C Exercises 3

C.1 Christoffel symbols for the Schwarzschild metric

We found for the spherically symmetric Schwarzschild metric. $A(r) = (1 - r_S/r)^{-1}$ and $B(r) = r_S/r - 1$. This allows writing out the Christoffel symbols explicitly with r_S, eliminating A, A', B, B'. Do that!

Solution: Section 8.5

C.2 Radial distance in Schwarzschild

Consider a radial line at constant t, θ, ϕ, hence $dt = d\theta = d\phi = 0$. Write down the line element ds and calculate the path length

$$s = \int_{r_1}^{r_2} ds$$

What would the same be in flat space-time?

Solution: Section 10.3, or explicitly:

$$ds = \frac{dr}{\sqrt{1 - \frac{r_S}{r}}} \tag{545}$$

$$\begin{aligned} s &= \int_{r_1}^{r_2} ds = \int_{r_1}^{r_2} \frac{1}{\sqrt{1 - \frac{r_S}{r}}} dr \\ &= \left[\sqrt{r(r - r_S)} + r_S \operatorname{atanh} \sqrt{\frac{r - r_S}{r}} \right]_{r_1}^{r_2} \\ &\neq r_2 - r_1 \end{aligned} \tag{546}$$

C.3 R_{33} for Schwarzschild

Show that the $R_{33} = 0$ equation, for our diagonal ansatz of the Schwarzschild metric with functions A and B, yields an identical equation to what we had from R_{22}.

- Spoiler: Lengthy, but not difficult
- Place a sheet with the Christoffel symbols next to you, it will be useful.
- And have the calculation for R_{22} next to you, as it works similarly in many places.

Solution: Equation 193

C.4 R_{0i} for Schwarzschild

Show that the $R_{0i} = R_{i0} = 0$ equation, for our diagonal ansatz of the Schwarzschild metric with functions A and B, is trivially true. Place a sheet with the Christoffel symbols next to you.

Solution: Equation 194

C.5 R_{12}, R_{13}, R_{23} for Schwarzschild

Show that the $R_{12} = R_{21} = 0$, $R_{13} = R_{31} = 0$, $R_{23} = R_{32} = 0$ equations, for our diagonal ansatz of the Schwarzschild metric with functions A and B, are trivially true. Place a sheet with the Christoffel symbols next to you.

Solution: Equations 195, 196, 197