

Resistivities in the Radioemission Region

A. Jessner (MPIfR)

J. Eilek (NMTech)

H. Lesch (LMU)

1. How can observations and plasmaphysics be reconciled?
2. Why are resistivities important?
3. What causes resistivity?
4. What are the typical magnitudes in the radio emission region?
5. Can one find a configuration that satisfies the constraints?

Observations and Plasmaphysics

**Electric charges in the inner parts of the PSR magnetosphere move only along the field lines,-
their plasma frequency determines the lower frequency limits of effective plasma waves and instabilities.**

$$\nu_{pl} = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{q_e^2}{\epsilon_0 \cdot \gamma \cdot m_e} \cdot \xi \cdot 2 \cdot \Omega \cdot \frac{B_0 \cdot \epsilon_0 \cdot r_{ns}^3}{q_e \cdot R^3}}$$

angular dependence neglected and using $\xi = \frac{n}{n_{GJ}}$
"Sturrock factor"

Depending on the emission process, the minimum observable frequency is

$$\nu_{min} = \frac{\gamma^{\alpha - \frac{1}{2}}}{2 \cdot \pi} \cdot \sqrt{\frac{q_e^2}{\epsilon_0 \cdot m_e} \cdot \xi \cdot 2 \cdot \Omega \cdot \frac{B_0 \cdot \epsilon_0 \cdot r_{ns}^3}{q_e \cdot R^3}}$$

$\alpha=1$ for curvature radiation and $\alpha=2$ for inverse compton scattering

Solving now for density and Lorentz factor we get a universal constraint for the emission height R , the Sturrock factor ξ and the Lorentz factor γ :

$$\xi \cdot \gamma^{2 \cdot \alpha - 1} = \frac{2 \cdot \pi^2 \cdot m_e \cdot v_{\text{min}}^2}{q_e \cdot r_{\text{ns}}^3 \cdot \Omega \cdot B_0} \cdot R^3 = 1.8 \cdot 10^{-3} \cdot \left(\frac{v_{\text{min}}}{100 \cdot \text{MHz}} \right)^2 \cdot \left(\frac{P}{\text{s}} \right) \cdot \left(\frac{10^{12} \cdot \text{G}}{B_0} \right) \cdot x^3$$

For a "standard pulsar": $v=400$ MHz, $B=10^{12}$ G, $P=0.5$ s, $x=50$

For 0531+21 (Crab): $v=160$ MHz, $B=3.8 \cdot 10^{12}$ G, $P=0.033$ s, $x_{\text{LC}}=157$

$$\xi \gamma = 1786$$

$$\xi \gamma = 163$$

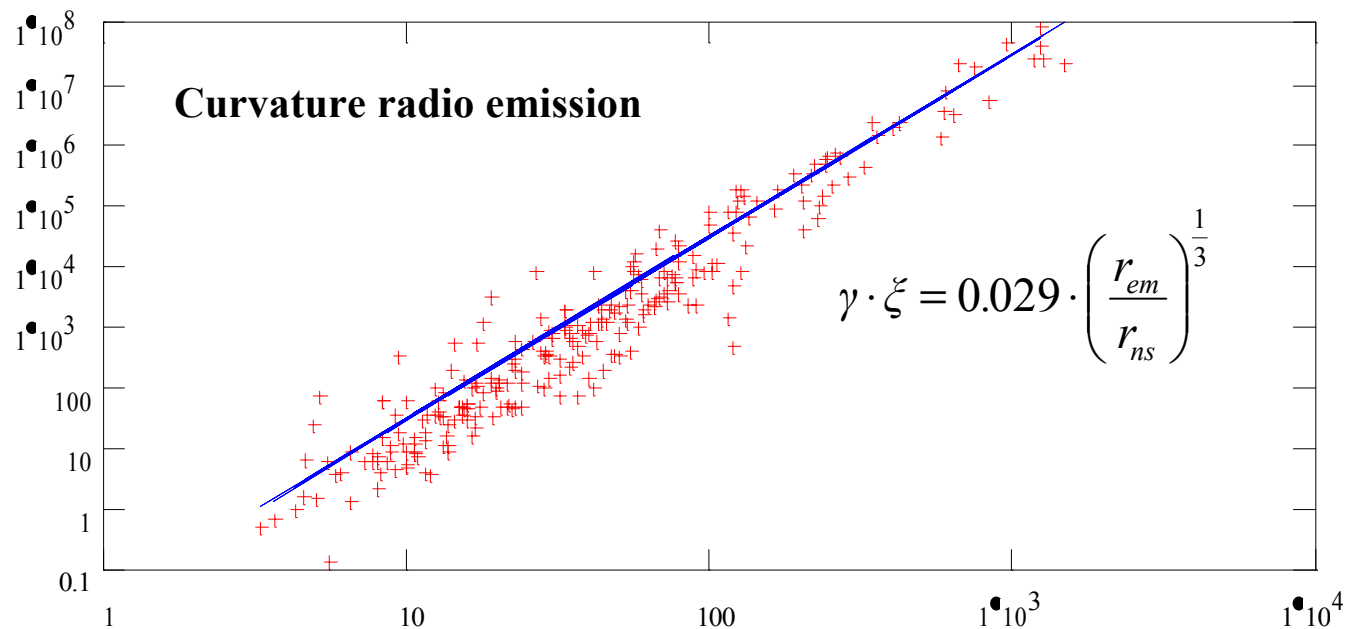
Kunzl, Lesch, Jessner, v. Hoensbroech
ApJ, 505, L139 (1998)

Standard (R&S-like) Models of the Pulsar Magnetosphere are not self-consistent!

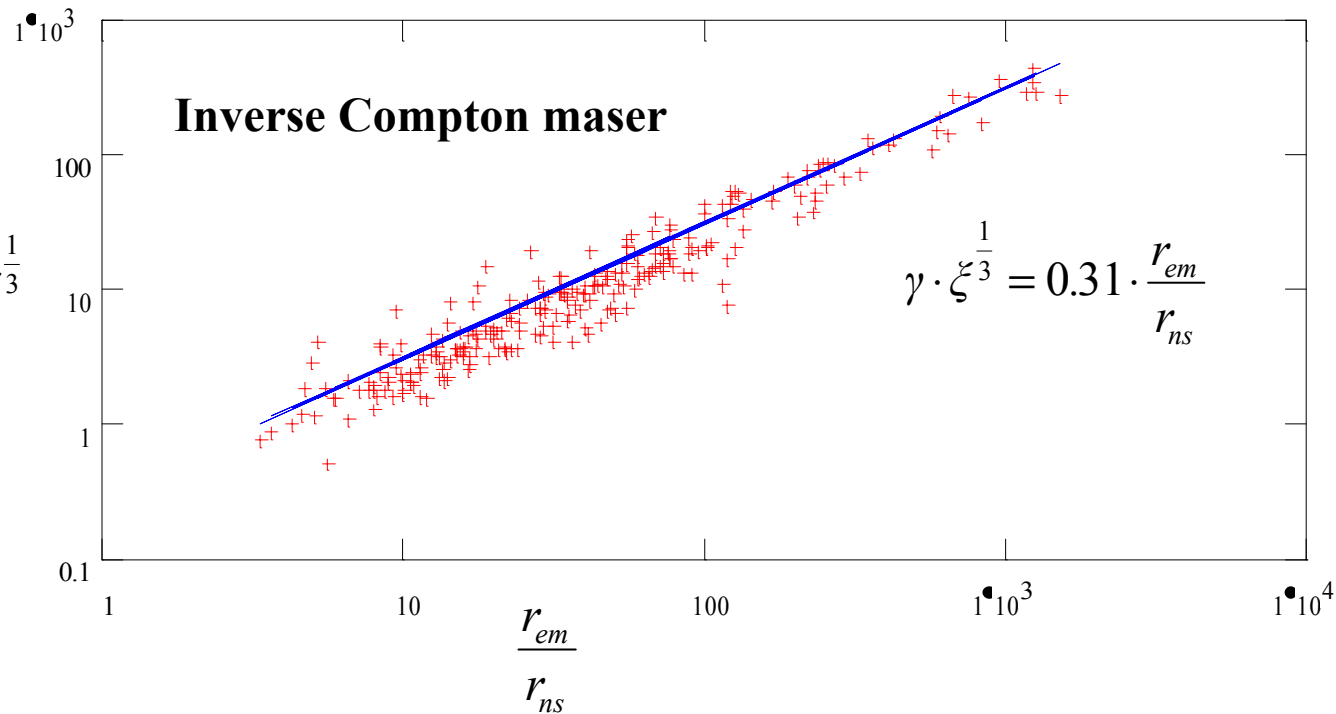
Pair creation must not influence the density where radio waves are emitted
(and may not even happen! Jessner et. al. 2001, Eilek et. al. 1999)

Upper Limits
on density and
Lorentzfactor
for 256 Pulsars
at $\nu=400$ MHz

$\xi\gamma$



$\gamma \cdot \xi^{\frac{1}{3}}$



The Observed Radio Luminosities are Dissipated Particle Energies!

Kramer, Xilouris, Jessner et. al.
A&A 306, 867 (1996)

Taylor, Manchester, Lyne,
ApJS, 88,529 (1993)

PSR 0355+54

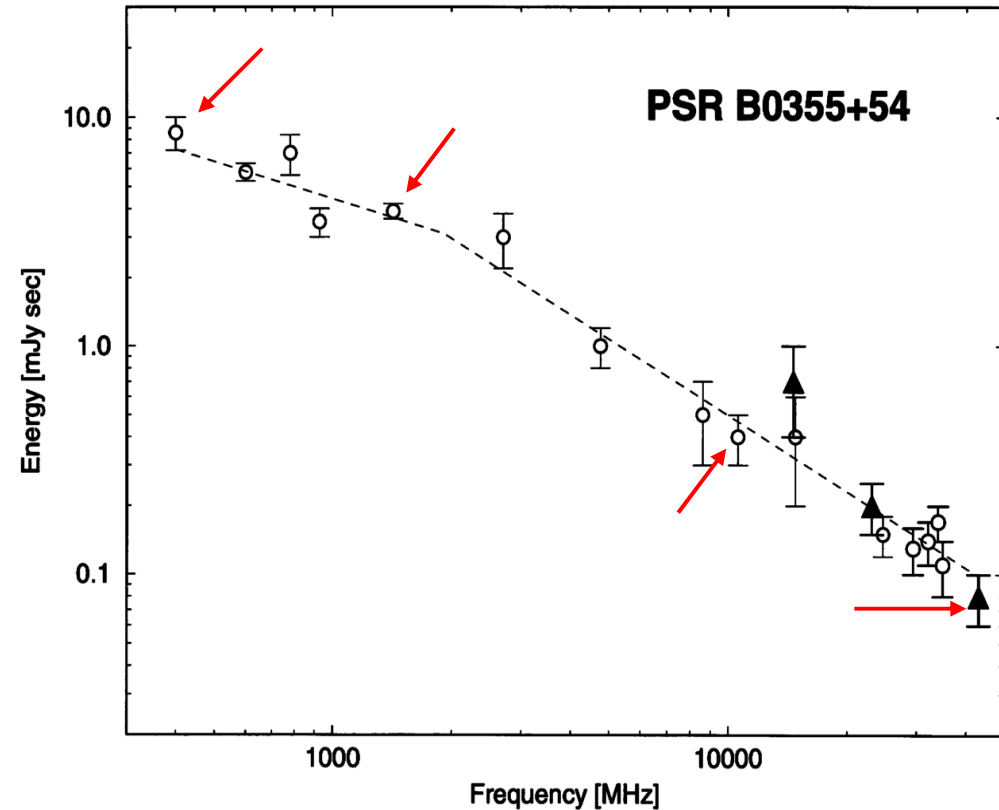
$\xi\gamma$

$$L_{408} := 352.3 \cdot 10^{26} \frac{\text{erg}}{\text{s}} \longrightarrow 1250$$

$$L_{1400} := 143 \cdot 10^{26} \frac{\text{erg}}{\text{s}} \longrightarrow 507$$

$$L_{10\text{GHz}} := 13.3 \cdot 10^{26} \frac{\text{erg}}{\text{s}} \longrightarrow 47$$

$$L_{33\text{GHz}} := 5 \cdot 10^{26} \frac{\text{erg}}{\text{s}} \longrightarrow 18$$

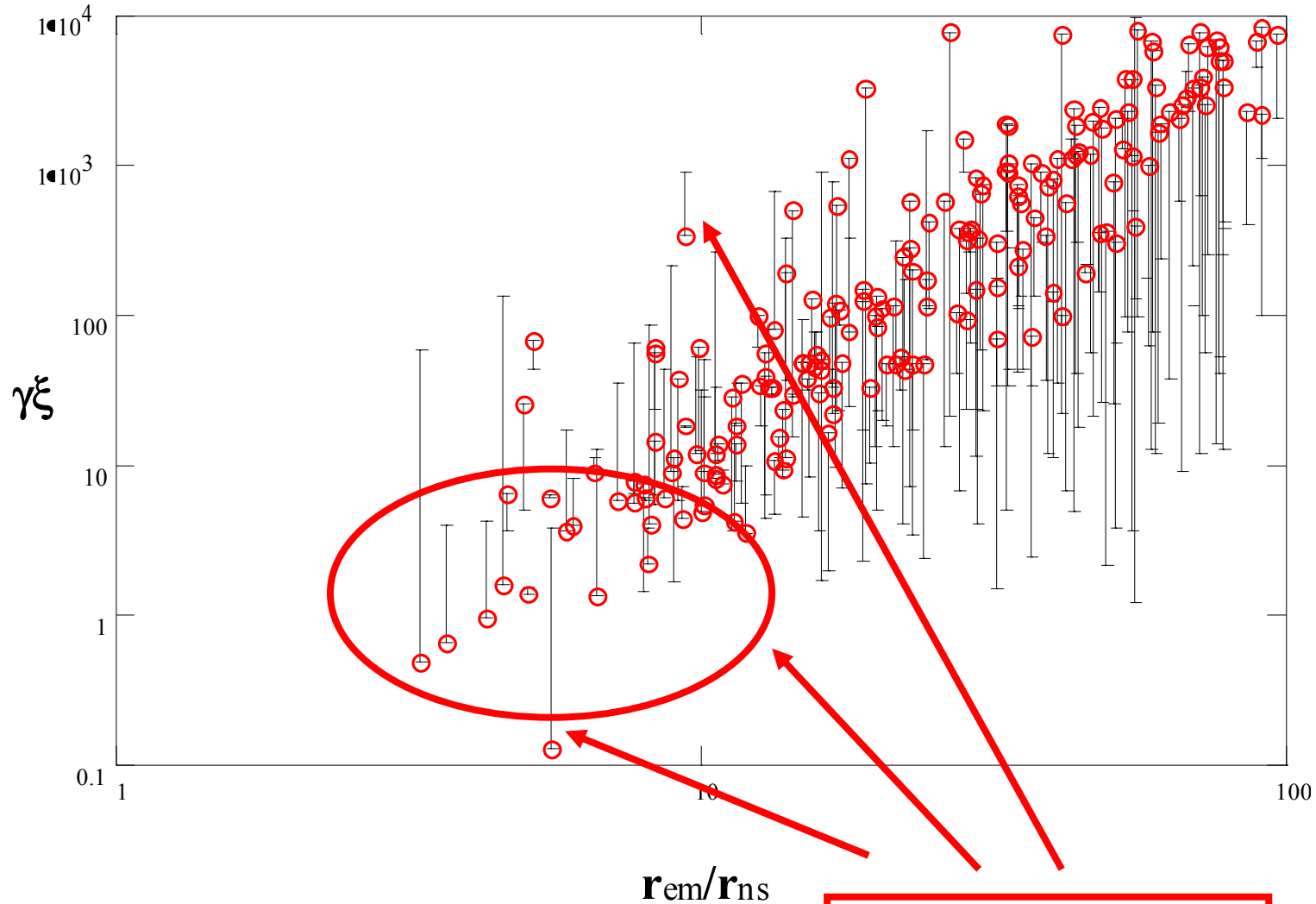


These are minimum estimates
as the width of the beam could be
greater!

$$\xi\gamma = \frac{L}{n_{GJ} \cdot m_e c^2 \cdot A_{cap} \cdot c}$$

Permitted and required Lorentz factors for 256 PSRs at $\nu=400\text{MHz}$

Red circles: Maximum permitted for curvature radiation mechanisms



With inv. Compton *all* PSRs would have

$$\gamma \cdot \xi|_{rad} > \gamma \cdot \xi|_{max}$$

Radio observations present us with a dilemma:

low particle energies

$$\gamma < 10^2$$

and densities around n_{GJ}

$$\xi \sim 1$$

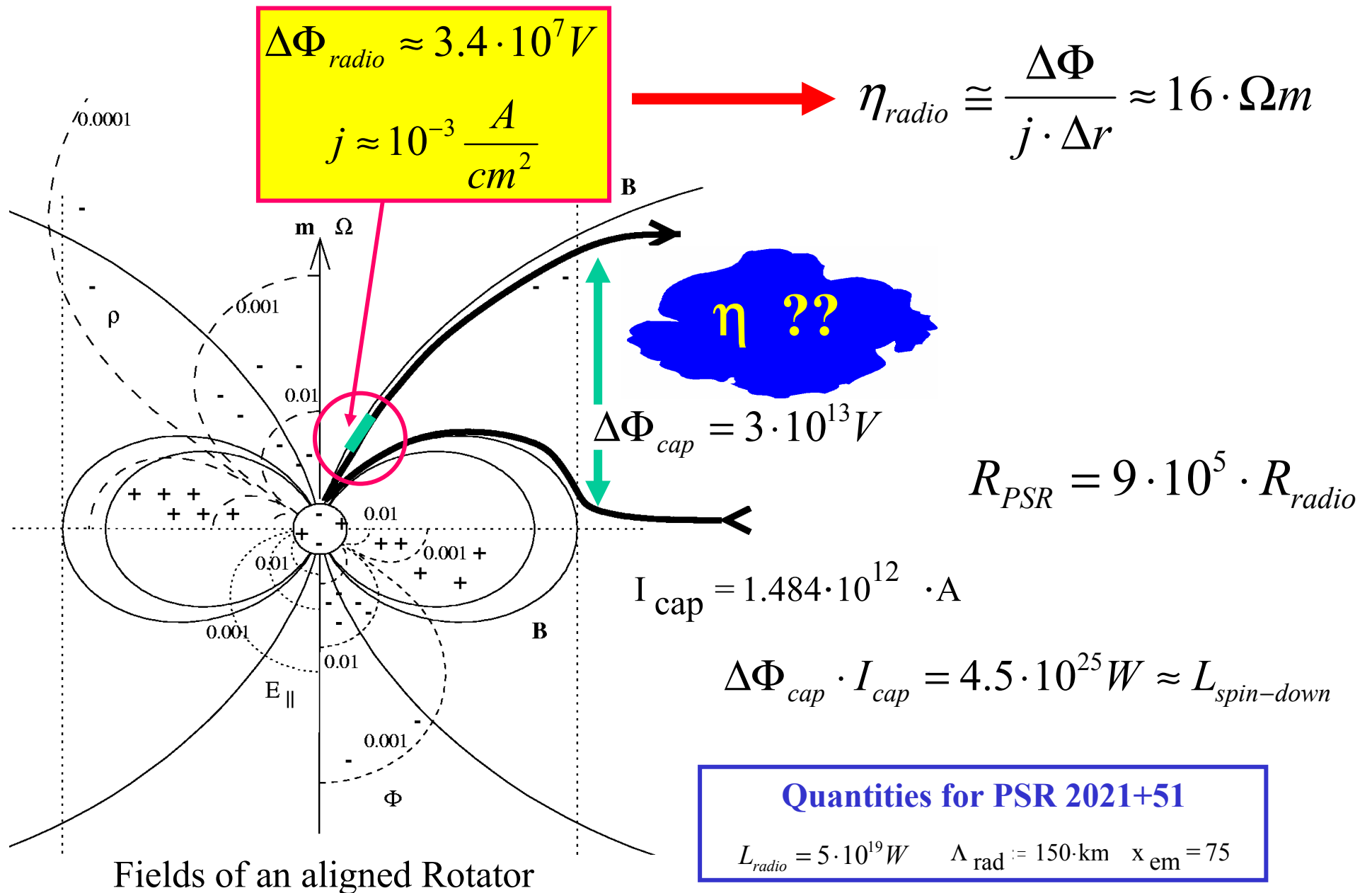
prevail in the radio emission zone,-

because low ν radiation has to escape!

But for the radio luminosities we need $\xi\gamma > 1000$

We have to find a model that fits the observations
without any violation of plasma physics !

Resistivities are Local Quantities that Determine the Global Current

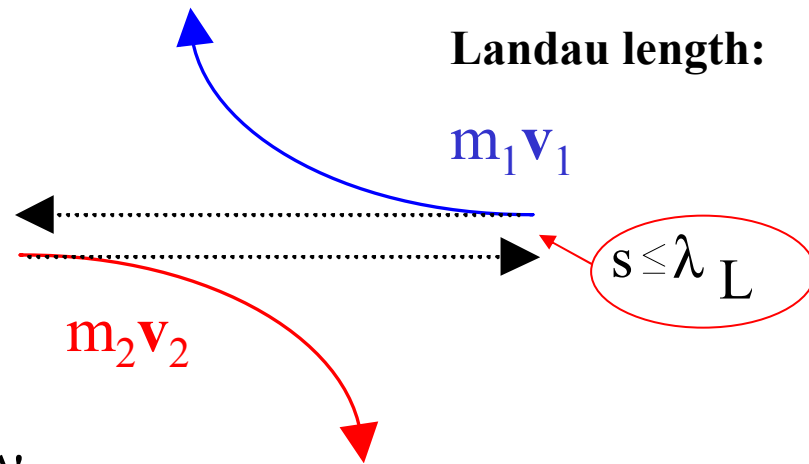


Dissipation corresponds to Resistivity !

Individual particle interactions:

Interparticle distance: $\lambda_n := n_e^{-\frac{1}{3}} = \mathbf{0.015 \text{ cm}}$

3-D Collisions:
(thermalisation)



Landau length: $\lambda_L = 5.57 \cdot 10^{-9} \cdot \text{cm}$

$v_{\text{coll}} = 0.011 \cdot \text{s}^{-1}$

$$\eta_{\text{coll}} := \frac{m_e \cdot v_{\text{coll}}}{q_e^2 \cdot n_e}$$

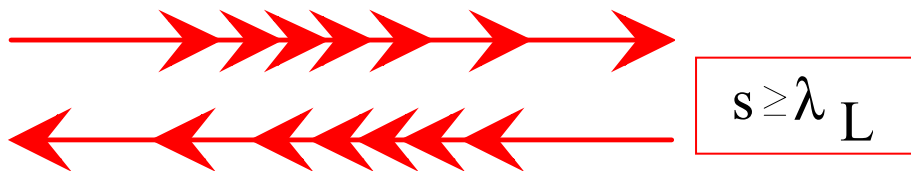
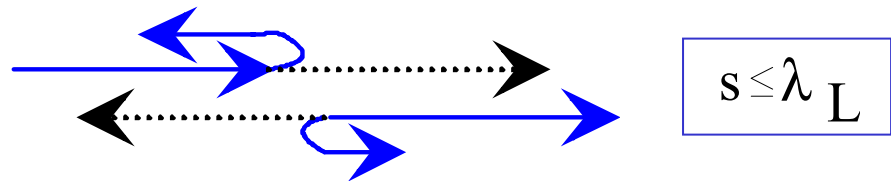
$\eta_{\text{coll}} = 1.3 \cdot 10^{-6} \cdot \text{Ohm} \cdot \text{m}$

$$L_{\text{coll}} = 10^{-7} \cdot L_{\text{radio}}$$

$\eta_{\text{Hg}} = 1 \cdot 10^{-6} \cdot \text{Ohm} \cdot \text{m}$

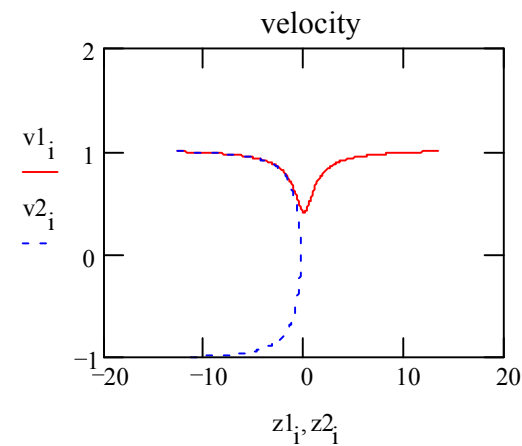
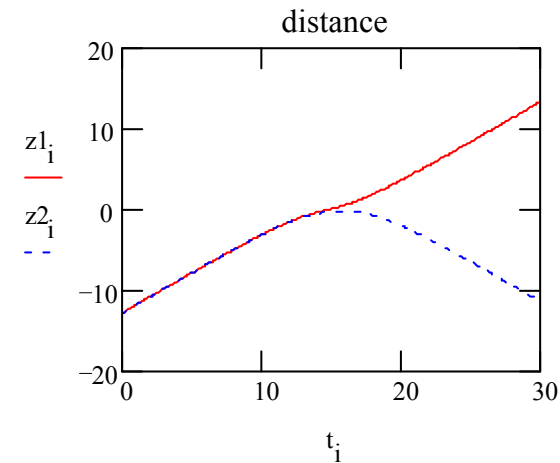
But the magnetic field constrains particle movement
and enforces an additional symmetry!

1-D collisions:



For particles of the similar mass:

Particle momenta are either **preserved**
or just **swapped**!

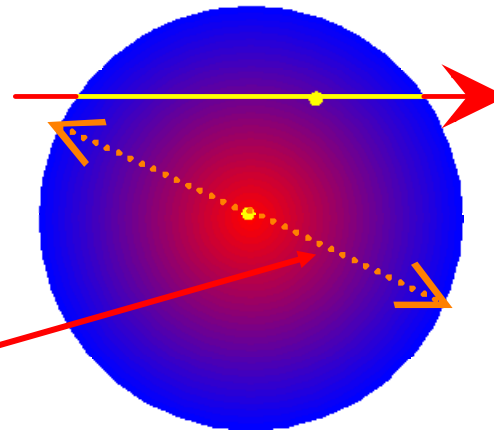


$\eta_{\text{coll}}=0$ for high B fields !

Collective Effects are important in a Plasma !

The shielding (Debye)-length

$$\lambda_D := \sqrt{\frac{\epsilon_0 \cdot k \cdot T_e}{q_e^2 \cdot n_e}} = 7 \text{ cm}$$



determines the range of el.mag. Interactions.

$$n_e \cdot \frac{4}{3} \cdot \pi \cdot \left(\frac{\lambda_D}{2}\right)^3 = 5.209 \cdot 10^7$$

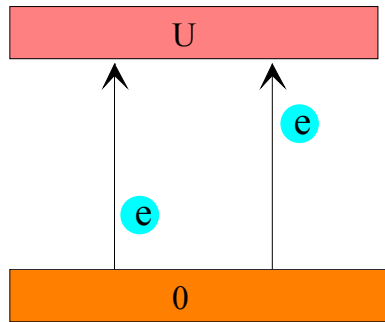
particles interact within a Debye sphere via electrostatic (Langmuir) waves

Their effect is certainly not small:

$$\eta_D = 2.8 \cdot \text{Ohm} \cdot \text{m}$$

$$L_{el.} \approx 0.3 L_{radio}$$

Even „empty“ space has a resistivity!

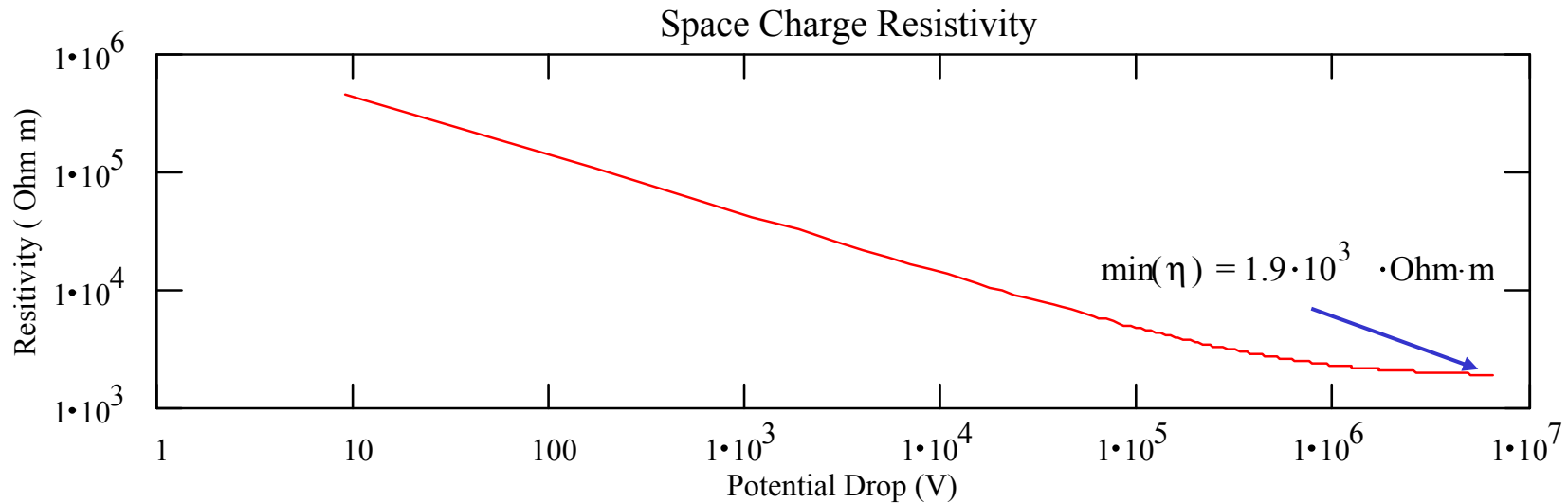


$U \ll 511 \text{ kV}$

$$j_{CL} = 7 \cdot 10^{-10} \cdot \left(\frac{x}{m}\right)^{-2} \cdot \left(\frac{U}{V}\right)^{\frac{3}{2}} \cdot \frac{A}{cm^2} \quad \eta_{CL} \cong 1.4 \cdot 10^5 \cdot \left(\frac{x}{m}\right) \cdot \left(\frac{U}{V}\right)^{-\frac{1}{2}} \cdot \text{Ohm m}$$

$U \gg 511 \text{ kV}$

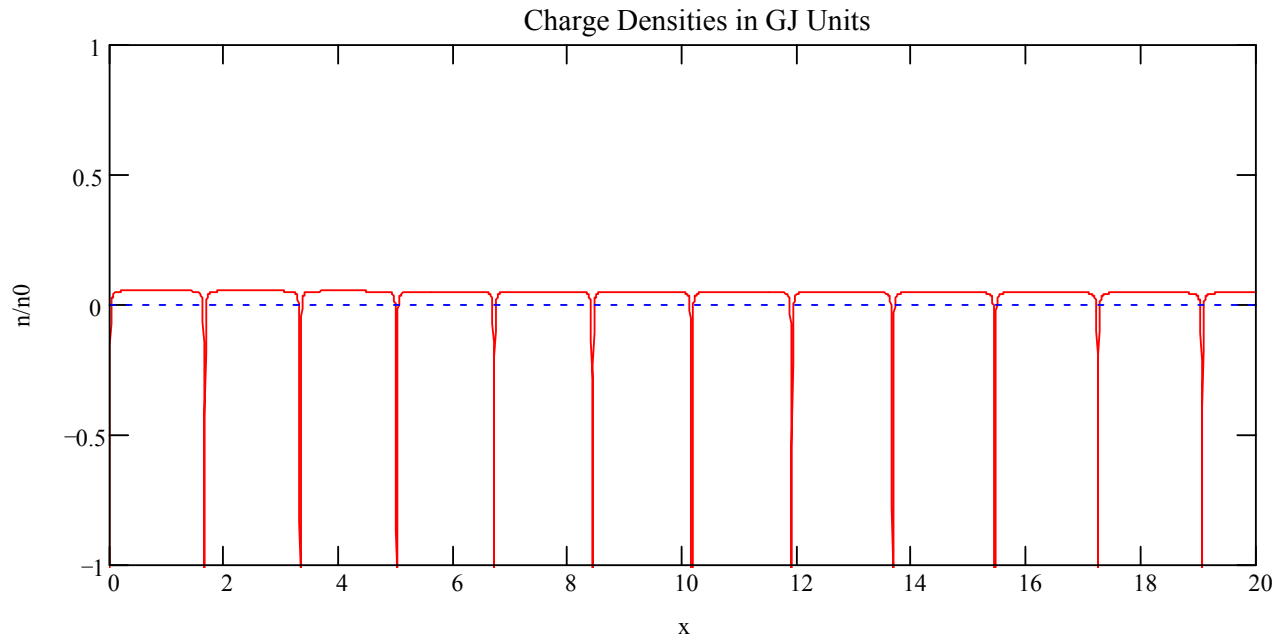
$$j_{CL} = 5.3 \cdot 10^3 \cdot \frac{A}{cm^2} \quad \eta_{CL} \cong 188 \cdot \left(\frac{x}{m}\right) \cdot \text{Ohm m}$$



over $\frac{c}{\omega_p} = 9.8 \cdot \text{m}$

$\omega_p = 3.1 \cdot 10^7 \cdot \text{s}^{-1}$

What happens if the injected current exceeds the space charge limits?



Such currents oscillate strongly, involving mildly relativistic potentials and strong density modulations.

These oscillations are common in diodes with overcritical currents

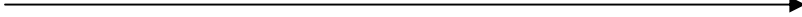
Discovery: Pierce (1940);

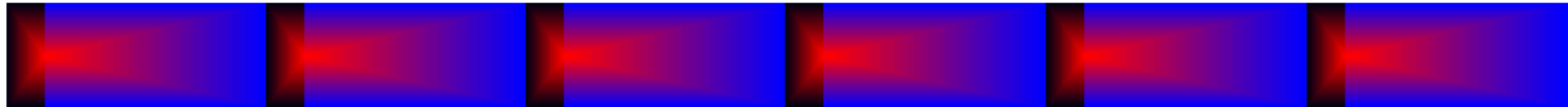
Analytical treatment: Mestel, Wang & Westfold; Shibata;

Numerical experiments: Eilek; Schopper & Lesch; Jessner

Anomalous Resistivity from Strong Density Modulations

A heuristic estimate of dissipation in the radio emission region:

Length of dissipation region $\Lambda_{\text{rad}} := 150 \cdot \text{km}$ 




Number of mini-gaps $N_{\text{CL}} := 70$

Length of acceleration region (mini-gap) $\lambda_{\text{CL}} := \frac{c}{\omega_p}$ $\lambda_{\text{CL}} = 9.8 \cdot \text{m}$ $\frac{\Lambda_{\text{rad}}}{N_{\text{CL}}} = 2.1 \cdot 10^3 \cdot \text{m}$

$\delta\gamma$ in gap: $\frac{\Phi_{\text{rad}} - n_e \cdot q_e \cdot c \cdot \eta_D \cdot (\Lambda_{\text{rad}} - N_{\text{CL}} \cdot \lambda_{\text{CL}})}{N_{\text{CL}}} \cdot \frac{q_e}{m_e \cdot c^2} = 0.78$

Langmuir losses in between two gaps: $\frac{n_e \cdot q_e \cdot c \cdot \eta_D \cdot (\Lambda_{\text{rad}} - N_{\text{CL}} \cdot \lambda_{\text{CL}})}{N_{\text{CL}}} \cdot \frac{q_e}{m_e \cdot c^2} = 0.16$

Total dissipated Lorentz factor: $\Delta\gamma = 65.9$  $\frac{L_{\text{rad}}}{A_{\text{cap}} \cdot n_{\text{GJ}} \cdot m_e \cdot c^3}$

Conclusions

- **Estimates of local resistivities can provide important clues on how the magnetospheric current system operates.**
- **Given the correct initial parameters, it is possible to obtain a self-consistent description of the radio emission region that will not violate the observed constraints!**