G Exercises 7

G.1 ABCD algorithm

The beam combiner of GRAVITY measures the complex visibilities via four intensities with small phase shifts:

$$\begin{aligned} I_A &= I_0 + I_0 A \cos(\phi) \\ I_B &= I_0 + I_0 A \cos(\phi + \pi/2) \\ I_C &= I_0 + I_0 A \cos(\phi + \pi) \\ I_D &= I_0 + I_0 A \cos(\phi + 3\pi/2) \end{aligned}$$

Derive the formulas for phase ϕ and amplitude A.

G.2 Relativistic aberration

A light source is moving with $\vec{u} = (u_x, 0, 0)$ with respect to some rest frame. The four-vector of a photon emitted is $p = (E/c, Ev_x/c^2, Ev_y/c^2, Ev_z/c^2)$ with $v_x^2 + v_y^2 + v_z^2 = c^2$, such that $E = |\vec{p}|c$. What is the angle θ' under which the light is seen in the rest-frame, as a function of emitting angle θ and relativistic $\beta = |\vec{u}|/c$?

$$\cos\theta' = \frac{p'_x}{|\vec{p'}|} = ...$$

Use a Lorentz boost for expressing the primed quantities in terms of original ones, the matrix is

$$\begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & \gamma & \\ 0 & 0 & 0 & \gamma \end{pmatrix}$$

G.3 Ricci tensor

For the Schwarzschild metric we found for the Ricci tensor

$$\begin{aligned} R_{00} &= \frac{B''}{2A} - \frac{A'B'}{4A^2} - \frac{B'^2}{4AB} + \frac{B'}{rA} \\ R_{11} &= -\frac{B''}{2B} + \frac{B'^2}{4B^2} + \frac{A'B'}{4AB} + \frac{A'}{rA} \\ R_{22} &= -\frac{1}{A} + \frac{rA'}{2A^2} + 1 - \frac{rB'}{2AB} \end{aligned}$$

What are these expressions when making the ansatz

$$\begin{aligned} B(r) &= e^{\nu}(r) \\ A(r) &= e^{\lambda}(r) \end{aligned}$$

G.4 Energy-momentum tensor for a perfect fluid

For a perfect fluid:

$$T_{\mu\nu} = \left(\rho + \frac{P}{c^2}\right) u_\mu u_\nu + P g_{\mu\nu}$$

The perfect fluid be at rest: $u^\alpha = (u_T, 0, 0, 0)$, and we have our ansatz for the Schwarzschild metric $g_{\mu\nu} = \mathrm{diag}(-B(r), A(r), r^2, r^2 \sin^2\theta)$, where A, B as in the previous exercise.

- From $u.u = -c^2$ derive the relation between u_T and ν.
- Evaluate $u_\alpha u_\beta$
- Write down the energy momentum tensor in terms of ν, λ, ρ, P