K Exercises 11

K.1 Salpeter time scale

Calculate the Salpeter time scale: After what time has a maximally accreting black hole radiated away an amount of energy equal to its rest mass? The result can be given in years.

K.2 Energy-driven feedback

An accreting black hole blows away material. From an energy argument a relation between black hole mass and velocity dispersion can be made. It is sufficient to work with proportionalities. The gas mass be M_g, and it is much larger than the black hole mass

- What is the gravitational energy of the gas?
- How does the velocity dispersion σ relate to the gas mass?
- Equate the energy radiated at Eddington luminosity to the gravitational energy, and eliminate M_g by using σ.
- The time scale can be estimated by r/σ.
- How does the black hole mass scale with velocity dispersion?

K.3 Quadrupole moment of a binary star

The quadrupole moment of a system is the traceless part of the moment of inertia I_{ij}:

$$Q_{ij} = \int \rho(x^i x^j - \frac{1}{3}\delta^{ij} r^2) dV$$

Assume a binary star with masses m_1 and m_2, orbiting with some angular velocity ω, in the xy-plane

- What are the distances r_1, r_2 from the mass center, if the two stars are separated by a?
- What are the positions x_1, y_1, x_2, y_2 as a function of time?
- Evaluate the nine components of Q_{ij}

K.4 Black hole evaporation

The Hawking temperature of a black hole is

$$T_H = \frac{\hbar c^3}{8\pi k_B G M}$$

- How hot is Sgr A* ?
- The luminosity is given by the Stefan-Boltzmann law $L_H = \sigma_{\rm SB} 4\pi r_S^2 T_H^4$. How long does it take, until it is completely evaporated? Note: $L_H = -\dot{M}c^2$ will yield a differential equation, one can easily solve.