F Exercises 6

F.1 Gravitational redshift for S2

How much gravitational redshift did the photons of the Galactic Center star S2 experience, when the star was at the pericenter of its orbit in 2018? The orbit has $a = 125\,\mathrm{mas}$, $e = 0.88$, the distance is $R_0 = 8.3\,\mathrm{kpc}$ and $M = 4.3 \times 10^6\,M_\odot$. Use equation 298. How many km/s is the redshift?
Further, there is the special relativistic Doppler effect. To leading order, it leads to a frequency shift of:

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{2}\frac{v^2}{c^2}$$

Use the vis-viva equation 321 to calculate the v at pericenter, and thereby the redshift due to the transverse Doppler effect. How does it compare to the gravitational redshift?

F.2 Relativistic precession for S2

Use equation 325 to evaluate the amount of relativistic precession expected for S2. By which angle on-sky does one apocenter differ from the next one?

F.3 Precession due to quadrupole moment

A body with radius R and and quadrupole moment Q_2 leads in the sense of a multipole expansion to a (small) perturbing potential of the form

$$V_{Q_2}(r) = \frac{Q_2}{2}\frac{GMR^2}{r^3}(3\sin^2\theta - 1)$$

Use section 7.2.1 to derive the amount of precession induced by that potential. Result:

$$\Delta\phi_{Q_2} = -3\pi Q_2 \frac{G^2M^2R^2}{l^4}(3\sin^2\theta - 1)$$

For the Sun, $Q_2 \approx 2 \times 10^{-7}$. How compares the quadrupole-moment induced precession on Mercury ($a = 0.387098\,\mathrm{AU}$, $e = 0.205630$) with the relativistic one? Hint: Use equation 320

F.4 Double-slit experiment

Consider Young's double slit experiment. The slit distance be B, the screen is in a distance D.

- Assume the light source is monochromatic and placed on the center line perpendicular ($\phi = 90°$) to the slit. Derive the interference pattern: At what screen positions x do maxima of the pattern appear?
- Under what angle ϕ' would one need to place a second light source with the same wavelength, such that the minima of the first interference pattern occur where the maxima of the second are?
- That angle is a measure of the resolution of the slit. How small is this number for $B = 100\,\mathrm{m}$ and $\lambda = 2.0\,\mu\mathrm{m}$ in milli-arcsec?
- Now removing the second light source again. What happens, if one uses as a (single) light source a laser with two wavelengths? What if there are N? What if it is a continuum source?