dark component. The model clearly belongs to the second class of models described in Paper I (§ 4.2; see middle frames of Fig. 2). Note how the two components “conspire” to give a fairly flat circular velocity profile.

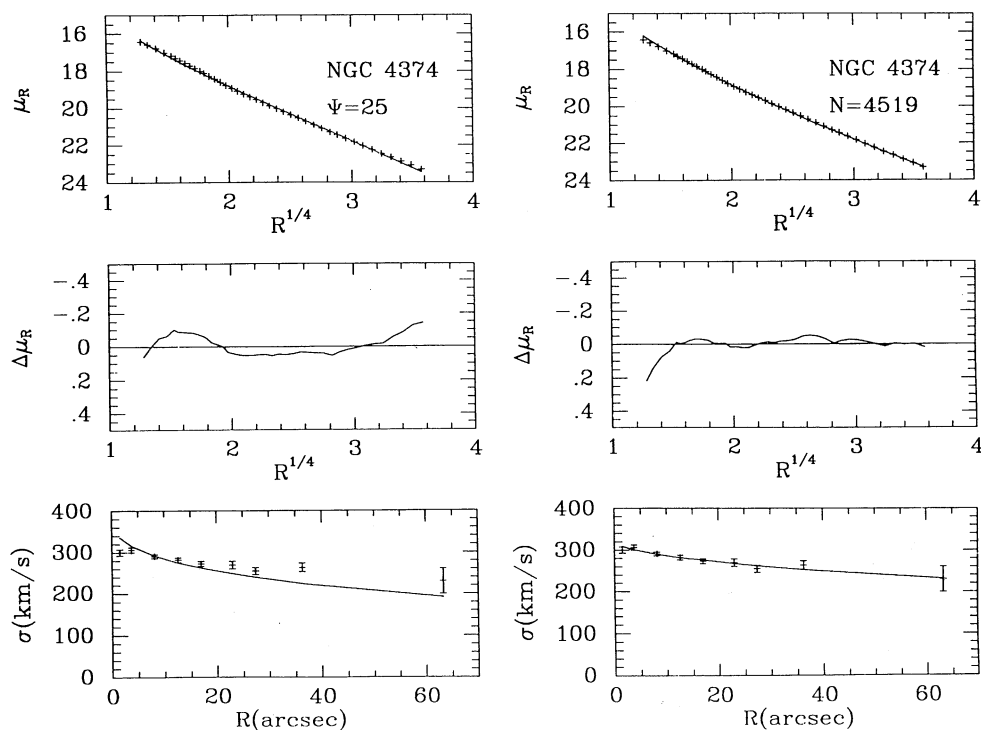


FIG. 8.—Best-fits to NGC 4374 with (*right*: model $N = 4519$) and without (*left*: $\Psi = 25$) dark matter, for the data P . Frames are shown as in Fig. 6.

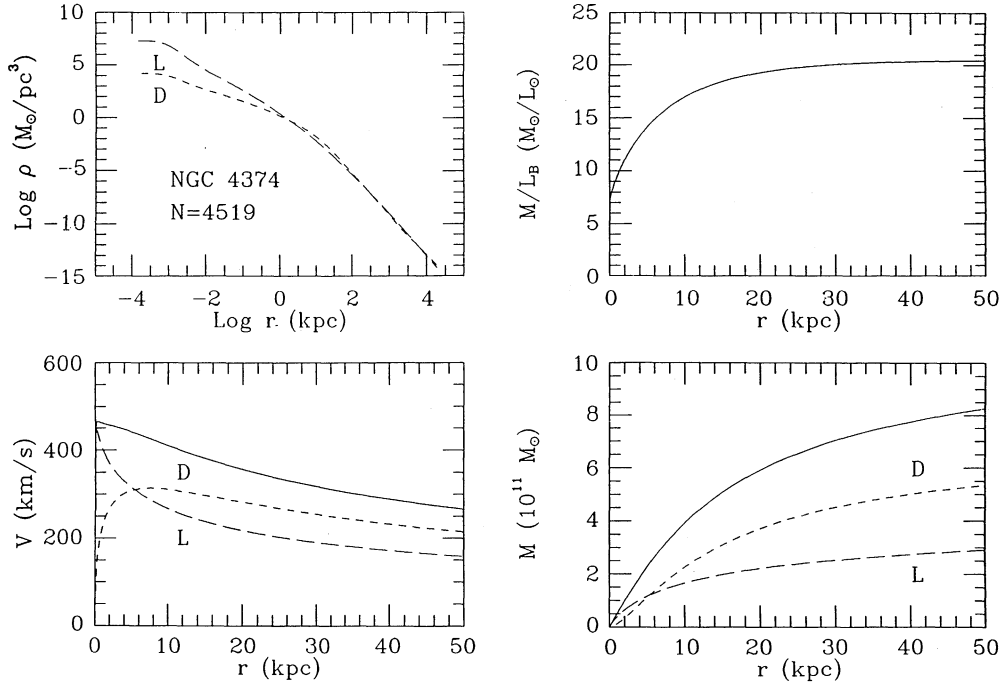


FIG. 9.—Intrinsic properties of the best-fit model ($N = 4519$) for NGC 4374 (data P). (See Fig. 7 for a description of the frames). This model belongs to a class of models intermediate between the first and the second of those described in Paper I (§ 4.2; see left and middle frames of Fig. 2). Here the two components have similar scale lengths so that the resulting circular velocity profile is not flat, and the mass-to-light ratio “saturates” the global value of 19 at $r \approx 20$ kpc.

5.2. The Statistical Significance of the Results

Some words have to be spent on the selection of the “best-fit” models. If we exclude the special cases of NGC 1399 and M87, where models with extended dark halos have been selected by taking into account that the peculiar kinematics of the central regions has increased the final value of χ^2_k , the upper limits for χ^2_p and χ^2_k have been chosen to be of the order of or lower than the values of χ^2_p and χ^2_k derived from the $M_D = 0$ one-component fit. Note that the photometric fit alone fixes “approximately” four of the six parameters that specify a two-component model. Indeed, by selecting models with a χ^2_p value lower than a given limit, we isolate a more or less narrow region in the $(r_L/r_D, M_L/M_D)$ plane (see Fig. 5 of Paper I). The kinematical fit limits this permitted region further. Therefore, while the one-component photometric and kinematical fits have two and one free parameters respectively, the two-component photometric and kinematical fits have “approximately” four and two free parameters, respectively. If we compare $\hat{\chi}^2_p$ and $\hat{\chi}^2_k$ (the χ^2_p and χ^2_k values normalized to the relevant number of the degrees of freedom, see Tables 2, 3, and 4), we see the following. The values of $\hat{\chi}^2_p$ are usually lower than 1, since our constant value of the photometric error (0.1 mag arcsec⁻²) is likely to overestimate the real errors on the data (a more realistic estimate of the photometric errors is described in the following) and also because the number of photometric data exceeds the actual number of independent data points. In the cases of NGC 4374, NGC 4472, and NGC 4636 only models with massive dark halos give $\hat{\chi}^2_k$ values lower than the “3 sigma” limit. Therefore, for these galaxies there is evidence for dark mass. For NGC 7626 models with massive halos have $\hat{\chi}^2_k$ lower by a factor of 2 than models without dark mass. From this statistical point of view, for the other galaxies the evidence for the existence of dark halos is less convincing.

Actually, a more rigorous selection from a statistical point of

view (say, select all the models with a given “confidence level”) is hard to implement, since the statistical meaning of the obtained values for the global $\bar{\chi}^2$ parameter (see eq. [4]) is hard to assess, given the uncertainties on the photometric error $\delta\mu$ and the number of independent data points. In addition, since usually we have $N_p \gg N_k$, the value of $\bar{\chi}^2$ is bound to measure the quality of the photometric fit only. This means that when it is possible to find models that minimize both χ^2_p and χ^2_k quantities, these minimize also $\bar{\chi}^2$, but models that give bad kinematical fits can nevertheless have a low $\bar{\chi}^2$ value. On the other hand, using photometric errors greater than or equal to 0.1, one derives values of $\bar{\chi}^2$ that are well below unity for the majority of the models. Consequently, if in many of the cases analyzed it is possible to identify a “best-fit” model, it is very hard to select rigorously a set of equally plausible models and thus to compute the relevant range of values for the physical parameters.

In an effort to overcome some of these difficulties, we have introduced a fiducial photometric error $\bar{\delta\mu}$ for a given galaxy by imposing that:

$$\bar{\chi}^2_f = \frac{\chi^2_p(\delta\mu^2/\bar{\delta\mu}^2) + \chi^2_k}{N_p + N_k - 6} \geq 1, \quad (11)$$

for all the models considered in the fit. In doing this we have in mind that the actual photometric errors are smaller than usually assumed, consistent with the small point to point scatter in the profiles. In general we find $\bar{\delta\mu} \approx 0.05$.

Using the derived values of $\bar{\delta\mu}$ as photometric errors, we have repeated the selection of the best-fit models and found that in the majority of cases they include the models selected according to the intuitive prescription mentioned before. However, the number of acceptable solutions within a nominal “3 sigma” confidence level is generally large and so is the

allowed range for the derived physical parameters. Only in the already mentioned cases of NGC 4472 and NGC 7626 the “3 sigma” limit excludes reverse models and requires $M_L/M_D < 1$. Since the kinematical profile of NGC 7626 is peculiar (see previous discussion), we claim a “3 sigma” detection of dark mass only for NGC 4472. For this galaxy *reverse* models are excluded at the 5 sigma level (see Fig. 4). The ranges of the parameters derived imposing the 3 sigma level do not differ significantly from those given in Tables 4–7 (see Fig. 5). Here we note that three of the models selected by the 3 sigma level are on the boundary of the parameter space covered by the set of 2939 accepted two-component models of the main survey (see Paper I). However, our conclusions are not expected to change if the fitting procedure is applied on a larger “data base” of two-component models, because models for which the dark component is too diffuse and not sufficiently massive would also be excluded on the basis of the simple argument given in Paper I (see § 5.1, especially eq. [15]). In fact, models selected by the photometric fit and with fairly flat velocity dispersion profile tend to cluster in a narrow strip in the $(r_L/r_D, M_L/M_D)$ plane (see eq. [21] and Fig. 5 of Paper I); in addition, we have checked that for NGC 4472 one-component ($M_D = 0$) models are rejected at the 7.4 sigma level.

Referring to the discussion of minimum halo models given in Paper I (§ 5), we note that all the best-fit models are often close to the relevant minimum halo solution (Paper I, eq. [15]); for NGC 4472 the minimum halo solutions compatible with the “3 sigma” requirement have $M_D \geq M_L$ (see Fig. 5).

6. M/L RATIOS

We now turn to the final discussion of our results and of their implications on the problem of dark matter in elliptical galaxies. The picture that the luminous stellar component of elliptical galaxies (see also discussion in Paper I) is usually embedded in a fairly massive and diffuse dark halo is supported by the results of our fits (see especially NGC 4472 and the cases of NGC 4636 and NGC 7626). However, we should reiterate that if this is indeed the real situation there is no way to make a stronger and more convincing case for the presence of dark matter in elliptical galaxies until accurate but, especially, extended kinematical data become available. Thus, not unexpectedly, one of the strongest cases of the present survey of ellipticals is that of NGC 4374 which appears to have a considerable amount of dark matter but *not* in diffuse form.

The M/L ratios derived from our best-fit models are listed in Table 7. If we exclude NGC 4278 (for this galaxy there can be a problem of distance determination) the derived values of the mass-to-light ratio for the luminous component M_L/L_B are not too large, (3–11 M_\odot/L_\odot with mean value 7), while the values for the sum of the two components referred to the sphere of radius R_e , $(M/L_B)_{R_e}$, are in the range 7–18 (mean value 12). All these numbers refer to the “best-fit” *normal* models. In contrast, the *global* (M/L_B) can be very large, up to 50 M_\odot/L_\odot . Note also that the mass-to-light ratios derived from one-component fits $(M/L_B)'$ (see Table 3) are usually intermediate, i.e., $M_L/L_B < (M/L_B)' < M/L_B$. Finally, we should stress that these values for the M/L ratios are based on the Hubble constant $H = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see Sandage & Tammann 1990, and references therein); they would double if we set $H = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

In Table 7 we also list for comparison the M/L ratios obtained in other dynamical studies; note that the authors of the first and second reference consider the same kinematical

data as those analyzed in this paper. The axisymmetric models of van der Marel, Binney, & Davies (1990) are characterized by (constant) M/L ratios that are slightly smaller than our $(M/L_B)_{R_e}$ ratios. Bicknell et al. (1989) find that an isotropic spherical model can explain the observed velocity dispersion profile of NGC 1399 if the M/L ratio increases with radius; alternatively, taking a constant M/L ratio requires that the stellar orbits become more tangential in the outer regions. The two numbers reported in Table 7 correspond to eastern and western kinematical profiles (for the constant M/L fits). Finally, we note that our results are within the ranges predicted by Katz & Richstone (1985) obtained for spherical isotropic models from an application of linear programming techniques in the presence of dark halos; their values are referred to the sphere of radius R_e .

In one respect our conclusions on the amount of dark matter present are on the *conservative* side. In fact, if galaxies were largely populated with radial orbits, as is sometimes argued by dynamicists, then the current data would already imply very large amounts of dark matter inside R_e .

7. CONCLUSIONS

The results of this paper add evidence to the picture that elliptical galaxies can be embedded in massive diffuse dark halos and suggest that dark matter can be present in significant amounts even inside the half-luminosity radius.

It has often been noted that stellar kinematical data may be explained without having to invoke the presence of dark matter if the underlying orbital structure is properly chosen, on a case by case basis; these arguments leave unresolved the physical justification of the various orbital structures that are found. In contrast, here we have shown that, under *simple physical assumptions* that lead to a well defined framework for stellar orbits, remarkably good fits are obtained by *fully self-consistent models*, and that these fits are able to discriminate cases where dark matter is required (and its amount is estimated) and cases where the data are well explained by one-component models. The impact of many limitations intrinsic to the adopted fitting procedure has been evaluated by means of extensive tests on simulated objects and good overall support to the method used has been found.

A considerable effort has also been made in order to give statistical significance to the results obtained, i.e., to establish a confidence level for the estimates of the amounts of dark matter derived for the objects of our survey. At least in one case, that of NGC 4472, we find that within our physical assumptions dark matter must be present ($M_L < M_D$) with a 3 sigma confidence level. Of course, statements of this kind are hard to make because the nature and the size of errors and the number of independent parameters in the fit of photometric and kinematical data are not fully under control. Even more severe limitations to making a strong statement on the presence of dark matter derive from the fact that the analyzed data are not as accurate and as radially extended as desired. In this latter respect, our best-fit models should be strengthened and improved upon by considering additional kinematical tracers, such as X-rays and the globular cluster system. In fact, the objects where we have the strongest indications for the presence of dark matter are also well-known X-ray sources; for the case of NGC 4472 the matching with the kinematical data of the globular cluster system is remarkable.

General trends in the amount and distribution of dark matter are hard to establish, given the small sample of objects

studied in this paper. A marginal correlation between amount of dark matter and size should be confirmed by application to smaller elliptical galaxies and to larger sets of fitted objects with confidence level comparable to or better than that obtained here for NGC 4374 and NGC 4472. In the near future stellar dynamical studies of this kind, coupled with studies of other kinematical tracers on statistically significant samples of ellipticals should be able to lead to solid statements of cosmological interest.

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